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Mathematical Modelling for PAWM Control Algorithm of the Mono-Phase Converter

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Abstract

The article presents a mathematical support of the control algorithm of the pulses modulation in amplitude and in width (PAWM) for the synthesis of the mono-phase converter output voltage, having as objectives the approaching of the effective value of the fundamental to the effective value of the proposed sinusoidal voltage at the terminals of the charge and diminishing of the weight of low frequency harmonics in the harmonic content of the voltage.. The commutation moments and the amplitude of each pulse are computed, imposing that the fundamental is equal to the proposed sinusoidal voltage, the high harmonics up to order 2m-1 are null and the distortion coefficients are minimal. For the numerical simulation of the model, the Matlab toolbox was used. The results of the simulation are numerically and graphically presented; they confirm the validity of the mathematical support of the control algorithm PAWM.

Key words: *PAWM (pulse amplitude width modulation) control algorithm, mono--phase converter, synthetic voltage.*

Introduction

In the adjustable electrical drives with asynchronous motors, the frequency static converter is the power supply which performs the variation of frequency and, correspondingly, the variation of the amplitude of the output voltage. The performances of the asynchronous motor are dependent on the capability of frequency static converter to ensure a power supply close to a sinusoidal form. Every deviation from the sinusoidal supply regime leads to: an increase of the currents in the windings, an increase of the power losses and implicitly a diminishing of the efficiency, an oscillating regime of the electromagnetic couple and implicitly a jerky movement of the rotor [1].

Depending on the strategy used to obtain variable frequency and voltage, two groups of frequency static converters could be identified [1]:

° one with variable direct voltage intermediate circuit, when the control of the output voltage and the frequency of the converter are divided between the commanded rectifier and the autonomous inverter; the converter functions after the principle of pulse amplitude and width modulation (PAWM);

° one with constant direct voltage intermediate circuit, when the inverter functions after the principle of pulse width modulation (PWM).

The converter functions in modulated regime if the output voltage is divided in several pulses, with the amplitude and width modulated after a sinusoidal function. The frequency static converter contains an inverter in modulated regime and a rectifier in commended regime, for regulate the frequency and the effective value of the voltage, correlated as needed by the adjustment of speed.

In this paper, one elaborates the mathematical model for optimal synthesising of the voltage for a mono-phase converter, based on the combination of the two pulse modulation techniques, in amplitude and in width. By numerical simulation one analyses the voltage synthesis; the commutation moments of the thyristors, the amplitude and the width of the pulses, the waveforms and the spectral analysis of the converter's voltage are determined.

Mathematical Model of the Synthetic Voltage

Below nominal frequency (f < 50 Hz) the static converter functions in modulated regime; the output voltage has the shape of rectangular pulses [1, 4, 5] with amplitude U_k, k =1, 2, …, m and different time lengths, where m is the number of steps (m pulses in the interval 0 – T / 4, where $T = 1/f$ is the period of the fundamental).

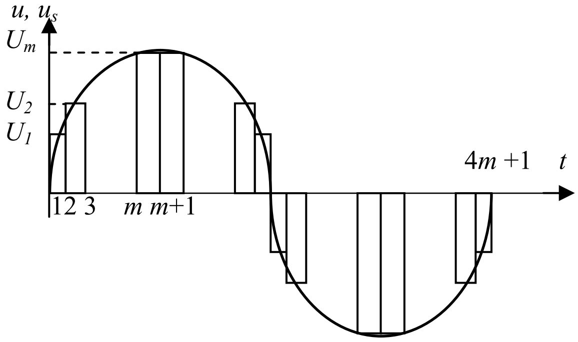

Fig. 1. The waveforms of the synthetic voltage and proposed voltage.

