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A New Interpretation of the Phasor Theory on Powers under Non-Sinusoidal Current

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Abstract

This paper is aimed to finding the corelation between the instantaneous complex powers components and the steady state powers under non-sinusoidal current (apparent power, active power, reactive power and distortion power). So, to ascertain that compensating of alternative parts of instantaneous active and reactive powers establishes a sinusoidal current in the parallel active filter using. The mathematical demonstration is found. Also, definitions for apparent power, distortion power and complex instantaneous distortion power are introduced. These definitions verify the orthogonal condition of active, reactive and distortion powers. The DC motor and full control rectifier driving system is used as study case.

Key words: *powers components, distortion powe, rectifier driving system.*

Introduction

For many years there has been a permanent debate within the international scientific community on the definition and interpretation of powers under non-sinusoidal conditions [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13]. The distortion power introduced by Budeanu in 1927 is particularly questioned.

Therefore, a special contribution was brought by the papers presented at the "International Workshops on Power Definitions and Measurement under Non-sinusoidal Conditions". One of the results of these meetings was the setting up, during the third conference that was held in Milan, of a working group formed by prestigious professors (L. S. Czarnecki, M. Depenbrock, A. E. Emanuel, A. Ferrero, R. Gretsch, P. Tenti, A. Ţugulea, J. D. van Wyk, J. L. Willems) with the purpose of analyzing the existing definitions of parameters under non-sinusoidal conditions and elaborating a single terminology in order to eliminate possible ambiguities.

The development of active filtering techniques has rendered topical the theory of instantaneous complex powers. This was first introduced, as a unitary concept, by V. Nedelcu [14], [15] and it was developed by other authors who used it for grounding certain active filtering techniques [16].

The theory of apparent instantaneous complex power and the active filtering

If $\underline{u}$ and $\underline{i}$ are the space phasors of the supply voltages of the distortion load and distorted three-phased current, defined in the following matrices [15]:

$$[u]=\begin{bmatrix}u_d\\ u_q\end{bmatrix}=\frac{2}{3}\begin{bmatrix}1 & -1/2 & -1/2\\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2}\end{bmatrix}\cdot\begin{bmatrix}u_a\\ u_b\\ u_c\end{bmatrix}=\frac{2}{3}[A]\cdot\begin{bmatrix}u_a\\ u_b\\ u_c\end{bmatrix}, \tag{1}$$

$$[i]=\frac{2}{3}[A]\cdot\begin{bmatrix}i_a\\ i_b\\ i_c\end{bmatrix}$$

the apparent instantaneous complex power results from

$$\underline{s}=p+jq=\frac{3}{2}\underline{u}\cdot\underline{i}=\frac{3}{2}\left[u_d i_d+u_q i_q+j\left(-u_d i_q+u_q i_d\right)\right]. \tag{2}$$

The direct and alternating components can be outlined in the instantaneous active and reactive powers, p and q:

$$\begin{aligned}p&=P+p\sim\\ q&=Q+q\sim\end{aligned}. \tag{3}$$

P and Q are the average values resulting from

$$\begin{aligned}P&=\frac{1}{T}\int_{t-T}^{t}pdt\\ Q&=\frac{1}{T}\int_{t-T}^{t}qdt\end{aligned}. \tag{4}$$

H. Akagi suggested the compensation of the alternating components, respectively the calculation of the reference currents in the active filter based on the relation resulting from developing the current in (2) [16], [17],

$$\underline{i}=i_d+ji_q=\frac{2}{3}\frac{1}{u_d^2+u_q^2}\underline{u}\cdot\underline{s^*}, \tag{5}$$

where $\underline{s}^*$ is the conjugate of $\underline{s}$. So, if $\underline{s_F}$ is the apparent instantaneous complex power corresponding to the active filter, the current absorbed by this is:

$$\underline{i}_F=i_{Fd}+ji_{Fq}=\frac{2}{3}\frac{1}{u_d^2+u_q^2}\underline{u}\cdot\underline{s^*}_F, \tag{6}$$

