BULETINUL Universităţii Petrol – Gaze din Ploieşti	Vol. LVIII No. 2 bis / 2006	71-77	Seria Tehnică

A multiphase model of induction motor with broken bars and end rings rotors

Kostas Gavros*, Mihai Priboianu**, Alexandru Fransua**, Răzvan Măgureanu**

* TEI Kozani, Greece,
** University Politehnica Bucharest, Romania

Abstract

In the paper, a new approach is proposed to model the broken rotor bars and rotor rings in induction motors. If there is any broken bar or broken ring, it will directly affect the stator current and motor torque by a certain content of harmonics. Interpreting different results for different model parameters allows building an expert system that can be used for real time monitoring and diagnosis.

Key words: *Fault diagnosis, induction machine, rotor-cage faults, motor modeling.*

Introduction

The induction motors operate frequently in highly corrosive and dusty environments. As a result, rotor failures account for a large percentage of total induction motor failures. The reasons for rotor bar and end-ring breakage can be caused by thermal stresses due to thermal overload and unbalance, hot spots or excessive losses, magnetic stresses caused by electromagnetic forces, unbalanced magnetic pull, electromagnetic noise and vibration, residual stresses due to manufacturing problems, dynamic stresses arising from shaft torques, centrifugal forces and cyclic stresses, environmental stresses caused, for example, by contamination and abrasion of rotor material due to chemicals or moisture, mechanical stresses due to loose laminations, fatigued parts, bearing failure, etc.[1].

The modeling of the induction machine is done by treating the current in each rotor bar loop as an independent variable. The model incorporates sinusoidal air-gap MMF, magnetic saturation of the machine is neglected, eddy-current, are not included there is a uniform air gap, and there are uniformly distributed insulated cage bars. The cage rotor can be viewed as identical and equally spaced rotor bars shorted at the two ends by two identical end rings. The rotor loop currents are coupled to each other and to the stator windings through mutual inductances. However, the end-ring loop current does not couple with the stator windings. It couples the rotor loop currents only through the end-ring leakage inductance and the end-ring resistance (Fig. 1, 2).

The motor equations

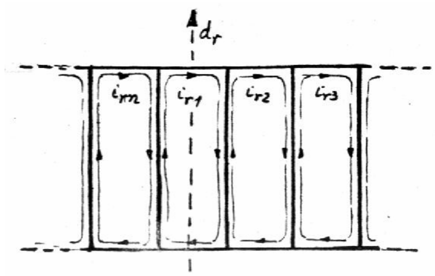


Fig. 1. The squirrel cage rotor model

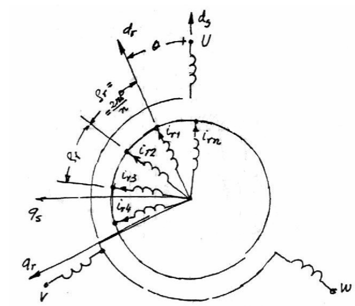


Fig. 2. The squirrel cage motor model

For the stator with the classical notations, one can write:

$$
\begin{aligned}
u_U &= R_1 i_U + L_{\sigma 1}\frac{\mathrm{d}i_U}{\mathrm{d}t} + \frac{\mathrm{d}\psi_U}{\mathrm{d}t} \\
u_V &= R_1 i_V + L_{\sigma 1}\frac{\mathrm{d}i_V}{\mathrm{d}t} + \frac{\mathrm{d}\psi_V}{\mathrm{d}t} \\
u_W &= R_1 i_W + L_{\sigma 1}\frac{\mathrm{d}i_W}{\mathrm{d}t} + \frac{\mathrm{d}\psi_W}{\mathrm{d}t} \\
i_U &+ i_V + i_W = 0
\end{aligned}
\tag{1}
$$

