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An Improvable Approach for PWM Control Algorithm of the Mono-Phase Inverter

- Part B -

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Abstract

The article presents a mathematical support of the PWM control algorithm for the improvable synthesis of the output voltage of the mono-phase inverter, having as objectives the approaching of the effective value of the fundamental to the effective value of the proposed sinusoidal voltage at the terminals of the charge and diminishing of the weight of low frequency harmonics in the harmonic content of the voltage.
The commutation moments of the pulses are computed, imposing that the fundamental of the synthetic voltage is equal to the proposed sinusoidal voltage and the high harmonics up to order 4m-2 are null. For the numerical simulation of the model, the Matlab toolbox was used. The results of the simulation are numerically and graphically presented; they confirm the validity of the mathematical support of the proposed PWM control algorithm. The aim of this paper is to obtain an inverter which to ensure a power supply close to a sinusoidal form. Some practical considerations are also presented.
The paper was split in two parts. Part B includes a numerical simulation example of the mathematical design of the proposed PWM control algorithm presented in part A and some practical considerations.

Key words: *PWM (pulse width modulation) control algorithm, mono--phase inverter, synthetic voltage.*

Results of the Numerical Simulation

For the numerical simulation of the proposed PWM control algorithm, we have used the Matlab toolbox [8], which offers facilities for solving non linear systems of equations, for the spectral analysis of the synthetic voltage, for the construction of the time – voltage vectors, for the graphical representation of the synthetic voltage and of the frequency spectrum.

The input data of the simulation program are:

- A, f – the amplitude and the frequency of the proposed sinusoidal voltage;
- E – the direct voltage of the intermediate circuit;
- m – the number of pulses in the interval $0 – T/4$;
- *itm* – the maximum number of iterations;
- *era* – the maximum allowable error;
- $t^{(0)} = \left[t_2^{(0)}, t_3^{(0)}, \ldots t_{2m}^{(0)}\right]$ – the initial approximation of the time vector. The elements of the vector $t^{(0)}$ are generated with a step of $1/(8fm)$. Taking into the account the practical possibilities to realize the commutation moments, the precision was limited to 0.2 ms (*era* = 2 e^{-4}).

The output data of the simulation program are:

- $t_2, t_4, \ldots t_{2m}$ – the commutation moments in the interval $0 - T/4$;
- the amplitude and the effective value of the harmonics up to order 25;
- k_{d1}, k_{d2} – the distortion coefficients;
- the graphs of the synthetic voltage and frequency spectrum of the harmonics up to order 25.

For the numerical solving of the non-linear system (11), the Newton – Raphson iterative method was used [6]. The approximation at iteration $j + 1$ has the following vectorial form:

$$t^{(j+1)} = t^{(j)} + dt^{(j)}, \tag{17}$$

where: $t^{(j)}$ is the approximation at the j^{th} iteration; $dt^{(j)}$ – the solution of the linear system of equations deducted with the method Newton – Raphson; $j = 0, 1, 2, 3, \ldots$ being the iteration counter.

The system of equations (11) could be written:

$$\begin{cases} f_1 = \sum\limits_{k=1}^{m} \left(\cos\omega t_{2k} - \cos\omega t_{2k+1}\right) - \dfrac{\pi A}{4E} \\ \cdot \\ f_i = \sum\limits_{k=1}^{m} \left(\cos(2i-1)\omega t_{2k} - \cos(2i-1)\omega t_{2k+1}\right) = 0 \end{cases} \tag{18}$$

$$i = 2, 3, \ldots, 2m-1$$

The jacobian matrix of the system is [6]:

