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Energetic Optimization with Arbitrary Terminal Moment of Electric Drives Acceleration for Static Torque with Constant Component and Speed Proportional Component

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Abstract

In the paper we consider an electric drive having static load torque with a constant component and a speed proportional component, in the hypothesis of constant inertia moment and of proportionality between the electromagnetic torque and the load current. Using variational calculus, optimally condition and expression of optimal control and extremal trajectory are determined, which ensures the minimum of energy losses caused by the load current through a Joule effect in the acceleration processes. Using numerical computer we can obtain graphical representation of these variables as time functions.

Key words: *electric drives, variational calculus, optimal control.*

Introduction

In the case of drives that work in continuous type service (S1), appears the necessity of achieving starting and braking processes, and in the case of those that work in uninterrupted type service with periodical change of speed (S8), appears the necessity of achieving speed variations. To asses these processes of acceleration and deceleration, the minimization of energy loss may be considered as a quality index, and the solving of this optimization problem can be obtained by using the classical variational calculus or the Euler – Lagrange algorithm and numerical computer.

Mathematical model

Considering an electrical drive load with static torque, having a constant component and a speed proportional component

$$M_S = M_0 + k_1\omega \tag{1}$$

that, in the hypothesis of ignoring the electromagnetic inertia in comparison to the mechanical inertia and a constant moment of inertia this driving system will be described by the general motion equation [2]

$$\mathrm{M} = \mathrm{M_S} + \mathrm{J}\frac{\mathrm{d}\omega}{\mathrm{dt}} \tag{2}$$

and by the dependence between speed and acceleration

$$\omega = \int \dot{\omega}\,\mathrm{dt}\,. \tag{3}$$

To extend the interpretations and the conclusions as to restraint the value intervals, relative coordinates will be used [7]. This way, considering as a reference for time, the mechanical time constant

$$\mathrm{T} = \frac{\mathrm{J}\omega_\mathrm{N}}{\mathrm{M_N}} \tag{4}$$

and for current, torque and speed their nominal values, relative values are obtained

$$\tau = \frac{\mathrm{t}}{\mathrm{T}}, \quad i = \frac{\mathrm{i}}{\mathrm{I_N}}, \quad \mu = \frac{\mathrm{M}}{\mathrm{M_N}}, \quad \mu_s = \frac{\mathrm{M_S}}{\mathrm{M_N}}, \quad \mu_0 = \frac{\mathrm{M_0}}{\mathrm{M_N}}, \quad k_1 = \frac{\mathrm{k_1}\omega_\mathrm{N}}{\mathrm{M_N}}, \quad \nu = \frac{\omega}{\omega_\mathrm{N}}, \tag{5}$$

resulting for the relative acceleration the following relation

$$\dot{\nu} = \frac{\dot{\omega}}{\omega_\mathrm{N}/\mathrm{T}} \tag{6}$$

In the hypothesis of proportionality between the electromagnetic torque and the load current, the equations (1), (2) and (3), in the relative coordinates, have the forms

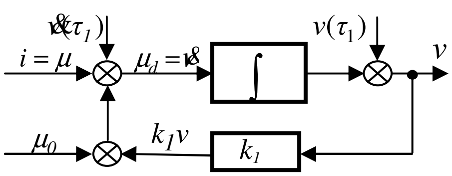

Fig. 1. Strucrural block diagram of electic drive.

$$\mu_s = \mu_0 + k_1\nu, \quad \mu_0 + k_1 = 1, \tag{7}$$

$$i = \mu = \mu_s + \dot{\nu} = \mu_0 + k_1\nu + \dot{\nu} \tag{8}$$

$$\nu = \int \dot{\nu}\,d\tau \tag{9}$$

with the initial and terminal conditions

$$\tau = \tau_1, \quad \nu(\tau_1) = \nu_1, \quad \tau = \tau_2, \quad \nu(\tau_2) = \nu_2. \tag{10}$$

that leads to the construction of the structural bloc diagram (fig.1).

The admissible controls set and the admissible trajectories set are considered limited and open sets.