By defining the new variables, one can write:

$$
\begin{aligned}
u_{ds} &= u_U; \qquad u_{qs} = \left(u_V - u_W\right)/\sqrt{3} \qquad & u_{ds} &= R_1 i_{ds} + L_{\sigma 1}\frac{\mathrm{d}i_{ds}}{\mathrm{d}t} + \frac{\mathrm{d}\psi_{ds}}{\mathrm{d}t} \\
i_{ds} &= i_U; \qquad i_{qs} = \left(i_V - i_W\right)/\sqrt{3} & & \\
\psi_{ds} &= \psi_U; \qquad \psi_{qs} = \left(\psi_V - \psi_W\right)/\sqrt{3} & u_{qs} &= R_1 i_{qs} + L_{\sigma 1}\frac{\mathrm{d}i_{qs}}{\mathrm{d}t} + \frac{\mathrm{d}\psi_{qs}}{\mathrm{d}t}
\end{aligned}
\tag{2}
$$

Defining the complex variables:

$$\underline{u}_{ss} = u_{ds} + \mathrm{j}u_{qs};\quad \underline{i}_{ss} = i_{ds} + \mathrm{j}i_{qs};\quad \underline{\psi}_{ss} = \psi_{ds} + \mathrm{j}\,\psi_{qs};\ u_{ds} = \sqrt{2}U\cos\omega t;\ u_{qs} = \sqrt{2}U\sin\omega t.$$

in a complex form in stator reference the stator equation for a five bar rotor is:

$$\underline{u}_{ss} = R_1\underline{i}_{ss} + L_{\sigma 1}\frac{\mathrm{d}\underline{i}_{ss}}{\mathrm{d}t} + \frac{\mathrm{d}\underline{\psi}_{ss}}{\mathrm{d}t} \tag{3}$$

where

$$\psi_{ds} = L_{m1}i_{ds} + L_{m12}[i_{r1}\cos\theta + i_{r2}\cos(\theta+\rho) + i_{r3}\cos(\theta+2\rho) + i_{r4}\cos(\theta+3\rho) + i_{r5}\cos(\theta+4\rho)]$$

$$\psi_{qs} = L_{m1}i_{qs} + L_{m12}[i_{r1}\sin\theta + i_{r2}\sin(\theta+\rho) + i_{r3}\sin(\theta+2\rho) + i_{r4}\sin(\theta+3\rho) + i_{r5}\sin(\theta+4\rho)]$$

As the rotor becomes asymmetrical when the bars or rings are broken, the best solution is to write the motor equations in rotor frames:

$$\underline{u}_{sr} = \underline{u}_{ss}\, e^{-\mathrm{j}\theta};\qquad \underline{i}_{sr} = \underline{i}_{ss}\, e^{-\mathrm{j}\theta};\qquad \underline{\psi}_{sr} = \underline{\psi}_{ss}\, e^{-\mathrm{j}\theta};\qquad \underline{u}_{sr} = u_{dr} + \mathrm{j}u_{qr} \tag{4}$$

$$\underline{u}_{sr} = R_1\underline{i}_{sr} + \left(\frac{\mathrm{d}}{\mathrm{d}t} + \mathrm{j}\dot{\theta}\right)\left(L_{\sigma 1}\underline{i}_{sr} + \underline{\psi}_{sr}\right) \tag{5}$$

$$\psi_{dr} = \psi_{ds}\cos\theta + \psi_{qs}\sin\theta = L_{m1}i_{ds} + L_{m12}(i_{r1} + i_{r2}\cos\rho + i_{r3}\cos 2\rho + i_{r4}\cos 3\rho + i_{r5}\cos 4\rho)$$

$$\psi_{qr} = -\psi_{ds}\sin\theta + \psi_{qs}\cos\theta = L_{m1}i_{qr} + L_{m12}(i_{r2}\sin\rho + i_{r3}\sin 2\rho + i_{r4}\sin 3\rho + i_{r5}\sin 4\rho)$$

For the rotor, for the loop 1, one can write:

$$0=(2R_b+2R_i)\,i_{r1}-R_b i_{r2}-R_b i_{r5}+(2L_{\sigma b}+2L_{\sigma i})\frac{\mathrm{d}i_{r1}}{\mathrm{dt}}-L_{\sigma b}\frac{\mathrm{d}i_{r2}}{\mathrm{dt}}-L_{\sigma b}\frac{\mathrm{d}i_{r5}}{\mathrm{dt}}+\frac{L_{m12}}{L_{m1}}\frac{\mathrm{d}\psi_{dr}}{\mathrm{dt}}$$