$$g(t) = \begin{bmatrix} \dfrac{\partial f_1}{\partial t_2} & \dfrac{\partial f_1}{\partial t_3} & \cdot\ \cdot\ \cdot\ \cdot & \dfrac{\partial f_1}{\partial t_{2m}} \\ \dfrac{\partial f_2}{\partial t_2} & \dfrac{\partial f_2}{\partial t_3} & \cdot\ \cdot\ \cdot\ \cdot & \dfrac{\partial f_2}{\partial t_{2m}} \\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot \\ \dfrac{\partial f_{2m-1}}{\partial t_2} & \dfrac{\partial f_{2m-1}}{\partial t_3} & \cdot\ \cdot & \dfrac{\partial f_{2m-1}}{\partial t_{2m}} \end{bmatrix} = \left[g_{ik}\right], \tag{19}$$

$$i = 1, 2, 3, \ldots, 2m-1; \quad k = 1, 2, 3, \ldots, 2m-1;$$

where the elements of the matrix are:

$$g_{ik} = \frac{\partial f_i}{\partial t_{k+1}} = (-1)^k (2i-1)\,\omega \sin(2i-1)\,\omega\, t_{k+1} \tag{20}$$

$$i = 1, 2, 3, \ldots, 2m-1; \quad k = 1, 2, 3, \ldots, 2m-1.$$

The linear system deducted with the Newton – Raphson at j^{th} iteration, in vectorial form is [6]:

$$g\left(t^{(j)}\right) d\,t^{(j)} = -f\left(t^{(j)}\right), \tag{21}$$

and its components:

$$\sum_{k=1}^{2m-1} g_{ik}\left(t^{(j)}\right) d\,t_{k+1}^{(j)} = -f_i\left(t^{(j)}\right), \tag{22}$$

$$i = 1, 2, 3, \ldots, 2m-1;$$

with the unknowns $d\,t_{k+1}^{(j)}$, $k = 1, 2, \ldots 2m - 1$.

The iteration process is finished when the estimated precision of the solution is attained, evaluated at the n^{th} iteration through the Euclidian norm of the vector $dt^{(n)}$:

$$\left\| dt^{(n)} \right\|_2 = \left[\sum_{k=1}^{2m-1} \left| dt_{k+1}^{(n)} \right|^2 \right]^{\frac{1}{2}} \leq era \quad , \tag{23}$$

or when the maximum number of iterations is attained, $n \leq itm$,where *era* and *itm* are given.

The final solution of the non-linear system (11) is:

$$t^{(n+1)} = t^{(n)} + dt^{(n)} . \tag{24}$$

The output data of the simulation program are computed.

For a proposed sinusoidal voltage with the amplitude A = 44V and frequency f = 10Hz the numerical results of the simulation are presented in table 2. Taking into account the limitations of a practical implementation, the values of the voltage steps in [V] are rounded at the second decimal, and the values of the commutation moments in [ms] are rounded at the first decimal.

The inverter functions in modulated regime with m = 1, 2, 3, …, 7 pulses in the interval 0 – T/4. The model is conceived such that the fundamental of the synthetic voltage is identical to the proposed sinusoidal voltage, and the odd and even harmonics up to order $4m-2$ are null. Through simulation, one can see that the harmonics of order $4km \pm 1$, k = 1, 2, 3, …, have a high amplitude (approximate to the fundamental), the others being practically negligible, which means that a high number of harmonics are eliminated.

One can remark the high weight of the low harmonics (of order 3, 5, 7, 9, 11) in the harmonic content of the voltage and the high values of the distortion coefficients, for m =1.

By increasing the number of pulses, one can achieve a better synthesis of the sinusoidal voltage, because the effective value of the fundamental of the synthetic voltage is identical to the effective value of the proposed sinusoidal voltage and the harmonics up to the $4m-2$ order nullify, so that the weight of low frequency harmonics decreases in the harmonic content of the voltage. The distortion coefficients are big, even if the number of pulses increases, due to the fact that the nullifying of the harmonics in domain of the low frequency implies the increase of the harmonics amplitude in other part of the spectrum. Because the high amplitude harmonics are shifted towards high frequencies, this is a advantage for that do not disturb the functioning of the asynchronous motor but is a disadvantage of the PWM voltage synthesis [1].

