The Heavy Plates Cold - Roll - Bending Modelling

Alexandru Traian Dumitrescu, Adrian Catangiu, Dan Ungureanu

Universitatea "Valahia", B-dul Unirii. No. 18, cod 130082, Târgoviște, Romania e-mail: atdumitrescu@gmail.com

Abstract

In the elementary bending theory it is usual to admit the hypothesis of small strains, small enough to neglect the transversal stresses induced by severe curvature. It is also admitted that the neutral surface coincides, during deformation, with the central plane of the plate. To carry out a mechanical modelling of heavy plates bending, in order to for obtain great diameter tube, it is necessary a general theory of plate bending, without restrictions regarding the magnitude of strains and curvature; it is also necessary to determine the neutral surface movement and the movement of each fibre across the plate thickness. The paper presents a model of strains and stresses calculus for a rigid-plastic material and for a hardenable material too. An important issue of these calculae is the width determination of the zone where the material suffer, during bending, an elongation and compression too and so, the Bauschinger effect has an important influence on the mechanical proprieties of tubes. In this zone the material strength after bending is less than the plate strength, the strength diminishing being proportional to the width of mentioned zone.

Key words: plate, cold forming, simulation, bending.

Notations

- a internal radius;
- b external radius;
- h plate thickness;
- σ_r radial stress;
- σ_{θ} tangential stress;
- c neutral surface radius;
- r current bending radius;
- *k* yielding strength;
- σ_c yield stress by uniaxial tension;
- M bending moment;
- α bending angle;
- *u* internal radial component of displacement vector;
- v tangential component of the displacement vector;
- L_0 initial length of the plate;
- θ angle of the radius and the symmetry plan;
- ε_r radial strain;
- ε_{θ} tangential strain;
- $\gamma_{r\theta}$ angular strain;
- m coefficient which described the initial position of the fibre with L_0 length;

 r_0 – bending radius correspondent to L_0 ;

- T tensile force (on width unit) which acts at the plate extremities, being normal to the extreme transversal sections;
- q uniform pressure applied on the internal surface of the plate;

s – the ratio T/(2 k h);

- $\overline{\sigma}$ average stress;
- $\overline{\epsilon}$ average strain.

Stresses and Strains Determination in a Rigid – Perfect Plastic Material Subjected to Bending

I shall consider the bending produced by the moments applied at plate extremities, the state being a plane strain one; the material behaves rigid-plastic, without hardening.

The main stresses in the bending are radial and tangential oriented, as a result of the deformation symmetry, and I shall designate them by σ_r , σ_{θ} respectively.

The equilibrium equation can be written in the radial direction as:

$$\frac{\mathrm{d}\sigma_r}{\mathrm{d}r} = \frac{\sigma_r - \sigma_{\theta}}{r} \tag{1}$$

I shall call by "c" the neutral surface radius, i.e. the radius of the cylindrical surface – including those fibres that do not modify their lengths when infinitesimal supplementary deformation takes place.

The fibres placed between the neutral surface and the external surface of the plate are subjected to elongation and those placed between the neutral surface and internal surface are subjected to compression.

The yielding condition for the plane strain state is:

$$\sigma_{\theta} - \sigma_r = 2k \quad \text{for} \quad c \le r \le b$$

$$\sigma_{\theta} - \sigma_r = -2k \quad \text{for} \quad a \le r \le c$$
(2)

where: $k = \frac{\sigma_c}{2}$ in the case of Tresca criterion, and $k = \frac{\sigma_c}{\sqrt{3}}$, in the case of Mises criterion, [1].

Having in view that $\sigma_r = 0$ for r = a, b, from (1) and (2) one can obtain

$$\sigma_r = 2k \ln \frac{r}{b} \quad \text{for} \quad c \le r \le b,$$

$$\sigma_r = 2k \ln \frac{a}{r} \quad \text{for} \quad a \le r \le c \tag{3}$$

As the equilibrium condition required σ_r to be continuous across the neutral surface, one can obtain:

$$2k\ln\frac{c}{b} = 2k\ln\frac{a}{c}$$

Thus, the neutral surface radius is given by

$$c = \sqrt{ab} \tag{4}$$

The other main stress component may be obtained from (2) and (3), as

$$\sigma_{\theta} = 2k \left(\ln \frac{r}{b} + 1 \right) \quad \text{if} \quad c \le r \le b$$

$$\sigma_{\theta} = 2k \left(\ln \frac{a}{r} - 1 \right) \quad \text{if} \quad a \le r \le c$$
(5)