and the current absorbed from the network is

$$\underline{i}_s=\underline{i}+\underline{i}_F. \tag{7}$$

Proposing the compensation of the alternating components of the powers required by the distortion load, the active filter power is

$$\underline{s}_F=-p_\sim-jq_\sim. \tag{8}$$

DC motor and full control rectifier driving system – case study

If the power of the rectifier supply transformer is high enough, the voltage will not be significantly distorted by the thyristors commutation and will practically be sinusoidal. The current on one phase has the well-known form and outlines the effect of the limited value of the

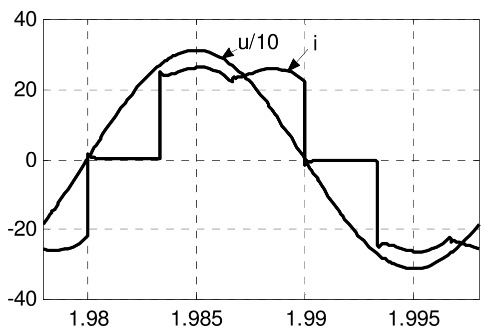

Fig. 1. The voltage and current on one phase of the rectifer.

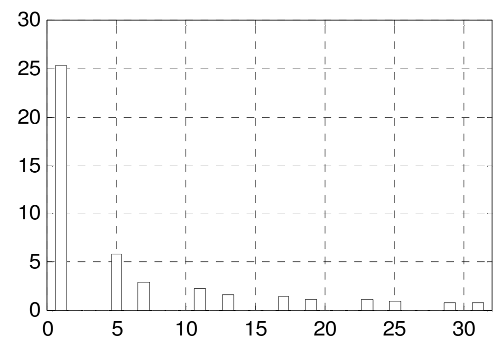

Fig. 2. The harmonics of the unfiltered rectifier current.

filtering inductivity (Fig. 1) and significantly contains $6n \pm 1$ order harmonics (Fig. 2).

There are two situations that have been studied for the calculation of the active filter currents, respectively the compensation of two apparent instantaneous powers [18], [19].

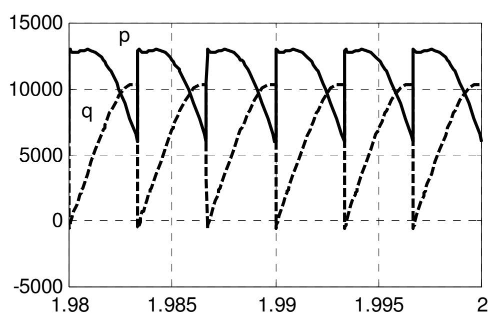

Fig. 3. The wave shapes of the instantaneous powers.

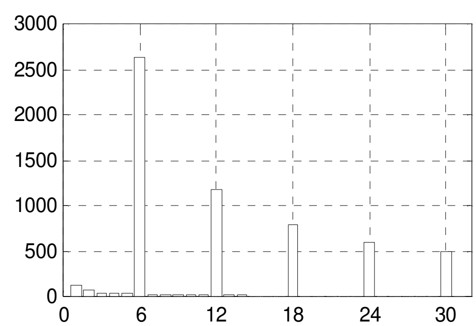

Fig. 4. The harmonics spectra of the instantaneous active power.

1. Only the alternating components of the real and imaginary parts are compensated,

$$s_{F1} = -p_{\sim} - jq_{\sim} \tag{9}$$

2. The alternating component of the real part and the entire imaginary part are compensated,

$$s_{F1} = -p_{\sim} - jq \tag{10}$$

The wave shapes of the instantaneous active and reactive powers are presented in Figure 3 and their alternating components significantly contain the 6n-order harmonics (Fig. 4, Fig. 5).