The graphical representations of the synthetic voltage and the frequency spectrum of the harmonics up to order 25 obtained by simulation for m = 1 and m = 4 are presented in figure 3 and figure 4. So, for a functioning of the asynchronous motor at low frequencies of 5 –10 Hz in conditions close to a sinusoidal source power supply, no less 7 pulses are needed for the voltage of the inverter.

Conclusions

- The voltage synthesis after PWM control algorithm consists in the generation of the commutation program for the power devices such that the widths of the output pulses of the inverter to be modulated after a sinusoidal function. The inverter assures the simultaneous control of frequency and amplitude of the fundamental of the applied voltage for the charge [1].
- The mathematical model of the proposed PWM control algorithm, is based on equality between the effective value of the fundamental of the synthetic voltage to the effective value of the proposed sinusoidal voltage at the terminals of the charge and on the assessment of a harmonics content of the synthetic voltage (the nullifying of low frequency harmonics up to order $4m$ – 2), according to relations (10).

Table 2. The results of the numerical simulation.

Input data / Output data								
	A [V]	44						
	f [Hz]	10						
	E	$55\sqrt{2}\ \pi$						
	itm	10						
	era	2 e – 4						
	m	1	2	3	4	5	6	7
Pulses commutation moments in 0 – T/4 [m s]	t_1	0	0	0	0	0	0	0
	t_2	22.7	11.7	7.9	6.0	4.9	4.1	3.5
	t_3	25.0	13.3	8.7	6.5	5.1	4.3	3.7
	t_4		23.9	16.0	12.1	9.7	8.1	7.1
	t_5		25.0	17.3	12.9	10.2	8.5	7.3
	t_6			24.2	18.2	14.6	12.2	10.6
	t_7			25.0	19.3	15.3	12.8	11.1
	t_8				24.4	19.6	16.3	14.3
	t_9				25.0	20.4	17.0	14.9
	t_{10}					24.5	20.5	18.4
	t_{11}					25.0	21.2	19.2
	t_{12}						24.6	22.5
	t_{13}						25.0	22.8
	t_{14}							24.7
	t_{15}							25.0
Amplitude/effective value of the first 25 harmonics [V]	b_1	44.00	44.00	44.00	44.00	44.00	44.00	44.00
	$U_{ef\,1}$	31.11	31.11	31.11	31.11	31.11	31.11	31.11
	b_3	42.83	0	0	0	0	0	0
	$U_{ef\,3}$	30.28	0	0	0	0	0	0
	b_5	40.54	0	0	0	0	0	0
	$U_{ef\,5}$	28.66	0	0	0	0	0	0
	b_7	37.24	42.83	0	0	0	0	0
	$U_{ef\,7}$	26.33	30.29	0	0	0	0	0
	b_9	33.09	41.68	0	0	0	0	0
	$U_{ef\,9}$	23.40	29.47	0	0	0	0	0
	b_{11}	28.28	1.15	42.83	0	0	0	0
	$U_{ef\,11}$	20.00	0.81	30.29	0	0	0	0
	b_{13}	23.04	1.13	41.68	0	0	0	0
	$U_{ef\,13}$	16.29	0.80	29.47	0	0	0	0
	b_{15}	17.60	38.36	1.14	42.82	0	9	0
	$U_{ef\,15}$	12.44	27.13	0.81	30.28	0	0	0
	b_{17}	12.20	36.24	0.01	41.69	0	0	0
	$U_{ef\,17}$	8.62	25.62	0.01	29.48	0	0	0
	b_{19}	7.06	3.14	0	1.14	42.83	0	0
	$U_{ef\,19}$	5.00	2.22	0	0.80	30.29	0	0
	b_{21}	2.39	3.05	1.14	0.01	41.68	0	0
	$U_{ef\,21}$	1.69	2.16	0.81	0.01	29.47	0	0
	b_{23}	1.65	31.28	38.36	0	1.14	42.83	0
	$U_{ef\,23}$	1.16	22.12	27.13	0	0.80	30.29	0
	b_{25}	4.91	28.51	36.23	0.02	0.01	41.68	0
	$U_{ef\,25}$	3.47	20.16	25.62	0.01	0.01	29.47	0
Tot. eff. value [V]	$U_{ef.t}$	73.44	80.58	81.78	82.20	82.39	82.50	82.55
Harm.tot.eff.val.[V]	$U_{ef.t.a}$	66.53	74.33	75.64	76.09	76.29	76.41	76.47
Distortion coefficients	k_{d1}	2.138	2.390	2.431	2.445	2.452	2.456	2.458
	k_{d2}	0.906	0.922	0.925	0.925	0.926	0.926	0.926