The variation of σ_r and σ_{θ} across the thickness of the plate is given in fig.1. It can be noticed that σ_r attains a maximum value on the neutral surface and the resultant force acting upon a section is given by

$$\int_{a}^{b} \sigma_{\theta} dr = \int_{a}^{b} \frac{d}{dr} (r\sigma_{r}) dr = r\sigma_{r} \bigg|_{a}^{b} = 0$$

where the equilibrium equation (1) is used.

The bending moment, corresponding to a width unit, is obtained as:

$$M = \int_{b}^{a} \sigma_{\theta} r dr = kc^{2} \ln \frac{ab}{c^{2}} + \frac{k}{2} \left(a^{2} + b^{2} - 2c^{2} \right)$$

and, using (4), on can write:

$$M = \frac{1}{2}k(b-a)^2 = \frac{1}{2}kh^2$$
(6)

Let *u* d α be the radial component, and *v* d α the tangential component of the displacement vector due to an infinitesimal strain; the bending angle α , calculated for the initial length L_0 , increases by d α .

Neglecting the elastic compressibility and taking into account that the associated deformation is an elongation for r > c, and a compression for r < c, we can consider the following expressions [3]:

$$u = \frac{1}{2\alpha} \left(r + \frac{c^2}{r} \right), \quad v = \frac{r\theta}{\alpha}$$
(7)

where θ is the angle between the radius and the symmetry plane. Then, the corresponding increments of strains have the following components:

$$d\varepsilon_{\theta} = -d\varepsilon_{r} = \frac{1}{2\alpha} \left(1 - \frac{c^{2}}{r^{2}} \right) d\alpha , \quad d\gamma_{r\theta} = 0$$
(8)

obtained from the relations [1]:



Fig.1. σ_r and σ_{θ} distribution in a plate subjected to bending without hardening

$$d\varepsilon_{\theta} = \frac{1}{r} \frac{\partial}{\partial \theta} (du_{\theta}) + \frac{du_{r}}{r}, \quad d\varepsilon_{r} = \frac{\partial}{\partial r} (du_{r})$$
$$d\gamma_{r\theta} = \frac{1}{r} \frac{\partial}{\partial \theta} (du_{r}) + \frac{\partial}{\partial r} (du_{\theta}) - \frac{du_{\theta}}{r}$$

Using relations (7), we get

$$(\mathrm{d}u_r)_{r=a} = -\frac{1}{2\alpha} \left(a + \frac{c^2}{a} \right) \mathrm{d}\alpha = -\frac{1}{2\alpha} (a+b) \mathrm{d}\alpha$$
$$(\mathrm{d}u_r)_{r=b} = -\frac{1}{2\alpha} \left(b + \frac{c^2}{b} \right) \mathrm{d}\alpha = -\frac{1}{2\alpha} (b+a) \mathrm{d}\alpha$$

and

$$\mathrm{d}h = (\mathrm{d}u_r)_{r=b} - (\mathrm{d}u_r)_{r=a} = 0$$

As a result, one can consider h being constant in every configuration, for a non-hardenable material; thus, according to (6), the moment of plastically bending does not depend on plastic deformation, for such a material.

From the equality of the final area and the initial area:

$$L_0 h = \frac{1}{2} \left(b^2 - a^2 \right) L_0 \alpha$$

the following relation is obtained (for the internal, external radii and the bending angle)

$$\alpha = \frac{2}{a+b} \tag{9}$$

The mechanical work, corresponding to the unit of width, is:

$$ML_0 \alpha = 2k \frac{h^2}{4} \cdot \frac{2L_0}{a+b} = k \frac{b-a}{b+a}$$
(10)

In spite of the constant thickness of the plate, the fibres are subjected to complex deformations. In (1.8), one can notice that the fibres having r < c are compressed, and those having r > c are subjected to an elongation.

If we consider a fibre in a non – bended plate, at a distance of $\frac{mh}{2}$ from the central plane, the following relation can be written, taking into account the equality of areas before and after bending:

$$\frac{1+m}{1-m} = \frac{r^2 - a^2}{b^2 - r^2} \qquad \text{or} \\ r = \sqrt{\frac{1}{2}(a^2 + b^2) + \frac{1}{2}(b^2 - a^2)m}$$
(11)

where $|m| \le 1$ and m > 0 for a fibre placed between the central plane and the convex part of the plate.