The proposed sinusoidal voltage at the output converter [4]:

$$u_s = A\sin\omega t \ , \qquad (1)$$

is approximated by the synthetic voltage, from figure 1, defined for a period like this:

° in the first quarter of period :

$$u(t) = U_k \ , \qquad (2)$$

for $t_k < t < t_{k+1}$, k = 1, 2, …, m, where k represents the number of pulse in the interval 0 – T /4; t_1 = 0, $t_{m+1} = T/4$ being the limits of the interval;

° during the second quarter of period the pulses are symmetrical with respect to the moment T /4:

$$u(t) = U_j = U_i \ , \ t_{j+1} = T/2 - t_i \ , \qquad (3)$$

for $t_j \le t \le t_{j+1}$, where $j = m + k$, $i = m - k + 1$, k = 1, 2, …m; $t_{m+1} = T/4$, $t_{2m+1} = T/2$ being the limits of the interval;

° during the second semi period the pulses are negative and symmetrical with respect to the moment T/2 :

$$u(t) = U_j = -U_k \ , \quad t_{j+1} = T/2 + t_{k+1} \ , \qquad (4)$$

for $t_j \le t \le t_{j+1}$, where j =2m + k, k =1, 2, …2m; $t_{2m+1} = T/2$, $t_{4m+1} = T$ the limits of the interval.

The voltage graphs $u_s(t)$ and $u(t)$ in the interval 0 – T are presented in figure 1; the notations on the time axis being the indexes of the commutation moments.

The Fourier series expansion of the synthetic voltage contains only odd harmonics in sinus [4, 5, 6]:

$$u(t) = \sum_{n=1,3,5}^{\infty} b_n \sin n\omega t = \sum_{n=1,3,5}^{\infty} \sqrt{2} U_{efn} \sin n\omega t \ , \qquad (5)$$

where U_{efn} is the effective value for the n^{th} harmonic and the series coefficients are:

$$b_n = \frac{2}{T}\int_0^T u(t)\sin n\omega t \, dt = \frac{4}{n\pi}\sum_{k=1}^{m} U_k \left(\cos n\omega t_k - \cos n\omega t_{k+1}\right). \qquad (6)$$

The commutation moments t_k, k =2, 3, …m are determined from the minimization of the mean square error between the functions $u_s(t)$ and $u(t)$, [3]:

$$\min E = \min \left[\frac{1}{T} \int_0^T \left(u_s(t) - u(t) \right)^2 \mathrm{d}t \right], \tag{7}$$

this represents the condition that the synthetic voltage has minimal distortion coefficients.

For a synthetic voltage with m step, the mean square error:

$$E = \frac{4}{T} \sum_{k=1}^{m} \int_{t_k}^{t_{k+1}} (A \sin \omega t - U_k)^2 \, \mathrm{d}t \ , \tag{8}$$

is minimal when the partial derivatives nullify:

$$\frac{\partial E}{\partial t_k} = 0 \, , \ k = 2, 3, \ldots m. \tag{9}$$

One obtains the system of equations:

$$\begin{cases} (A \sin \omega t_2 - U_1)^2 - (A \sin \omega t_2 - U_2)^2 = 0 \\ . \\ (A \sin \omega t_m - U_{m-1})^2 - (A \sin \omega t_m - U_m)^2 = 0 \end{cases} \tag{10}$$

One determines the commutation moments:

$$t_k = \frac{1}{\omega} \arcsin \frac{u_{k-1} + u_k}{2}, \quad k = 2, 3, \ldots m, \tag{11}$$

where $u_k = U_k / A$ is the reported voltage.

From the conditions that the fundamental equals the proposed voltage and that the 3rd, 5th, $(2m - 1)$th harmonics nullify [4, 5], the following system of equations is obtained:

$$\begin{cases} \sum_{k=1}^{m} u_k \left(\cos \omega t_k - \cos \omega t_{k+1} \right) = \frac{\pi}{4} \\ \sum_{k=1}^{m} u_k \left(\cos 3\omega t_k - \cos 3\omega t_{k+1} \right) = 0 \\ . \\ \sum_{k=1}^{m} u_k \left[\cos (2m-1) \omega t_k - \cos (2m-1) \omega t_{k+1} \right] = 0. \end{cases} \tag{12}$$

From relations (11) and (12) one obtains the non-linear system of transcendental equations:

$$\begin{cases} u_1 + \sum_{k=2}^{m} (u_k - u_{k-1}) \cos \left(\arcsin \frac{u_{k-1} + u_k}{2} \right) = \frac{\pi}{4} \\ u_1 + \sum_{k=2}^{m} (u_k - u_{k-1}) \cos \left(3 \arcsin \frac{u_{k-1} + u_k}{2} \right) = 0 \\ . \\ u_1 + \sum_{k=2}^{m} (u_k - u_{k-1}) \cos \left((2m-1) \arcsin \frac{u_{k-1} + u_k}{2} \right) = 0 \end{cases} \tag{13}$$

with the unknowns: $u_1, \ u_2, \ldots u_m$.