Optimization criterion

The assessing of acceleration and deceleration process will be done considering the quality index of energy losses minimization caused by the load current through Joule effect, during those processes, expressed by the integral

$$\Delta w = \int_{\tau_1}^{\tau_2} \Delta p\, d\tau = \int_{\tau_1}^{\tau_2} \rho i^2 d\tau\;. \tag{11}$$

In this case, taking into account the general motion equation (8), the optimization functional-criterion has the expression [7]

$$J\left[v(\tau)\right]=\int_{\tau_1}^{\tau_2} i^2 d\tau=\int_{\tau_1}^{\tau_2}\left(\mu_S+\dot{v}\right)^2 d\tau . \tag{12}$$

Formulation of the optimization problem

The optimization problem consists in determination the admissible optimal control function $i^*(\tau)$ or $\mu^*(\tau)$, which is able to transfer the system from the initial condition $\left(\tau_1, v\left(\tau_1\right)\right)$ to the terminal conditions $\left(\tau_2, v\left(\tau_2\right)\right)$, on an the admissible extremale trajectory $v^*\left(\tau_1\right)$, ensuring the minimum of the optimality criterion (12)

$$J\left[v(\tau)\right]=\int_{\tau_1}^{\tau_2}\left(\mu_s+\dot{v}\right)^2 d\tau=min \tag{13}$$

for a fixed value of speed variation expressed, by the integral

$$\Delta v=v_2-v_1=\int_{\tau_1}^{\tau_2}\dot{v}(\tau)\,d\tau, \tag{14}$$

effectuated is in a given interval of time

$$\tau_2-\tau_1=\int_{\tau_1}^{\tau_2} 1\, d\tau , \tag{15}$$

and satisfying the restrictions

$$\left|i(\tau)\right|<i_{max}, \quad \left|\mu(\tau)\right|<\mu_{max}, \quad \left|v(\tau)\right|<v_{max}, \quad \left|\dot{v}(\tau)\right|<\dot{v}_{max} . \tag{16}$$

In conformity with the principle of reciprocity, the given formulation is equivalent with the formulation through which every isoperimetric condition (14) and (15) can become optimization criteria ($v_2-v_1=$ max, $\tau_2-\tau_1=$ min) or a linear combination of them [6].

So, it results a linear – quadratic optimization problem of isoperimetric extremum. To solve this issued problem, the primal problem of conditional extremum will be reduced to a dual problem of unconditional extremum by a Lagrange adjoint function based on Lagrange multiplier λ_0 [6], [7].

$$L=\left(\mu_S+\dot{v}\right)^2+\lambda_0\dot{v}=\left(\mu_0+k_1 v+\dot{v}\right)^2+\lambda_0\dot{v} \tag{17}$$

and by determining the unconditional extremum with the functional

$$J\left[v(\tau)\right]=\int_{\tau_1}^{\tau_2} L(\tau)\,d\tau=min \tag{18}$$

on the same extremals as those of the primal problem [6].

Necessary condition of extremum

The necessary condition of extremum is expressed by the Euler – Lagrange equation [6], [7]

$$\frac{\partial L}{\partial v}-\frac{d}{d\tau}\frac{\partial L}{\partial \dot{v}}=0\,, \tag{19}$$

that leads to the linear differential equation of the second order

$$\ddot{v}-k_1^2 v=k_1\mu_0\,. \tag{20}$$

Because the condition of extremum expressed by the differential equation (20) does not contain the Lagrange multiplier λ_0, it corresponds to that which might result in the case that Euler–Lagrange equation would be applied directly for the functional of the criterion (13).