As an example, the final radius of the central fibre, in a non-deformed state, which corresponds to m = 0 (see fig. 2), is given by:

$$r_f = \sqrt{\frac{1}{2}(a^2 + b^2)} > \frac{a+b}{2}$$

Considering the formula (11) and r = c, one can notice that the fibre which, in the final configuration, coincides with the neutral surface, has "m" given by:

$$m = \frac{a-b}{a+b}$$

Thus, the neutral surface, which before the bending has coincided with the central plane, is moving towards the plate inner plane during the plastically bending.

All the fibres having $m \ge 0$ are subjected to elongation and the fibres for which r < c are subjected to compression; the fibres for which

$$0 > m > \frac{a-b}{a+b}$$

$$c < r < \sqrt{\frac{1}{2}\left(a^2 + b^2\right)},$$
(12)

have been surpassed by the neutral surface, so the fibres being subjected firstly to compression and afterwards to an elongation. In this zone, the Bauschinger effect can have a remarkable importance.



Fig.2. The relative movement of longitudinal fibres, during the bending: 1 - the central fibre (initial); 2 - the fibre having L_0 length; 3 - the neutral surface

Every moment, there is a fibre subjected to a compression followed by an elongation; so, the present length equals L_0 (the initial length). The radius of this fibre is obtained using (9) and is given by

$$r_0 = \frac{1}{\alpha} = \frac{a+b}{2} = a + \frac{h}{2}$$
(13)

and its initial position – obtained using (11) – is:

$$m = \frac{1}{2} \left(\frac{a-b}{a+b} \right)$$

Now, we shall consider that tensile forces T (on width unit) are acting at the plate extremities, being normal to the extreme transversal sections.

The resultant forces must be balanced by an uniform pressure, q, applied on the internal surface

of the plate; from the equality of the internal components we can write:

$$T = a q \tag{14}$$

The stress components are given, in this case, by:

Taking into account the continuity of σ_r across the neutral surface, it is obtained:

$$c = \sqrt{abe^{-\frac{q}{2k}}} \tag{16}$$

So, the neutral surface is moving more, in this case, towards the plate inner planes.

Further, the displacements are given by the relations (7), but the plate thickness does not remain constant because

$$(\mathrm{d}u_r)_{r=a} = -\frac{1}{2\alpha} \left(a + \frac{c^2}{a} \right) \mathrm{d}\alpha = -\frac{1}{2\alpha} \left(a + be^{\frac{q}{2k}} \right) \mathrm{d}\alpha,$$
$$(\mathrm{d}u_r)_{r=b} = -\frac{1}{2\alpha} \left(b + \frac{c^2}{b} \right) \mathrm{d}\alpha = -\frac{1}{2\alpha} \left(b + ae^{\frac{-q}{2k}} \right) \mathrm{d}\alpha,$$

from where it can be written

$$\frac{dh}{da} = \frac{(du_r)_{r=b} - (du_r)_{r=a}}{da} = \frac{b + ae^{\frac{-q}{2k}} - a - be^{\frac{-q}{2k}}}{a + be^{\frac{-q}{2k}}},$$

$$\frac{dh}{da} = \frac{h(e^{\frac{q}{2k}} - 1)}{h + a(e^{\frac{q}{2k}} + 1)}$$
(17)

The equation (17) expresses the fact that $\frac{dh}{da} > 0$, so – during plastic bending – the plate thickness is decreasing.

If $\frac{h}{a}$ is less than $\frac{1}{5}$, for example, the thickness variation can be approximated by neglecting σ_r as well as the modification of the state from compression to elongation, for the fibres placed between the initial position and the present position of the neutral surface.

By putting "s" as the ratio T/(2 k h), it is obtained

$$\frac{q}{2k} = \frac{T}{2ka} = \frac{sh}{a}, \quad \text{with} \quad 0 \le s \le 1$$

Using (16) and Taylor serial developments for $e^{-sh/a}$ and for $\sqrt{1+(1-s)\frac{h}{a}}$, the neutral surface results to be at a distance of sh/2 to the central surface.