For loop 2:

$$0=-R_b i_{r1}+(2R_b+2R_i)\,i_{r2}-R_b i_{r3}+L_{\sigma b}\frac{\mathrm{d}i_{r1}}{\mathrm{dt}}+(2L_{\sigma b}+2L_{\sigma i})\frac{\mathrm{d}i_{r2}}{\mathrm{dt}}-L_{\sigma b}\frac{\mathrm{d}i_{r3}}{\mathrm{dt}}+\frac{L_{m12}}{L_{m1}}\frac{\mathrm{d}}{\mathrm{dt}}\left(\psi_{dr}\cos\rho+\psi_{qr}\sin\rho\right)$$

+ … loop 3, 4, 5… where ψ_{dr} and ψ_{qr} are previously defined. In a matrix form, the motor equations are presented bellow.

$$\boldsymbol{u}=\boldsymbol{C}\cdot\boldsymbol{x}+\boldsymbol{D}\cdot\frac{\mathrm{d}}{\mathrm{dt}}\boldsymbol{x}\,,\quad \text{where } \boldsymbol{u},\boldsymbol{i}\in\mathbb{R}^{7},\ \boldsymbol{C},\boldsymbol{D},\boldsymbol{A} \text{ and } \boldsymbol{B}\in\mathbb{R}^{7x7} \tag{6}$$

$$\boldsymbol{u}=\boldsymbol{C}\cdot\boldsymbol{x}+\boldsymbol{D}\cdot\dot{\boldsymbol{x}} \text{ or } \dot{\boldsymbol{x}}=-\boldsymbol{D}^{-1}\boldsymbol{C}\,\boldsymbol{x}+\boldsymbol{D}^{-1}\boldsymbol{u}=\boldsymbol{A}\,\boldsymbol{x}+\boldsymbol{B}\,\boldsymbol{u} \tag{7}$$

For constant speed and the slip *s*, for the voltage vector in rotor coordinates is:

$$\boldsymbol{u}=\left[U\sqrt{2}\cos s\omega_1 t\quad U\sqrt{2}\sin s\omega_1 t\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\right]^{\mathrm{T}}=\left[\boldsymbol{u}_{sr}{}^{\mathrm{T}}\boldsymbol{0}^{\mathrm{T}}\right]^{\mathrm{T}} \tag{8}$$

where $\boldsymbol{u}_{sr}\in\mathbb{R}^{2}$ and $\boldsymbol{0}\in\mathbb{R}^{5}$.

The particular form of the voltage vector allows simplifying the integration process.

$$\dot{\boldsymbol{u}}=\sqrt{2}Us\omega_1\left[-\sin s\omega_1 t\quad \cos s\omega_1 t\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\right]^{\mathrm{T}}=\sqrt{2}Us\omega_1\left[\boldsymbol{u}_{sr}\boldsymbol{0}\right]^{\mathrm{T}} \tag{9}$$

$$\frac{\mathrm{d}}{\mathrm{dt}}\begin{bmatrix}u_{dr}\\ u_{qr}\end{bmatrix}=\begin{bmatrix}0 & -s\omega_1\\ s\omega_1 & 0\end{bmatrix}\cdot\begin{bmatrix}u_{dr}\\ u_{qr}\end{bmatrix}=\boldsymbol{E}\,\boldsymbol{u}_{sr} \tag{10}$$

Due to this property, equation (7), $\begin{bmatrix}\boldsymbol{i}\\ \boldsymbol{u}_{sr}\end{bmatrix}=\boldsymbol{x}_e\in\mathbb{R}^{9}$ the extended states:

$$\text{and}\begin{bmatrix}\boldsymbol{A} & \boldsymbol{B}\\ \boldsymbol{0} & \boldsymbol{E}\end{bmatrix}=\boldsymbol{A}_e\in\mathbb{R}^{9x9};\qquad \frac{\mathrm{d}}{\mathrm{dt}}\begin{bmatrix}\boldsymbol{i}\\ \boldsymbol{u}_{sr}\end{bmatrix}=\begin{bmatrix}\boldsymbol{A} & \boldsymbol{B}\\ \boldsymbol{0} & \boldsymbol{E}\end{bmatrix}\begin{bmatrix}\boldsymbol{i}\\ \boldsymbol{u}_{sr}\end{bmatrix}\qquad \dot{\boldsymbol{x}}=\boldsymbol{A}_e\boldsymbol{x} \tag{11}$$

where the significances of the matrices are evident. This relation represents, for constant speed, the standard form of the equations that characterized a linear control system in state variables. It should be remarked that the input functions are included in the column vector $\boldsymbol{x}_e$, the new extended system state variables.