Optimal solution with arbitrary terminal moment for constant static torque [4]

In the case of the constant static torque, considering the particularizations

$$\mu_0>0, \qquad k_1=0, \qquad \mu_S=\mu_0\,, \tag{21}$$

the condition of extremum, expressed by the differential equation (20), becomes

$$\ddot{v}=0 \tag{22}$$

and successively integrating, we obtain the trajectories family

$$\dot{v}=C_1\,, \qquad v=C_1\tau+C_2\;. \tag{23}$$

From fixed initial condition, we determine one of the integration constants

$$\tau=\tau_1, \qquad v(\tau_1)=C_1\tau_1+C_2=v_1, \quad \Rightarrow \quad C_2=v_1-C_1\tau_1\,, \tag{24}$$

resulting the trajectories fascicle

$$\dot{v}(\tau,C_1)=C_1, \quad v(\tau,C_1)=C_1(\tau-\tau_1)+v_1\,. \tag{25}$$

The terminal moment being arbitrary, the terminal extremity of the trajectory (τ_2, v_2) will be mobile on the transversal, that means

$$v_2=\varphi(\tau_2)=\text{const} \qquad \text{and} \qquad \dot{\varphi}(\tau_2)=0\,, \tag{26}$$

and the necessary condition of the extremum existence implies satisfying the condition of transversality [6]

$$\left[L+(\dot{\varphi}-\dot{v})\frac{\partial L}{\partial \dot{v}}\right]_{\tau=\tau_2}=0, \tag{27}$$

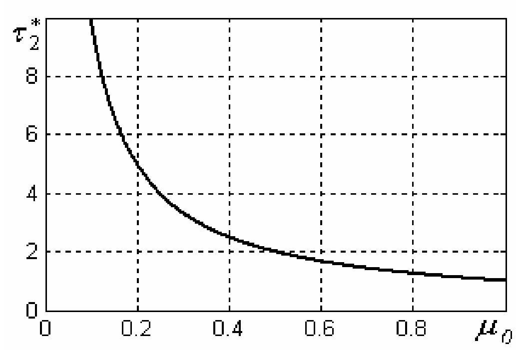

Fig. 2. Dependence of optimum terminal mo-ment from static torque during starting with constant static torque ($\tau_1=0$, $v_1=0$

that leads to equation

$$\mu_0^2-\dot{v}^2=0 \quad \text{or} \quad \mu_0^2-C_1^2=0\,, \tag{28}$$

from which the second arbitrary constant is determined

$$C_1=\pm\,\mu_0\;. \tag{29}$$

Taking into consideration the arbitrary constant values (24) and (29), the contact condition between the extremal trajectory and transversal implies the equation

$$\tau=\tau_2, \quad v(\tau_2)=\pm\mu_0(\tau_2-\tau_1)+v_1=v_2 \tag{30}$$

where permits determining the optimum terminal moment (fig,2).

$$\tau_2^* = \pm \frac{\nu_2 - \nu_1}{\mu_0} + \tau_1 \ . \tag{31}$$

Terminal moment is inversely proportional with the value of the static torque, resulting a terminal moment which tends to infinite, when the static torque tends to zero, that means non functioning of the drive without load.

Substituting the value of integration constant (29) in the solution (25), we can determine the extremal trajectory [3]

$$\nu^*(\tau) = \pm \mu_0 (\tau - \tau_1) + \nu_1, \quad \dot{\nu}^*(\tau) = \mu_d^*(\tau) = \pm \mu_0, \quad \forall \tau \in \left[\tau_1, \tau_2^*\right], \tag{32}$$

resulting a linear speed function of time and a constant acceleration (fig.3 and fig.4).

Taking into consideration the motion equation (8) and the acceleration (32), the optimal control results

$$i^*(\tau) = \mu^*(\tau) = \mu_0 + \mu_d = \begin{cases} 2\mu_0, & \text{acceleration} \\ 0, & \text{deceleration} \end{cases}, \qquad \forall \tau \in \left[\tau_1, \tau_2^*\right] \tag{33}$$

equal to the double of the static torque value for acceleration (fig.3) and null for deceleration (fig.4). So, the absolute minimum of optimization criterion is ensured

$$J_{min}^*\left[\nu(\tau)\right] = \int_{\tau_1}^{\tau_2} \left(\mu_0 + \dot{\nu}^*\right)^2 d\tau = \begin{cases} 4\mu_0^2\left(\tau_2^* - \tau_1\right) \\ 0 \end{cases} \tag{34}$$

or, substituting the optimal terminal moment (31), we obtain

$$J_{min}^* = \begin{cases} 4\mu_0 (\nu_2 - \nu_1), & \text{acceleration} \\ 0, & \text{deceleration.} \end{cases} \tag{35}$$