The system of equations (13) is solved numerically, the voltage steps are obtained and from relations (11) one determines the commutations moments. The synthetic voltage is completely defined.

Spectral Analysis of the Synthetic Voltage

The effective values of voltage harmonics and the distortion coefficients are given by the following formulas [1, 4, 5, 7]:

- the effective values of the n^{th} harmonic:

$$U_{efn} = \frac{b_n}{\sqrt{2}},\ n = 1,\ 3,\ 5, \ldots ; \tag{14}$$

- the total effective value:

$$U_{ef.t} = \left[\frac{1}{T}\int_0^T u^2(t)\,\mathrm{d}t\right]^{\frac{1}{2}} = \left[\frac{4}{T}\sum_{k=1}^{m} U_k^2\left(t_{k+1} - t_k\right)\right]^{\frac{1}{2}} ; \tag{15}$$

- the total effective value of the high harmonics :

$$U_{ef.t.a} = \left[\frac{1}{T}\int_0^T \left(\sum_{n=3,5,..}^{\infty} b_n \sin n\omega t\right)^2 \mathrm{d}t\right]^{\frac{1}{2}} = \left[\frac{1}{2}\sum_{n=3,5,..}^{\infty} b_n^2\right]^{\frac{1}{2}} = \left[U_{ef.t}^2 - U_{ef.1}^2\right]^{\frac{1}{2}} ; \tag{16}$$

- the distortion coefficient k_{d1}, defined as the square root of the ratio between the conducting power in harmonics and the conducting power in the fundamental:

$$k_{d1} = \frac{U_{ef.t.a}}{U_{ef.1}} = \left[\frac{U_{ef.t}^2}{U_{ef.1}^2} - 1\right]^{\frac{1}{2}} ; \tag{17}$$

- the distortion coefficient k_{d2}, defined as the square root of the ratio between the conducting power in harmonics and the conducting power in the synthetic voltage:

$$k_{d2} = \frac{U_{ef.t.a}}{U_{ef.t}} = \frac{k_{d1}}{\left(1 + k_{d1}^2\right)^{\frac{1}{2}}} \tag{18}$$

Results of the Numerical Simulation

For the numerical simulation of the optimal voltage synthesis, one has used the Matlab toolbox [2], which has facilities for solving non linear systems of equations, for the spectral analysis of the synthetic voltage, for the construction of the time – voltage vectors, for the graphical representation of the synthetic voltage and of the frequency spectrum.

The input data are: $A, f, m, u^{(0)} = \left[u_1^{(0)}, u_2^{(0)}, \ldots . u_m^{(0)}\right]$, where $u^{(0)}$ is the initial approximation of the reported voltage vector. The elements of the vector $u^{(0)}$ are positive and are generated with the step $1/m$.

The output data of the simulation program are:

- the amplitude of the voltage steps: $U_1, U_2, \ldots U_m$;
- the commutation moments in the interval $0 - T/4$: $t_1, t_2, \ldots t_{m+1}$;
- the amplitude and the effective value of the harmonics up to order 25;
- the distortion coefficients k_{d1}, k_{d2};

° the graphs of the synthetic voltage and of the spectrum of the harmonics up to order 25.

For the numerical solving of the non-linear system (13), one has used an iterative method [2, 3] from the optimisation toolbox of Matlab. The system (13) could be written like in the following:

$$\begin{cases} f_1(u) = u_1 + \sum\limits_{k=2}^{m} (u_k - u_{k-1}) \cos\left(\arcsin \dfrac{u_{k-1} + u_k}{2} \right) - \dfrac{\pi}{4} = 0 \\ f_2(u) = u_1 + \sum\limits_{k=2}^{m} (u_k - u_{k-1}) \cos\left(3 \arcsin \dfrac{u_{k-1} + u_k}{2} \right) = 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ f_m(u) = u_1 + \sum\limits_{k=2}^{m} (u_k - u_{k-1}) \cos\left((2m-1) \arcsin \dfrac{u_{k-1} + u_k}{2} \right) = 0 \ , \end{cases} \tag{19}$$

where $f_1, f_2, \dots f_m$ are non-linear functions with the variables $u_1, u_2, \dots u_m$.