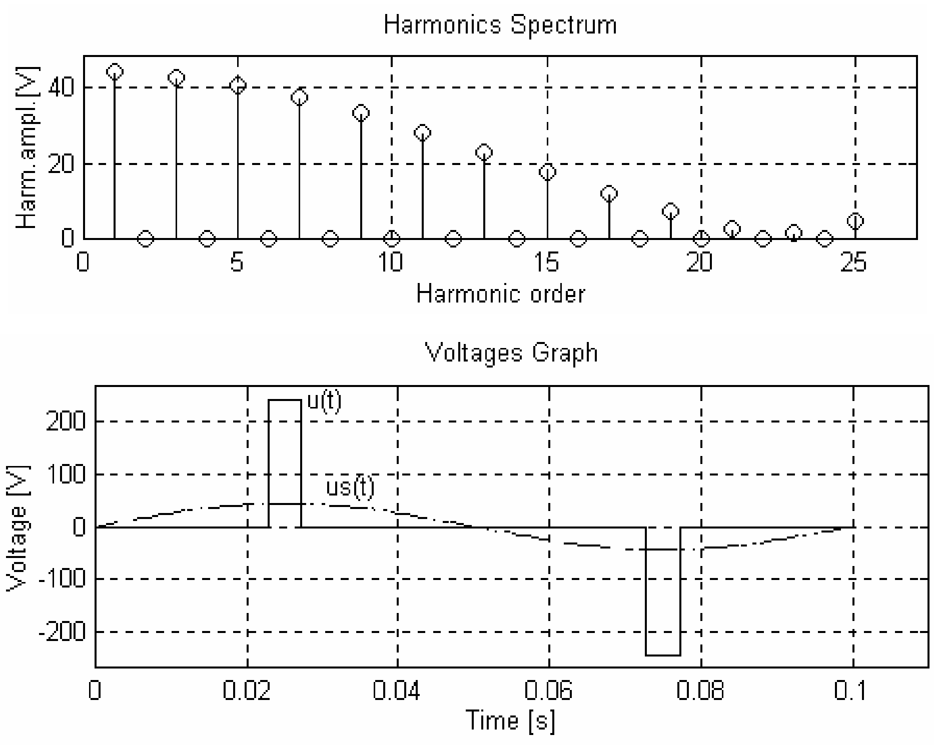

Fig. 3. The graphical results of the simulation for *m* =

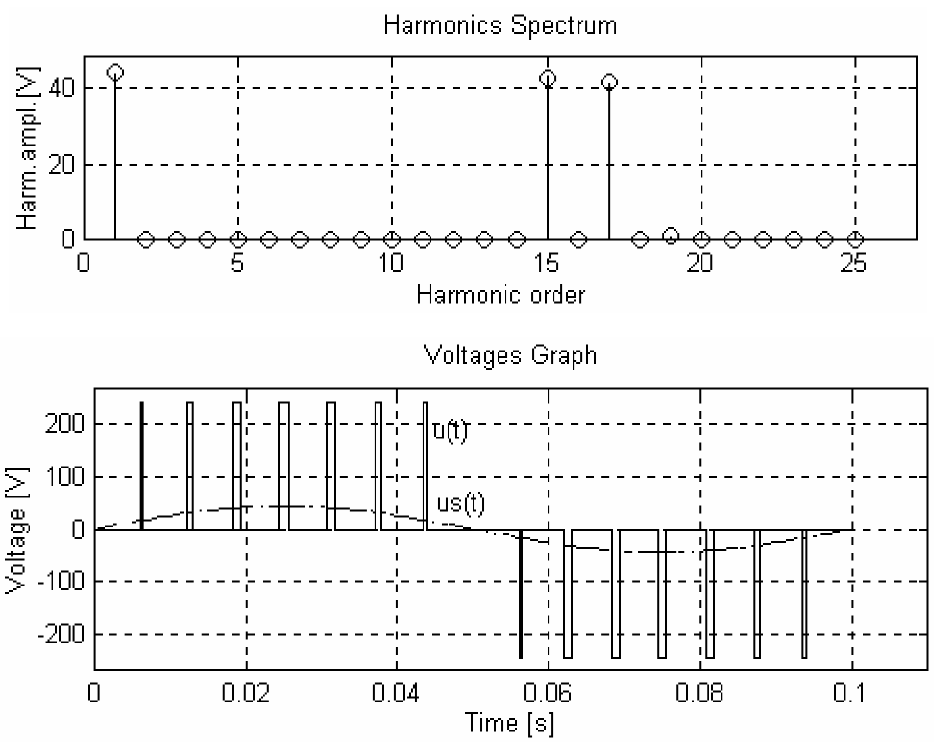

Fig. 4. The graphical results of the simulation for *m* =

- On the simulation model one determines the sequence and the commutation moments of the power devices, the pulses durations, the amplitude and the effective value of the harmonics and the spectral analysis of the synthetic voltage.
- The results obtained through numerical simulation on model, using the Matlab toolbox, show that the fundamental of the synthetic voltage is equal to the proposed sinusoidal voltage and that the first harmonics up to order $4m-2$ are null. To the later, the distortion coefficients are big due to the fact that the nullifying of the harmonics in some part of the spectrum implies the increase of the harmonics in other part of the spectrum. The synthesis of the sinusoidal voltage based on the proposed PWM control algorithm is improvable, because one diminishes of the weight of low frequency harmonics in the harmonic content of the voltage. The results confirm the validity of the model.
- In the domain of the adjustable electrical drives with alternative current motors, the proposed PWM control algorithm could be implemented in software on a microprocessor in order to obtain an on-line command system.