In case of the alternating components compensation, the filtered current is sinusoidal and has the same phase as the load current fundamental, respectively its phase is delayed relative to the voltage, by the control angle (30^0), (Fig. 6). The superior harmonics spectra of the unfiltered and filter currents are identical and reflect a very good filtering (Fig. 7, Fig. 8).

In case of the compensation of the instantaneous active power alternating components and of the entire instantaneous reactive power, the filtered current is also sinusoidal and has the same phase as the voltage (Fig. 9), and the current delivered by the active filter has, besides the harmonics of the previous case, an important fundamental component (Fig. 10).

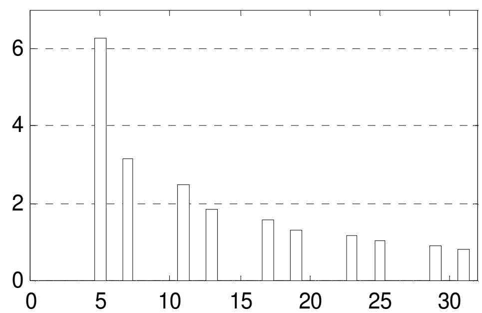

Fig. 7. The unfiltered rectifier current harmonics.

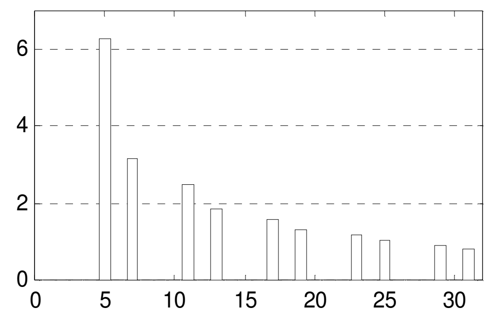

Fig. 8. The filter current harmonics in the case of alternating components compensation.

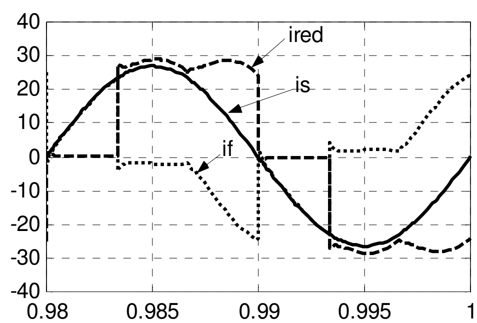

Fig. 9. Rectifier, active filter and network currents on one phase in the case of the compensation of entire reactive power and alternating component of active power.

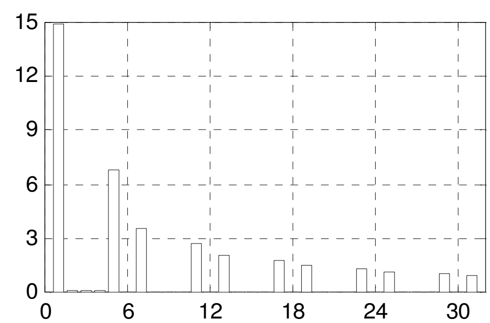

Fig. 10. The filter current harmonics in the case of the compensation of the entire instantaneous reactive power.

The following conclusions are drawn regarding the instantaneous powers:

1. The alternating components of the instantaneous active and reactive powers directly contribute to the current wave distortion and therefore it represents a direct measurement of the fluctuating power;
2. The average value of the instantaneous reactive power represents the reactive power.