The function modules are subsumed and the target function results:

$$f(u) = \sum_{k=1}^{m} \left| f_k(u) \right| \ , \tag{20}$$

defined and with values in the positive real space.

If, for the vector u^*, the target function $f(u^*) = 0$, then u^* represents the solution of the system of equations (19).

The solution u^* is calculated with the Matlab function "*fmins*" which minimizes the target function in the neighbourhood of the initial estimation $u^{(0)}$ with the defined precision 1.e-4.

For the solution u^*, one verifies if $f_i(u^*) = 0$, $i = 1, 2, \dots m$; on the contrary, the initial estimation is changed. If solution u^* is accepted, one determines the pulses commutation moments with the relations (11).

For a sinusoidal voltage proposed with the amplitude 44V and f =10 Hz, the numerical results of the simulation for m =1, 2, …7 pulses in the interval 0 – T/4 are presented in the table I. Taking into account the limitations of a practical implementation, the values of the voltage steps in [V] are rounded at the second decimal, and the values of the commutation moments in [ms] are rounded at the first decimal.

The graphical representations of the synthetic voltage and the spectrum of the harmonics up to order 25 obtained by simulation for m = 1 and m = 4 are presented in figure 2 and 3.

For m =1, the converter functions in non-modulated regime, one can remark the high weight of the low harmonics (of order 3, 5, 7, 9, 11) in the harmonic content of the voltage and the high values of the distortion coefficients.

By increasing the number of steps, one can achieve a better synthesis of the sinusoidal voltage, because the harmonics up to the $2m$ – 1 order nullify, and the distortion coefficients and the amplitude of the other harmonics decrease. For m =7, one can observe that the effective value of the harmonics represents 6,6% from the effective value of the fundamental, and the distortion coefficients equal 0.066.

So, for a functioning of the asynchronous motor at low frequencies of 5 –10 Hz in conditions close to a sinusoidal source power supply, 4 – 7 approximation steps are needed for the voltage of the inverter.

Table1. The results of the numerical simulation.

Input data / Output data	Umax [V]	44						
	f [Hz]	10						
	m	1	2	3	4	5	6	7
The relative amplitude of pulse	u_1	0,785	0,490	0,266	0,199	0,163	0,097	0,139
	u_2		1,050	0,615	0,566	0,397	0,375	0,297
	u_3			0,977	0,847	0,679	0,540	0,461
	u_4				1,000	0,831	0,763	0,582
	u_5					0,990	0,870	0,654
	u_6						0,992	0,866
	u_7							0,989
The pulses amplitude [V]	U_1	34,55	21,56	11,72	8,75	7,19	4,30	6,12
	U_2		46,20	27,10	24,91	17,46	16,49	13,70
	U_3			43,00	37,28	29,87	23,76	20,29
	U_4				44,00	36,57	33,54	25,59
	U_5					43,56	38,26	28,77
	U_6						43,66	38,12
	U_7							43,53
Pulses commutation moments in 0 – T/4 [ms]	t_1	0	0	0	0	0	0	0
	t_2	25	14	7,3	6,2	4,5	3,8	3,5
	t_3		25	14,7	12,5	9,0	7,6	6,2
	t_4			25	18,7	13,6	11,3	8,7
	t_5				25	18,2	15,2	10,6
	t_6					25	19	13,7
	t_7						25	18,9
	t_8							25
Amplitude/effective value of the first 25 harmonics [V]	b_1	44,00	47,47	44,73	44,57	44,30	44,22	44,18
	$U_{ef\,1}$	31,11	33,56	31,63	31,52	31,33	31,27	31,24
	b_3	14,67	0	0	0	0	0	0,06
	$U_{ef\,3}$	10,37	0	0	0	0	0	0,04
	b_5	8,80	3,53	0	0	0	0	0,03
	$U_{ef\,5}$	6,22	2,50	0	0	0	0	0,02
	b_7	6,29	8,36	2,19	0	0	0	0,40
	$U_{ef\,7}$	4,44	5,92	1,55	0	0	0	0,28
	b_9	4,89	2,85	0,53	0	0	0	0
	$U_{ef\,9}$	3,46	2,02	0,37	0	0	0	0
	b_{11}	4,00	0,27	0,52	0	1,11	0	0
	$U_{ef\,11}$	2,83	0,19	0,36	0	0,78	0	0
	b_{13}	3,38	3,12	3,86	0,01	0,02	1,08	0
	$U_{ef\,13}$	2,39	2,20	2,73	0	0,01	0,76	0
	b_{15}	2,93	3,54	2,51	2,96	0,06	0,24	1,07
	$U_{ef\,15}$	2,07	2,50	1,77	2,09	0,04	0,17	0,76
	b_{17}	2,59	0,29	0,21	2,63	0,05	0,02	0,06
	$U_{ef\,17}$	1,83	0,20	0.15	1,86	0,03	0.01	0,04
	b_{19}	2,32	0,54	0,27	0	0,09	0,06	0,37
	$U_{ef\,19}$	1,64	0,38	0,19	0	0,06	0,05	0,26
	b_{21}	2,10	2,69	0,64	0	2,08	0,11	1,14
	$U_{ef\,21}$	1,48	1,90	0,45	0	1,47	0,08	0,80
	b_{23}	1,91	1,47	0,36	0	1,95	0,24	0,34
	$U_{ef\,23}$	1,35	1,04	0,26	0	1,38	0,17	0,24
	b_{25}	1,76	0,16	0,52	0	0,07	1,41	0,25
	$U_{ef\,25}$	1,24	0,11	0,36	0	0,05	1,00	0,18
Total effective value[V]	$U_{ef.t}$	34,55	34,65	31,96	31,72	31,44	31,36	31,31
Total effective value of harmonics [V]	$U_{ef.t.a}$	15,04	8,59	4,56	3,58	2,73	2,32	2,08
Distortion coefficients	k_{d1}	0,483	0,256	0,144	0,114	0,087	0,074	0,066
	k_{d2}	0,435	0,248	0,143	0,113	0,087	0,074	0,066