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O abordare îmbunătățită a algoritmului de comandă PWM al invertorului monofazat

- Partea B -

Rezumat

Articolul prezintă un suport matematic al algoritmului de comandă PWM pentru sinteza îmbunătățită a tensiunii de ieşire a invertorului monofazat, având ca obiective aproximarea valorii efective a fundamentalei cu valoarea efectivă a tensiunii sinusoidale propuse la bornele sarcinii şi diminuarea ponderii armonicelor de joasa frecvența în conținutul armonic al tensiunii.
Momentele de comutație ale pulsurilor sunt calculate în condițiile în care fundamentala tensiunii sintetice este egală cu tensiunea sinusoidală propusă şi armonicele superioare până la ordinul 4m – 2 sunt nule. Pentru simularea numerică a modelului s-a folosit pachetul Matlab. Rezultatele simulării sunt prezentate numeric şi grafic; se confirmă validitatea suportului matematic al algoritmului de comandă PWM propus. Scopul acestui articol este să se obțină un invertor care să asigure o sursă de alimentare apropiată de forma sinusoidală. De asemenea sunt prezentate câteva considerații practice.
Articolul este împărțit în două părți. Partea B conține un exemplu de simulare numerică a modelului matematic al algoritmului de comandă PWM propus prezentat în partea A şi câteva considerații practice.