Identically, from (17) it is obtained

$$\frac{\mathrm{d}h}{\mathrm{d}a} \cong \frac{sh^2}{2a^2}$$

If "s" is kept constant during the bending, one can obtain:

$$\Delta h = -\frac{sh^2}{2a},$$

relation which allows the calculation of Δh in the considered approximation.

The Determination of Strains and Stresses in a Rigid Plastic Hardenable Material Subjected to Bending

We consider the plate made of a rigid plastic hardenable material. As we have already mentioned previously, the fibres can be subjected to complex deformations. According to the nature of these deformations, 3 zones can be marked on the plate thickness (fig. 2) i.e.: zone I in which the fibres are elongated, zone II where the fibres are subjected to a compression and zone III in which the fibres are subjected firstly to a compression, and afterwards to an elongation.

To calculate the deformation intensity, the way of charging of each fibre must be studied; the stress intensity is calculated using the hardening law, which we presume to be of the general form:

where

and

 $\sigma_{c} = \overline{\sigma} = \overline{\sigma}(\overline{\epsilon}),$ $\overline{\sigma} = \sqrt{\sigma_{r}^{2} + \sigma_{\theta}^{2} - \sigma_{r}\sigma_{\theta}} \quad (\text{von Mises})$ $d\varepsilon = \frac{2}{\sqrt{3}} |d\varepsilon_{\theta}'|, \quad \overline{\varepsilon} = \int d\varepsilon$ (19)

For the elongation itself or for the compression itself, one can write:

$$\varepsilon_{\theta}^{'} = \int d\varepsilon_{\theta}^{'} = \ln \frac{r}{r_{0}}$$

where r_0 is the radius of the fibre having L_0 length. So, the strain intensity (for the fibres in zones I and II, suffering a one-direction deformation) is calculated by:

$$\overline{\varepsilon} = \overline{\varepsilon}(r) = \frac{2}{\sqrt{3}} \left| \ln \frac{r}{r_0} \right| \qquad (\text{von Mises}) \tag{20}$$

The yield condition can be written, for a hardenable material, as:

$$\sigma_{\theta} - \sigma_r = 2k(\overline{\sigma}) \quad \text{for} \quad c \le r \le b$$

$$\sigma_{\theta} - \sigma_r = -2k(\overline{\sigma}) \quad \text{for} \quad a \le r \le c \tag{21}$$

where $k = \frac{\overline{\sigma}}{2}$ for Tresca's criterion and $k = \frac{\overline{\sigma}}{\sqrt{3}}$ for von Mises' criterion.

The σ_r component is obtained from (1) and (21) by integration, as follows: $\circ~$ in zone I

$$\sigma_r^l(r) = \int_r^b \frac{\mathrm{d}\sigma_r}{\mathrm{d}r'} \mathrm{d}r' = \int_r^b \frac{\sigma_r - \sigma_\theta}{r'} \mathrm{d}r' = 2 \int_r^b \frac{k(\overline{\sigma})}{r'} \mathrm{d}r' = 2 \int_r^b \frac{k(\overline{\epsilon})}{r'} \mathrm{d}r' = 2 \int_r^b k(\overline{\epsilon}) \mathrm{d}\overline{\epsilon}$$
(22)

where $c_0 \le r \le b$, and o in zone II

$$\sigma_r^{II}(r) = \int_a^r \frac{\mathrm{d}\sigma_r}{\mathrm{d}r} \mathrm{d}r' = \int_a^r \frac{\sigma_r - \sigma_\theta}{r'} \mathrm{d}r' = -2 \int_a^r \frac{k(\overline{\sigma})}{r'} \mathrm{d}r' = -2 \int_a^r \frac{\hat{k}(\overline{\epsilon})}{r'} \mathrm{d}r' = -2 \int_a^r k(\overline{\epsilon}) \mathrm{d}\overline{\epsilon}$$
(23)

where $a \le r \le c$, and $\overline{\epsilon}$ is calculated according to (20) in both zones.

For a hardenable material, c is not known, and for the integration of the following relations

$$\frac{\mathrm{d}\sigma_r}{\mathrm{d}r} = \frac{\sigma_r - \sigma_{\theta}}{r} = \frac{2k(\overline{\sigma})}{r}, \qquad c \le r \le c_0 \tag{24}$$

each fibre deformation must be determined. The integration is carried out starting from the continuity condition of σ_r across the surface $r = c_0$, σ_r being given in (22).