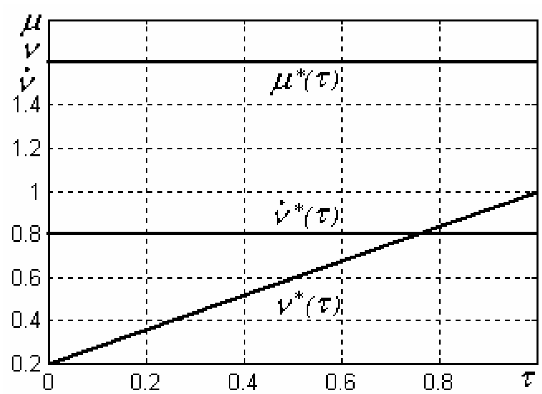

Fig. 3. Optimal control and extremal trajectory during acceleration witch constant static torque *(μ_0=0.8, τ_1=0, ν_1=0.2 şi ν_2=1)*

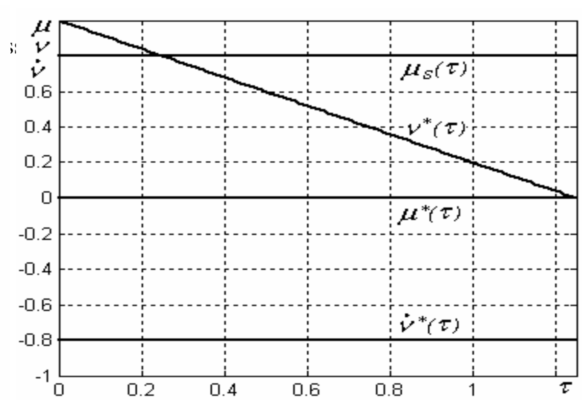

Fig. 4. Optimal control and extremal trajectory during deceleration witch constant static torque *(μ_0=0.8, τ_1=0, ν_1=1 şi ν_2=0)*

Optimal solution with arbitrary terminal moment for statique torque with constant component and speed proportional component [1]

In the case of such a static torque, having the particularization

$$\mu_0 > 0, \qquad k_1 > 0 \qquad \text{and} \qquad \mu_0 + k_1 = 1, \tag{36}$$

the static torque takes the form

$$\mu_s = \mu_0 + k_1 v, \tag{37}$$

and extremum condition expressed by the differential equation (20) becomes

$$\ddot{v} - k_1^2\, v = k_1\, \mu_0\,. \tag{38}$$

Based on solving the characteristic equation attached to the homogeneous differential equation (38)

$$r^2 - k_1^2 = 0, \qquad \Rightarrow \qquad r_{1,2} = \pm k_1 \tag{39}$$

we obtain the general solution of homogeneous differential equation

$$v_g = C_1\, e^{k_1 \tau} + C_2\, e^{-k_1 \tau} \tag{40}$$

which, together with the particular solution of nonhomogeneous differential equation

$$v_p = -\frac{\mu_0}{k_1}\,, \tag{41}$$

lead to the general solution of nonhomogeneous differential equation, representing the family of trajectories

$$v = v_p + v_g = -\frac{\mu_0}{k_1} + C_1\, e^{k_1 \tau} + C_2 e^{-k_1 \tau} \tag{42}$$