The solution of equation (11) for known initial conditions $\boldsymbol{x}(0)$ for any point in the interval $0\le s\omega_1 t\le 2\pi$ is $\boldsymbol{x}\left(s\omega_1 t\right)=e^{s\omega_1 t A_e}\boldsymbol{x}\left(0\right)$.

$$
\underbrace{\begin{bmatrix} u_{dr} \\ u_{qr} \\ \\ \\ \\ \\ \\ \end{bmatrix}}_{\boldsymbol{u}}
=
\underbrace{\begin{bmatrix}
R_1 & \dot{\theta}(L_{\sigma 1}+L_{m1}) & & -\dot{\theta} L_{m12}\sin\rho & -\dot{\theta} L_{m12}\sin 2\rho & -\dot{\theta} L_{m12}\sin 3\rho & -\dot{\theta} L_{m12}\sin 4\rho \\
\dot{\theta}(L_{\sigma 1}+L_{m1}) & R_1 & \dot{\theta} L_{m12} & \dot{\theta} L_{m12}\cos\rho & \dot{\theta} L_{m12}\cos 2\rho & \dot{\theta} L_{m12}\cos 3\rho & \dot{\theta} L_{m12}\cos 4\rho \\
 & & 2R_b+2R_i & -R_b & & & -R_b \\
 & & -R_b & 2R_b+2R_i & -R_b & & \\
 & & & -R_b & 2R_b+2R_i & -R_b & \\
 & & & & -R_b & 2R_b+2R_i & -R_b \\
 & & -R_b & & & -R_b & 2R_b+2R_i
\end{bmatrix}}_{\boldsymbol{C}}
\cdot
\underbrace{\begin{bmatrix} i_{dr} \\ i_{qr} \\ i_{r1} \\ i_{r2} \\ i_{r3} \\ i_{r4} \\ i_{r5} \end{bmatrix}}_{\boldsymbol{x}}
$$

$$
+
\underbrace{\begin{bmatrix}
L_{\sigma 1}+L_{m1} & & L_{m12} & L_{m12}\cos\rho & L_{m12}\cos 2\rho & L_{m12}\cos 3\rho & L_{m12}\cos 4\rho \\
 & L_{\sigma 1}+L_{m1} & & L_{m12}\sin\rho & L_{m12}\sin 2\rho & L_{m12}\sin 3\rho & L_{m12}\sin 4\rho \\
L_{m12}\cos\varepsilon & L_{m12}\sin\varepsilon & L_{m2}+2L_{\sigma b}+L_{\sigma i} & L_{m2}\cos\rho-L_{\sigma b} & L_{m2}\cos 2\rho & L_{m2}\cos 3\rho & L_{m2}\cos 4\rho-L_{\sigma b} \\
L_{m12}\cos\rho & L_{m12}\sin\rho & L_{m2}\cos 4\rho-L_{\sigma b} & L_{m2}+2L_{\sigma b}+L_{\sigma i} & L_{m2}\cos\rho-L_{\sigma b} & L_{m2}\cos 2\rho & L_{m2}\cos 3\rho \\
L_{m12}\cos 2\rho & L_{m12}\sin 2\rho & L_{m2}\cos 3\rho & L_{m2}\cos 4\rho-L_{\sigma b} & L_{m2}+2L_{\sigma b}+L_{\sigma i} & L_{m2}\cos\rho-L_{\sigma b} & L_{m2}\cos 2\rho \\
L_{m12}\cos 3\rho & L_{m12}\sin 3\rho & L_{m2}\cos 2\rho & L_{m2}\cos 3\rho & L_{m2}\cos 4\rho-L_{\sigma b} & L_{m2}+2L_{\sigma b}+L_{\sigma i} & L_{m2}\cos\rho-L_{\sigma b} \\
L_{m12}\cos 4\rho & L_{m12}\sin 4\rho & L_{m2}\cos\rho-L_{\sigma b} & L_{m2}\cos 2\rho & L_{m2}\cos 3\rho & L_{m2}\cos 4\rho-L_{\sigma b} & L_{m2}+2L_{\sigma b}+L_{\sigma i}
\end{bmatrix}}_{\boldsymbol{D}}
\cdot\frac{\mathrm{d}}{\mathrm{d}t}
\underbrace{\begin{bmatrix} i_{dr} \\ i_{qr} \\ i_{r1} \\ i_{r2} \\ i_{r3} \\ i_{r4} \\ i_{r5} \end{bmatrix}}_{\boldsymbol{x}}
$$