The procedure is identically applied starting from (23). The calculation finishes if the continuity of σ_r for r = c is verified (see [3]).

It is necessary to notice that some geometrical relations, established in part 1 are not yet valid. Thus, considering the constancy of the area and h as variable, one can write

and

$$L_{0}h_{0} = \frac{1}{2}(b^{2} - a^{2})L_{0}\alpha,$$

$$\alpha h^{2} + 2a\alpha h - 2h_{0} = 0$$
(25)

The relation (25) allows the calculus of plate thickness in the present configuration, if one knows α , *a* and h_0 . For a rigid-perfect plastic material, where $h = h_0$, the relation (25) implies (9) which can be written

$$a = a(\alpha) = \frac{1}{\alpha} - \frac{h_0}{2} \tag{26}$$

All this result for α as the only parameter of plastically bending.

Using (.25), one can obtain: $h < h_0$, if

$$\alpha h_0 + 2\alpha a - 2 > 0 \tag{27}$$

The condition (27) must be observed in all the cases where a decrease of plate thickness takes place.

Numerical Results

Firstly, we shall calculate the tube radii, the stresses and strains produced after the bending of a plate having $L_0 = 3780$ mm and $h_0 = 14.3$ mm, the plate is made of a steel with the following characteristics: E = 205 GPa, v = 0.29 and $\sigma_c = 475$ MPa (a perfect plastic rigid material).

If we use Tresca's criterion, then we can write $\sigma_c = 2k$ and $\overline{\epsilon} = \left| \ln \frac{r}{r_0} \right|$. At the end of the bending,

 $\alpha = \frac{2\pi}{L_0} = 0.0017$ the final internal radius is $a_{fin} = 594.45$ mm, according to (26); from $b_{fin} = a_{fin} + b_{fin}$

 h_0 , one obtain $b_{fin} = 608.75$ mm.

The final radius of central fibre in a non-deformed state, is $r_f = 601.64$ mm.

The neutral fibre is described by (4) and one can obtain $c_{fin} = 601.56$ mm, and the radius of the fibre having the initial length L_0 is $r_0 = 601.6$ mm.

The stress has the following components, at the end of the bending (the values are given in MPa):

$$\sigma_r = \begin{cases} 475 \ln \frac{r}{608.75} & \text{if} \quad 601.56 \le r \le 608.75 \\ 475 \ln \frac{594.45}{r} & \text{if} \quad 594.45 \le r \le 601.56 \end{cases}$$

$$\sigma_{\theta} = \begin{cases} 475(\ln\frac{r}{608.75} + 1) & \text{if} \quad 601.56 \le r \le 608.75 \\ 475(\ln\frac{594.45}{r} - 1) & \text{if} \quad 594.45 \le r \le 601.56 \end{cases}$$

Taking into account the $\overline{\epsilon}$ expression and usind Tresca's criterion, for $r = b_{fin}$ one obtains:

$$\overline{\varepsilon} = \overline{\varepsilon}_{\theta} = \ln \frac{b_{fin}}{r_0} = 0.011$$
$$\overline{\varepsilon} = -\overline{\varepsilon}_{\theta} = -\ln \frac{a_{fin}}{r} = 0.012$$

and, for $r = a_{fin}$

The zone I, in which the fibres are subjected to elongation, is described by 601.64 mm $\leq r$ \leq 608.75 mm. Zone II, of compression, is described by 594.45 mm $\leq r \leq$ 601.56 mm, and zone III, of mixed stresses, delimited by 601.56 mm $\leq r \leq$ 601.64 mm, has a very little width, of 0.08 mm, i.e. 0.5% of h_0 .