Being fixed the initial extremity of the trajectory $\left(\tau_1, v\left(\tau_1\right)\right)$, we can determine one of the integration (arbitrary) constants

$$\tau = \tau_1, \quad v\left(\tau_1\right) = -\frac{\mu_0}{k_1} + C_1\, e^{k_1 \tau_1} + C_2 e^{-k_1 \tau_1} = v_1, \quad \Rightarrow \quad C_1 = \left(\frac{\mu_0}{k_1} + v_1\right) e^{-k_1 \tau_1} - C_2 e^{-2k_1 \tau_1} \tag{43}$$

which, being substituted in the solution (42), we obtain the trajectories fascicle

$$v\left(\tau, C_2\right) = -\frac{\mu_0}{k_1} + \left[\left(\frac{\mu_0}{k_1} + v_1\right) e^{-k_1 \tau_1} - C_2 e^{-2k_1 \tau_1}\right] e^{k_1 \tau} + C_2 e^{-k_1 \tau} \quad \text{and} \tag{44}$$

$$\dot{v}\left(\tau, C_2\right) = \left[\left(\mu_0 + k_1 v_1\right) e^{-k_1 \tau_1} - k_1 C_2 e^{-2k_1 \tau_1}\right] e^{k_1 \tau} - k_1 C_2 e^{-k_1 \tau}\,. \tag{45}$$

The terminal moment being arbitrary, the terminal extremity of the trajectory $\left(\tau_2, v\left(\tau_2\right)\right)$ is mobile on the transversal, that means

$$v_2 = \varphi\left(\tau_2\right) = const. \qquad \text{and} \qquad \dot{\varphi}\left(\tau_2\right) = 0, \tag{46}$$

and the necessary condition of the extremum existence implies satisfying the condition of transversality [6]

$$\left[L + \left(\dot{\varphi} - \dot{v}\right) \frac{\partial L}{\partial \dot{v}}\right]_{\tau = \tau_2} = 0\,, \tag{47}$$

Having

$$L=\left(\mu_0+k_1 v+\dot{v}\right)^2+\lambda_0\dot{v}, \qquad \frac{\partial L}{\partial \dot{v}}=2\left(\mu_0+k_1 v+\dot{v}\right)+\lambda_0 \tag{48}$$

the equation is obtained

$$\left|\left(\mu_0+k_1 v\right)^2-\dot{v}^2\right|_{\tau=\tau_2}=0 \tag{49}$$

or, taking into consideration speed expression (44) and the expression of the corresponding acceleration (45)

$$\begin{aligned}&\left[\left(\left(\mu_0+k_1v_1\right)e^{-k_1\tau_1}-k_1C_2\,e^{-2k_1\tau_1}\right)e^{k_1\tau_2}+k_1C_2\,e^{-k_1\tau_2}\right]^2-\\&-\left[\left(\left(\mu_0+k_1v_1\right)e^{-k_1\tau_1}-k_1C_2\,e^{-2k_1\tau_1}\right)e^{k_1\tau_2}-k_1C_2\,e^{-k_1\tau_2}\right]^2=0\end{aligned} \tag{50}$$

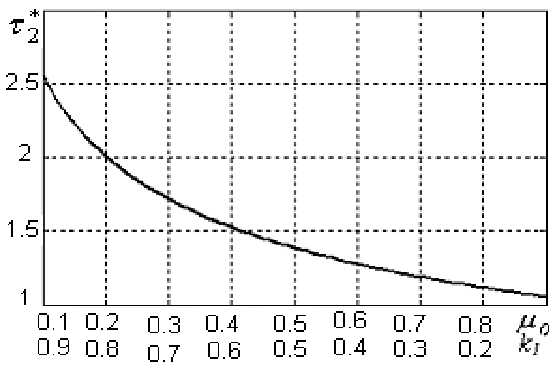

Fig. 5. Dependence of optimal terminal moment from static torque during starting and static torque with constant component and speed proportional component.

finally resulting the from

$$4k_1C_2\left[\left(\mu_0+k_1v_1\right)e^{-k_1\tau_1}-k_1C_2\,e^{-2k_1\tau_1}\right]=0 \tag{51}$$

from which the second integration constant is determined

$$C_2=\begin{cases}0, & \text{acceleration}\\ \left(\dfrac{\mu_0}{k_1}+v_1\right)e^{k_1\tau_1}, & \text{deceleration}\end{cases} \tag{52}$$

substituting (52) into (43) we get

$$C_1=\begin{cases}\left(\dfrac{\mu_0}{k_1}+v_1\right)e^{-k_1\tau_1}, & \text{acceleration}\\ 0, & \text{deceleration}.\end{cases} \tag{53}$$