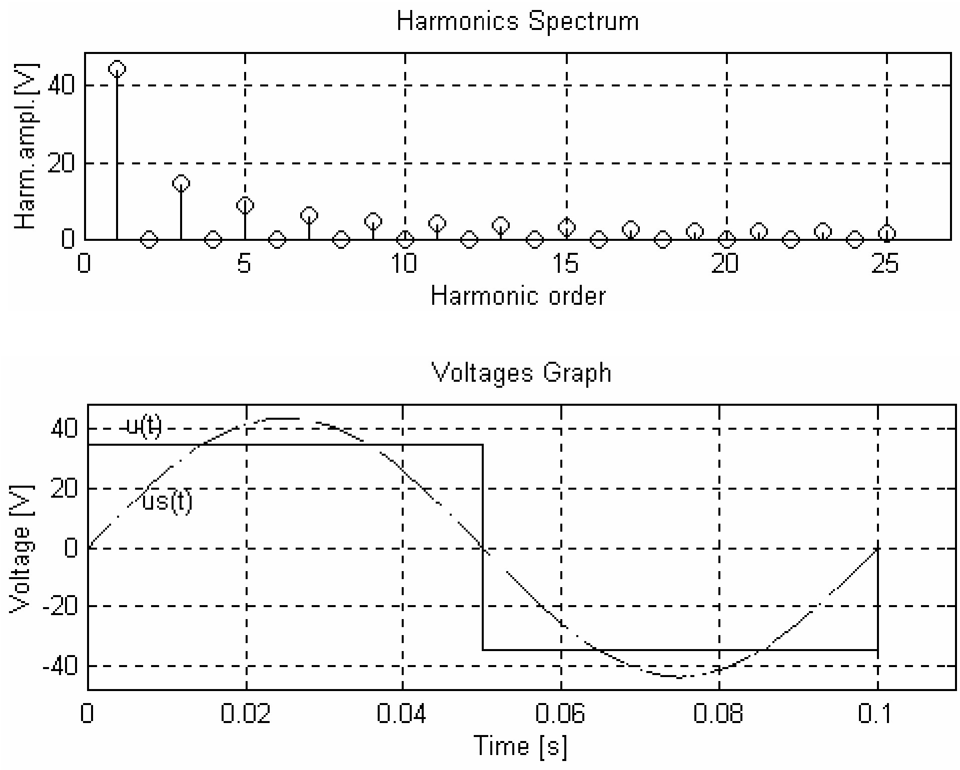

Fig. 2. The graphical results of the simulation for m = 1.