This relation can also be used for the determination of the initial conditions. The permanent regime being a succession of periodic transitory regimes, the initial condition should be recovered at the end of that period: $\boldsymbol{i}(2\pi)=\boldsymbol{i}(0)$ and the final transformed voltages can be linked by the initial values through a relation:

$$\boldsymbol{u}_{sr}(2\pi)=\boldsymbol{u}_{sr}(0) \text{ or, in a compact form:} \begin{bmatrix} \boldsymbol{i}(2\pi) \\ \boldsymbol{u}_{sr}(2\pi) \end{bmatrix} = e^{2\pi \boldsymbol{A}_e} \cdot \begin{bmatrix} \boldsymbol{i}(0) \\ \boldsymbol{u}_{sr}(0) \end{bmatrix} = \boldsymbol{G}\,\boldsymbol{x}(0) \qquad (12)$$

The final $\boldsymbol{x}$ value can be found from (12) as:

$$\boldsymbol{x}(2\pi)=e^{2\pi \boldsymbol{A}_e}\cdot \boldsymbol{x}(0) \qquad (13)$$

or, replacing in the equality (12):

$$\left(\boldsymbol{I}-e^{2\pi \boldsymbol{A}_e}\right)\boldsymbol{x}(0)=\boldsymbol{0} \text{ or } \begin{bmatrix} \boldsymbol{w}_1 & \boldsymbol{w}_2 \\ \boldsymbol{w}_3 & \boldsymbol{w}_4 \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{i}(0) \\ \boldsymbol{u}_{sr}(0) \end{bmatrix} = 0 \qquad (14)$$

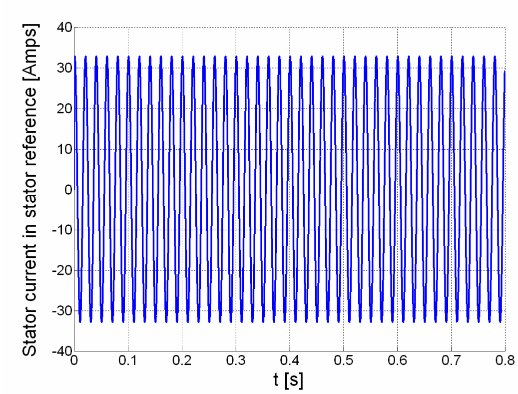
where $\boldsymbol{w}=\boldsymbol{I}-e^{2\pi \boldsymbol{A}_e}$ is a 9 x 9 matrix, and $\boldsymbol{w}_1, \boldsymbol{w}_2, \boldsymbol{w}_3, \boldsymbol{w}_4$ are sub-matrices with 7 x 7, 4 x 2, 2 x 4 and 2 x 2 elements, respectively. It can easily be seen that $[w_3]$ is an identical null matrix, and the product:

$$\boldsymbol{w}_4 \cdot \boldsymbol{u}_{sr}(0)=\boldsymbol{0}, \qquad (15)$$

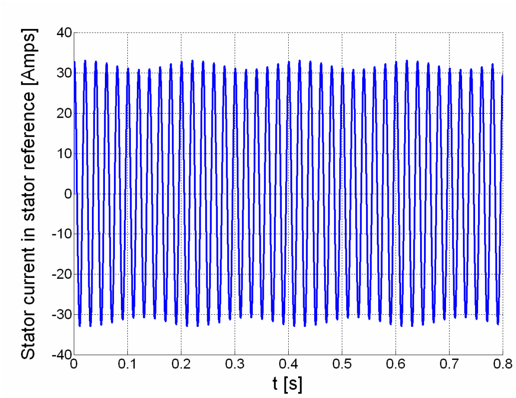
is identically satisfied, representing relations between the initial values of the transformed voltages, and relation (15) becomes:

$$\boldsymbol{i}(0)=-\boldsymbol{w}_1^{-1}\cdot \boldsymbol{w}_2 \cdot \boldsymbol{u}_{sr}(0). \qquad (16)$$

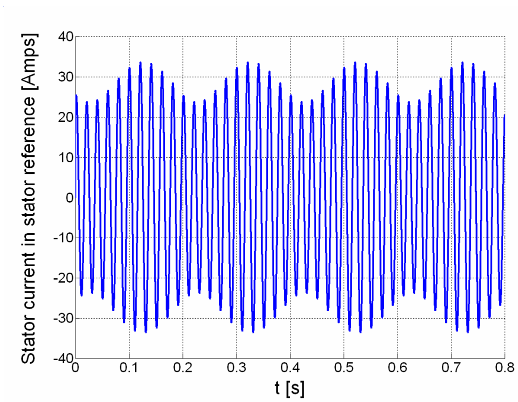
Knowing the values for $\boldsymbol{u}_{sr}(0)$ the values of the initial transformed currents $\boldsymbol{i}(0)$ can be obtained by simple algebraic means. To evaluate the function of matrix $e^{s\omega_1 T \boldsymbol{A}_e}$ one can use the Matlab® program. To solve this problem, we need to compute only once the function $e^{\boldsymbol{A}_e s\omega_1 T/n_2}$, $\boldsymbol{x}$ for different times being calculated through a recursive method. Extending these equations for a motor with 21 bars, $\rho=2\pi/21$ one can obtain the following results presented in figures 3, 4, 5 and 6. The calculations were done for a slip of 5% (exaggerated in order to emphasize the effect of rotor asymmetries). The broken bars were modeled by 1000 increase of the specified bar electrical resistance and similar for the broken end ring [2, 3].