Taking into consideration the values of the arbitrary constants (52) and (53), the contact condition behveen the extremal trajectory and transversal, implies

$$\tau=\tau_2, \quad v\left(\tau_2\right)=-\frac{\mu_0}{k_1}+\left(\frac{\mu_0}{k_1}+v_1\right)e^{\pm k_1(\tau_2-\tau_1)}=v_2 \tag{54}$$

$$\text{or} \quad e^{\pm k_1(\tau_2-\tau_1)}=\frac{\mu_0+k_1v_2}{\mu_0+k_1v_1} \tag{55}$$

from which, we determine, through logarithmation, the optimum terminal moment (fig.5)

$$\tau_2^*=\pm\frac{1}{k_1}ln\,\frac{\mu_0+k_1v_2}{\mu_0+k_1v_1}+\tau_1\ . \tag{56}$$

Substituting the integration constants (52) and (53) in solution (42) of the differential equation, we obtain the evolution in time (fig.6) and (fig.7) of the extremal trajectory, for speed

$$\nu^{*}(\tau) = -\frac{\mu_0}{k_1} + \left(\frac{\mu_0}{k_1} + \nu_1\right) e^{\pm k_1(\tau - \tau_1)} \qquad \forall \tau \in \left[\tau_1, \tau_2^{*}\right], \tag{57}$$

for acceleration and deceleration, dynamic torque respectively

$$\dot{\nu}^{*}(\tau) = \mu_d^{*}(\tau) = \pm\left(\mu_0 + k_1 \nu_1\right) e^{\pm k_1(\tau - \tau_1)} \quad \forall \tau \in \left[\tau_1, \tau_2^{*}\right], \tag{58}$$

and for shock

$$\ddot{\nu}^{*}(\tau) = k_1\left(\mu_0 + k_1 \nu_1\right) e^{\pm k_1(\tau - \tau_1)} \qquad \forall \tau \in \left[\tau_1, \tau_2^{*}\right]. \tag{59}$$

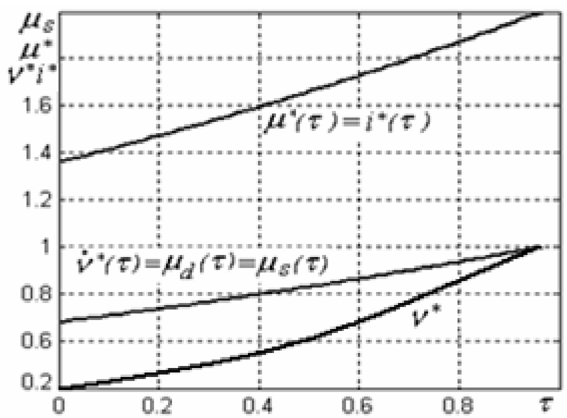

Fig. 6. Optimal control and extremal trajectory during acceleration and static torque with constant component and speed proportional component *(μ_0=0.6, k_1=0.4, ν_1=0.2, ν_2=1, τ1=0)*

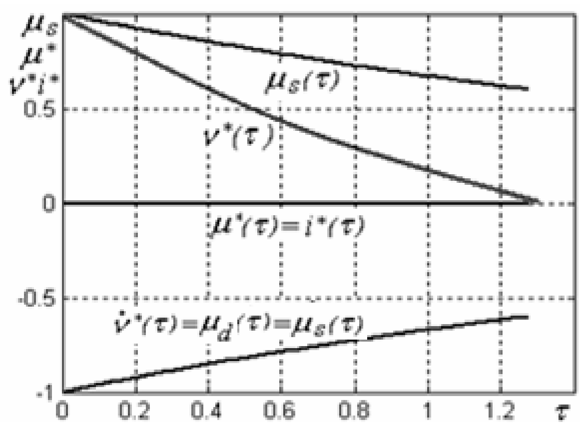

Fig. 7. Optimal control and extremal trajectory during deceleration and static torque with constant component and speed proportional component *(μ_0=0.6, k_1=0.4, ν_1=0.2, ν_2=1, τ_1=0)*