Mathematical description

In case of a parallel active filter, the current absorbed from the network results as the sum of the load and filter currents (7), and considering the dependences of the currents on the instantaneous complex power (5), (6) the network current phasor is obtained for the two compensation methods

$$\underline{i}_{s1} = \frac{2}{3|u|^2}\underline{u} \cdot (P - jQ), \tag{11}$$

$$\underline{i}_{s2} = \frac{2}{3|u|^2}\underline{u} \cdot P. \tag{12}$$

If the voltage system is symmetrical and balanced sinusoidal, the space phasor components and the square of its modulus are:

$$\begin{aligned} u_d &= \sqrt{2}U \sin(\omega t + \alpha) \\ u_q &= -\sqrt{2}U \cos(\omega t + \alpha). \\ |u|^2 &= 2U^2 \end{aligned} \tag{13}$$

Making the replacements in (11), the following expressions are obtained:

$$\underline{i}_1 = \frac{\sqrt{2}}{3U}\sqrt{2}[\sin(\omega t + \alpha) + j\cos(\omega t + \alpha)] \cdot (P - jQ). \tag{14}$$

The a-phase current is the real part

$$i_a = \frac{\sqrt{2}}{3U}\sqrt{P^2 + Q^2}\sin(\omega t + \alpha - \beta), \tag{15}$$

where $\beta = arctg\frac{Q}{P}$.

The expression (15) shows that, by the compensation of the instantaneous active and reactive powers alternating components, all the current harmonics are compensated and the current is sinusoidal but its phase is delayed relative to the voltage by a β angle.

Similarly, in the case of compensating the entire instantaneous reactive power, the network a-phase current is sinusoidal and has the same phase as the voltage

$$i_a = \frac{\sqrt{2}P}{3U}\sin(\omega t + \alpha). \tag{16}$$

Harmonic expressions of the instantaneous powers

If the load is distorted but symmetrical and balanced, the currents won't have a direct component and will contain N harmonics, and the space phasor components will be

$$\begin{aligned} i_d &= \sqrt{2}\sum_{k=1}^{N} I_k \sin(k\omega t + \beta_k) \\ i_q &= -\sqrt{2}\sum_{k=1}^{N} I_k \cos(k\omega t + \beta_k) \end{aligned}. \tag{17}$$

Using relation (15) and (17), the instantaneous active power is obtained

$$p = 3U\sum_{k=1}^{N} I_k \left[\sin(\omega t + \alpha)\cdot \sin(k\omega t + \beta_k) + \cos(\omega t + \alpha)\cdot \cos(k\omega t + \beta_k)\right] =$$
$$= 3U\sum_{k=1}^{N} I_k \cos\left[(k-1)\omega t + \beta_k - \alpha\right] = 3UI_1 \cos(\beta_1 - \alpha) + \tag{18}$$
$$+ 3U\sum_{k=2}^{N} I_k \cos\left[(k-1)\omega t + \beta_k - \alpha\right]$$

Similarly the instantaneous reactive power is obtained

$$q = -3UI_1 \sin(\beta_1 - \alpha) - 3U\sum_{k=2}^{N} I_k \sin\left[(k-1)\omega t + \beta_k - \alpha\right]. \tag{19}$$

The square of the instantaneous complex power modulus will be

$$|s|^2 = p^2 + q^2 = |u|^2 |i|^2 = 9U^2\left[\sum_{k=1}^{N} I_k^2 + 2\sum_{k=1}^{N-1}\sum_{i=k+1}^{N} I_k I_i \cos\left[(i-k)\omega t + \beta_i - \beta_k\right]\right] \tag{20}$$

Using the above expressions, the powers present in the specialized literature can be computed

1. Theactive power

$$P = \frac{1}{2\pi}\int_0^{2\pi} p d(\omega t) = 3UI_1 \cos\phi_1 \; ; \tag{21}$$

2. Theaverage value of the instantaneous reactive power

$$Q = \frac{1}{2\pi}\int_0^{2\pi} q d(\omega t) = 3UI_1 \sin\phi_1 \; . \tag{22}$$

The relation obtained for the reactive power coincides with the relation directly introduced in the harmonic theory of powers.