If the material is hardenable, following the law: $\sigma_c = 826.22(\overline{\epsilon})^{0.15}$ then, assuming that the radii a_{fin} , b_{fin} and c_{fin} are those previously calculated, σ_r can be expressed by:

$$\sigma_{r}^{I}(r) = -\int_{r}^{b_{fm}} \frac{d\sigma_{r}}{dr'} dr' = -\int_{r}^{b_{fm}} \frac{\sigma_{r} - \sigma_{\theta}}{r'} dr' = -826.22 \int_{r}^{b_{fm}} \frac{1}{r'} \left(\ln \frac{r'}{r_{0}} \right)^{0.15} dr' = -\frac{826.22}{1.15} \int_{r}^{b_{fm}} \frac{d}{dr'} \left(\ln \frac{r'}{r_{0}} \right)^{1.15} = 718.45 \left[\left(\ln \frac{r}{r_{0}} \right)^{1.15} - \left(\ln \frac{b_{fm}}{r_{0}} \right)^{1.15} \right];$$

o in zone II
$$\sigma_{r}^{II}(r) = \int_{a_{fm}}^{r} \frac{d\sigma_{r}}{dr'} dr' = 718.45 \left[\left(\ln \frac{a_{fm}}{r_{0}} \right)^{1.15} - \left(\ln \frac{r}{r_{0}} \right)^{1.15} \right].$$

Now, we shall consider how the third zone dimension will vary as function of the plate length (L_0) and thickness (h_0) , for some examples given in figure 3 and table 1.



Fig.3. Third zone variation for different plates, $L_0 = 780-3780$ mm and $h_0 = 14.3$ and 25 mm

Tabl	e 1.	The	variation	of thi	rd zone	thickn	ess (III)) divide	d by	$h_{0,1}$	function	of L	$_0$ and	h_0
------	------	-----	-----------	--------	---------	--------	-----------	----------	------	-----------	----------	------	----------	-------

L_0 [mm]	h_0 [mm]	$(III/h_0)*100$			
3780	14.3	0.59			
2780	14.3	0.81			
1780	14.3	1.26			
780	14.3	2.88			
3780	25	1.04			
2780	25	1.41			
1780	25	2.21			
780	25	5.03			

From table 1 and figure 3 it is obvious that the third zone thickness increases with the plate length decreasing and the plate thickness increasing. So, the contribution of this zone, where the Bauschinger effect takes place, to the mechanical properties of rolled tubes becomes important for small diameter tubes and large plate thickness.

Conclusions

- 1. It is proposed a model for strains and stresses calculation during the heavy plates bending. The strains and stresses are calculated by considering two types of materials: a rigid-plastic one and a hardenable one.
- 2. The model allows the plate fibres movement description in every moment of the processing. So, it is described the evolution of three zones across the plate thickness: zone I in which the fibres are elongated, zone II where the fibres are subjected to a compression and zone III in which the fibres are subjected firstly to a compression, and afterwards to an elongation, in this zone, the Bauschinger effect can have a remarkable importance.
- 3. The third zone thickness increases with the plate length decreasing and the plate thickness increasing. So, the contribution of this zone, where the Bauschinger effect takes place, to the mechanical properties of rolled tubes becomes important for small diameter tubes and large thickness plate.

References

- 1. Hill, R. The mathematical theory of plasticity. Clarendon Press, Oxford, 1950.
- 2. Katchanov, L.-Elements de la theorie de la plasticite. Editions Mir, Moscou, 1975.
- 3. Verguts, H., Sowerby, R. The pure plastic bending of laminated sheet metals, *Int. J. Mech. Sci.*, **17**, 1975.

Modelarea formării la rece a tuburilor din table groase

Rezumat

În teoria clasică a încovoierii, se folosește de obicei ipoteza micilor deformații, suficient de reduse pentru a neglija tensiunile transversale induse de încovoierea severă. De asemenea, se consideră că suprafața neutră în timpul deformării coincide cu planul central al tablei. Pentru modelarea matematică a încovoierii tablelor groase destinate fabricării țevilor de diametre mari, se folosește teoria generală a încovoierii plăcilor fără restricții legate de mărimea deformațiilor și curburii, dar este necesară determinarea deplasării suprafeței neutre respectiv, a fiecărei fibre de-a lungul grosimii plăcii. Lucrarea prezintă modelul de calcul al tensiunilor și deformațiilor pentru un material rigid-plastic și pentru unul durificabil. Un capitol important al acestor calcule este constituit de determinarea zonei în care materialul, în timpul încovoierii, suferă o alungire urmată de compresiune și ca urmare efectul Bauschinger are o importantă influență asupra caracteristicilor mecanice ale țevii. După încovoiere, în această zonă, rezistența materialului este mai redusă decât rezistența globală a plăcii, reducerea rezistenței fiind proporțională cu lățimea zonei menționate.