Undamaged rotor

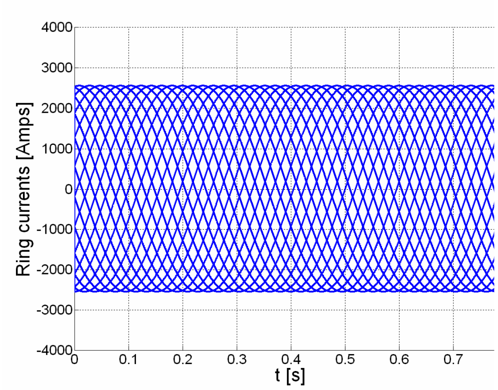


Broken bar

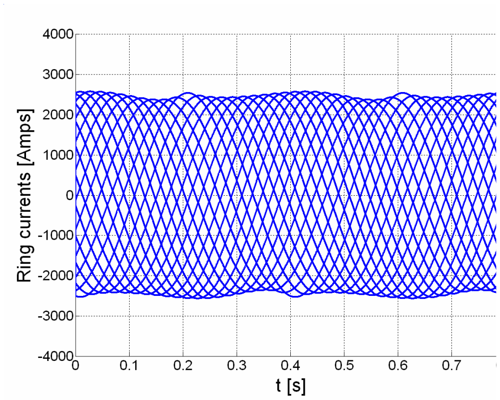


Broken ring

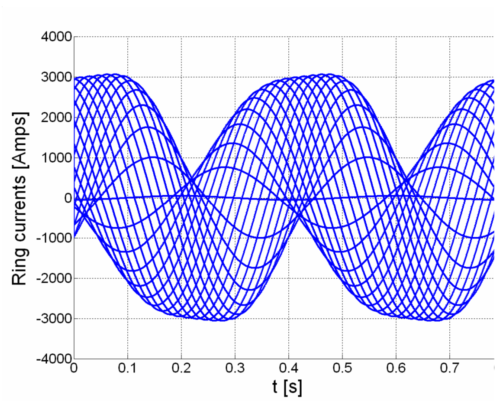
Fig. 3. Stator currents



Undamaged rotor

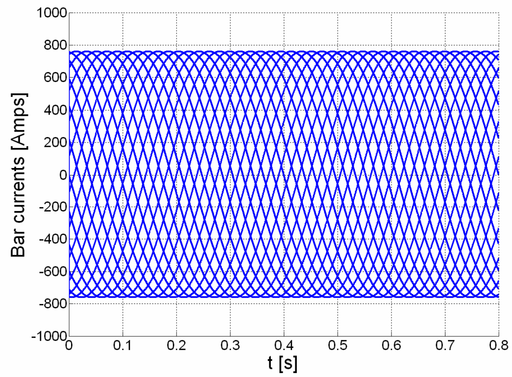


Broken bar

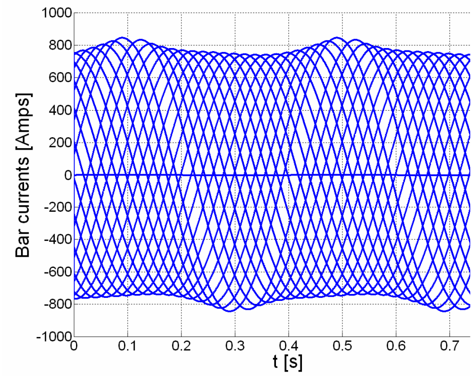


Broken ring

Fig. 4. Ring currents (loop currents)



a) Undamaged rotor



b) Broken bar

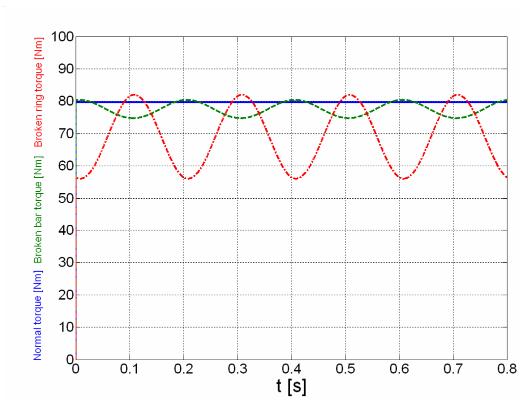
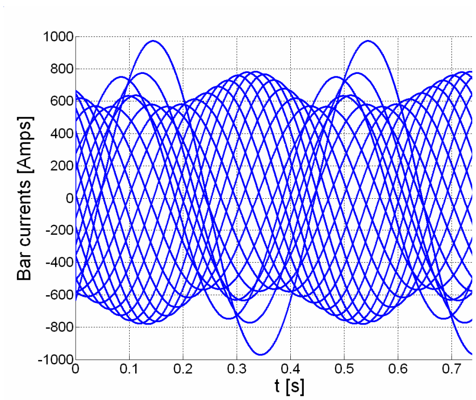


Fig. 5. Electromagnetic torque



c) Broken ring

Fig. 6. Rotor bar currents

Conclusions

The results presented above allow the representation and understanding of the rotor currents in and the effects on the stator currents and electromagnetic torque. A real time monitoring will permit to diagnose the motor health and motor defects [4].

References

1. Constantinescu-Simon, L., Fransua, A., Saal, K., Elektrische Machinen und Antriebssysteme, *Vieweg, Braunschweig*, 1999.
2. Milimonfared, J., Kelk, H. M., Nandi, S., Minassians, A. D., Toliyat, H. A., *A* Novel Approach for Broken-Rotor-Bar Detection in Cage Induction Motors, *IEEE Transactions On Industry Applications*, Vol. 35, No. 5, September/October 1999.
3. Bonnett, A. H., Soukup, G. C., Rotor failures in squirrel cage induction motors, *IEEE Transactions On Industry Applications.*, vol. IA-22, pp. 1165–1173, Nov./Dec. 1986.
4. Douglas, H., Pillay, P., Ziarani, A. K., Broken Rotor Bar Detection in Induction Machines With Transient Operating Speeds, *IEEE Transactions on Energy Conversion*, vol. 20, no. 1, march 2005