In the active and reactive powers expressions, the angle ϕ_1 represents the phase difference between the voltage and the current fundamental,

$$\phi_1 = \alpha - \beta_1$$

3. Theeffective value of the instantaneous complex power modulus is

$$\sqrt{\frac{1}{2\pi}\int_0^{2\pi} |s|^2 d(\omega t) = 9U^2 \frac{1}{2\pi}\int_0^{2\pi}\left[\sum_{k=1}^{N} I_k^2 + 2\sum_{k=1}^{N-1}\sum_{i=k+1}^{N} I_k I_i \cos\left[(i-k)\omega t + \beta_i - \beta_k\right]\right] d(\omega t)} =$$
$$= \sqrt{9U^2 \sum_{k=1}^{N} I_k^2} = 3UI = S \tag{23}$$

because for the terms in the double sum $i \neq k$, and the integrals are zero..

It is noticed that the effective value of the instantaneous complex power modulus is the apparent power itself, and its expression has been introduced to the specialized literature as definition.

If in the square of the instantaneous complex power modulus the instantaneous powers are separated by components

$$|s|^2 = p^2 + q^2 = (P + p_\sim)^2 + (Q + q_\sim)^2 = P^2 + Q^2 + p_\sim^2 + q_\sim^2 + 2(Pp_\sim + Qq_\sim), \quad (24)$$

and the square of the effective value is calculated, the following relation is obtained

$$S^2 = P^2 + Q^2 + \frac{1}{2\pi}\int_0^{2\pi}\left(p_\sim^2 + q_\sim^2\right)d(\omega t), \quad (25)$$

because,

$$\int_0^{2\pi} 2(Pp_\sim + Qq_\sim)d(\omega t) = 0. \quad (26)$$

Then, at least in the case when the voltage is not distorted, the relation introduced by C Budeanu is correct and, in the phasor theory on power, the distortion power is represented by the average value of the sum of the squares of the instantaneous active and reactive powers alternative components,

$$D^2 = \frac{1}{2\pi}\int_0^{2\pi}\left(p_\sim^2 + q_\sim^2\right)d(\omega t). \quad (27)$$

Conclusions

The analysis performed for the case when the supply voltage of the distortion load is sinusoidal allows the outlining of new issues and brings new arguments in favor of the distortion power relation introduced by C Budeanu.

1. Because, by compensating the alternating components of the instantaneous active and reactive powers, the current becomes sinusoidal, the conclusion is that these contribute directly to the current distortion. Therefore, they represent the direct measurement of the current distortion degree.

2. By expressing the active and reactive powers as the average values of the instantaneous powers, the complex powers can be defined.

- The instantaneous distortion complex power,

$$\underline{d} = p_\sim + jq_\sim ; \quad (28)$$

- The average apparent complex power,

$$\underline{S}_m = P + jQ \quad (29)$$

3. The apparent instantaneous complex power is the sum of the two powers,

$$\underline{s} = \underline{S}_m + \underline{d} . \quad (30)$$

4. The apparent power is defined as the effective value of the apparent instantaneous complex power modulus, thus resulting the relation introduced as definition by the specialized literature.

5. The distortion power is defined as the effective value of the instantaneous distortion complex power modulus (27).

6. Using the above definition the relation $S^2 = P^2 + Q^2 + D^2$ is found.

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O nouă interpretare a teoriei fazoriale a puterilor în curent nesinusoidal

Rezumat

Lucrarea şi-a propus găsirea unor relaţii între componentele puterii complexe instantanee în regim ne sinusoidal şi puterile aparentă, activă, reactivă şi deformantă. Se arată, prin simulare şi analitic, că, în cazul unui filtru activ paralel, prin compensarea componentelor alternative ale puterilor instantanee activă şi reactivă, curentul absorbit de la reţea devine sinusoidal. De asemenea, se propun definiţii pentru puterea aparentă, puterea deformantă şi puterea deformantă complexă instantanee şi se regăsesc expresiile cunoscute, în funcţie de componentele armonice. Aceste definiţii verifică şi condiţia de ortogonalitate a puterilor introdusă de C. Budeanu. Un sistem de acţionare cu motor de c.c. şi redresor trifazat complet comandat este utilizat ca şi studiu de caz.