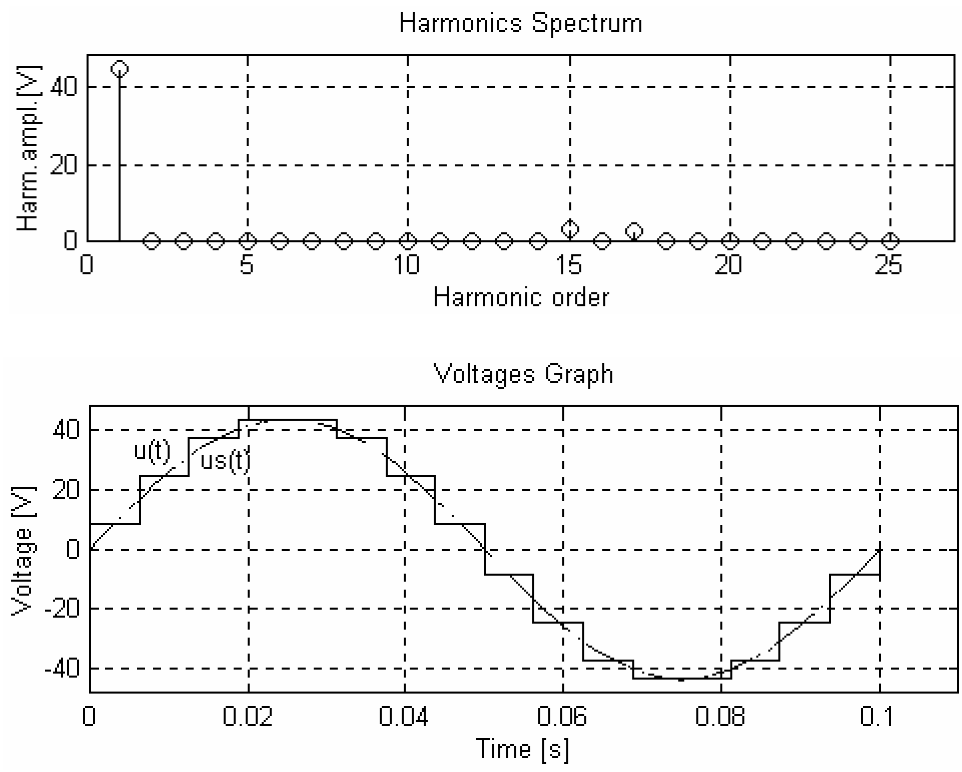

Fig. 3. The graphical results of the simulation for m =4.

Conclusions

° The synthesis of the voltage based on the principle of pulse amplitude and width modulation is optimal, because the distortion coefficients are minimized and the first $2m-1$ harmonics nullify.

° The results obtained through numerical simulation on the model, using the Matlab toolbox, show that the fundamental of the synthetic voltage is identical to the proposed sinusoidal voltage and that the high harmonics up to the $2m-1$ order are null. To the later, the distortion coefficients are big due to the fact that the nullifying of the harmonics in some part of the spectrum implies the increase of the harmonics in other part of the spectrum.

° In the domain of the power electronics, the presented synthesis method could be implemented on a microprocessor, which commands the rectifier and the autonomous inverter after a program obtained from the simulation.

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Modelarea matematică a algoritmului de comandă PAWM al convertorului monofazat

Rezumat

Articolul prezintă un suport matematic al algoritmului de comandă a modulaţiei pulsurilor in amplitudine şi (PAWM, acronimele de la Pulse Amplitude Width Modulation) pentru sintetizarea tensiunii de ieşire a convertorului monofazat, având ca obiective aproximarea valorii efective a fundamentalei cu valoarea efectivă a tensiunii sinusoidale propuse la bornele sarcinii şi diminuarea ponderii armonicelor de joasa frecvenţa în conţinutul armonic al tensiunii. Momentele de comutaţie şi amplitudinea fiecărui puls sunt calculate în condiţiile în care fundamentala tensiunii sintetice este egală cu tensiunea sinusoidală propusă, armonicele superioare până la ordinul 2m – 1 sunt nule şi coeficienţii de distorsiuni sunt minimi .Pentru simularea numerică a modelului s-a folosit pachetul Matlab. Rezultatele simulării sunt prezentate numeric şi grafic; se confirmă validitatea suportului matematic al algoritmului de comandă PAWM.