Taking into account the speed extremal (44), load static torque will have the expression

$$\mu_s = \mu_0 + k_1 \nu = \left(\mu_0 + k_1 \nu_1\right) e^{\pm k_1(\tau - \tau_1)}, \qquad \forall \tau \in \left[\tau_1, \tau_2^{*}\right] \tag{60}$$

and, substituting the static torque expression (47) and dynamic torque expression (45) into motion equation (8), we can determine the optimal control variable [5]

$$\begin{aligned} i^{*}(\tau) &= \mu^{*}(\tau) = \mu_s + \mu_d = \left(\mu_0 + k_1\nu_1\right) e^{\pm k_1(\tau-\tau_1)} \pm \left(\mu_0 + k_1\nu_1\right) e^{\pm k_1(\tau-\tau_1)} = \\ &= \begin{cases} 2\left(\mu_0 + k_1\nu_1\right) e^{k_1(\tau-\tau_1)}, & \text{acceleration} \\ 0, & \text{deceleration} \end{cases}, \qquad \forall \tau \in \left[\tau_1, \tau_2^{*}\right], \end{aligned} \tag{61}$$

resulting an electromagnetic torque equal to the double of the static torque value for acceleration (fig.6) and null for deceleration (fig.7). Absolute minimum of optimization criterion (12) is

$$J^*_{min} = \int_{\tau_1}^{\tau_2} i^2(\tau)\, d\tau = \int_{\tau_1}^{\tau_2} \left[2\left(\mu_0 + k_1 v_1\right) e^{k_1(\tau-\tau_1)} \right]^2 d\tau = \frac{2}{k_1}\left(\mu_0 + k_1 v_1\right)^2 \left(e^{2k_1\left(\tau_2^* - \tau_1\right)} - 1 \right), \quad (62)$$

and taking into consideration the optimum terminal moment(55), it becomes

$$J^*_{min} = \frac{2}{k_1}\left(\mu_0 + k_1 v_1\right)^2 \left[\left(\frac{\mu_0 + k_1 v_2}{\mu_0 + k_1 v_1} \right)^2 - 1 \right] = 4\mu_0\left(v_2 - v_1\right) + 2k_1\left(v_2^2 - v_1^2\right). \quad (63)$$

The sign"+" applies for acceleration process and the sign"+" applies for deceleration process

Conclusions

For constant static torque, the speed is a linear function of time, and the others variables of the problem are constant. For static torque with a component proportional to speed, ale variables of the problem are exponential functions of time.

Electromagnetic torque of the driving motor, necessary for the optimal control, is equal to the double of the static torque in acceleration processes and is null in the deceleration processes (considering a brake with recuperation of energy).

Obtained results, determined for a variation of speed, are valid-through particularization-both for starting, considering $v_1=0$, and also for braking until stopping considering $v_2=0$.

The problem in which static torque is proportional to speed ($\mu_0=0,\ k_1>0$), optimization with arbitrary terminal moment doesn't have an optimal solution in the starting or braking until stopping cases.

Obtained results can be used both in designing and also in optimal control of the electric driving systems with constant static torque or depending on speed which works in service-type continuous S_1 or in uninterrupted service with periodical modification of speed S_8. Through energy savings obtained in the starting, braking and periodical modification of speed processes obtain a quality and efficiency increasing of electric driving system.

Optimal control for system transportation from initial to terminal condition with minimal energy losses, is obtained in open-loop or with program which is applied to the system input. This solution presents some disadvantages concerning both implementing the program and also its exact effectuation. For this reason, we preferred a solution in closed-loop that can be obtained directly through Ricati equation.

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Comanda optimală cu timp final liber a accelerării acţionărilor electrice cu cuplu static cu componentă constantă şi componentă proporţională cu viteza după criteriul energetic

Rezumat

În această lucrare, considerând o acţionare cu cuplu static cu o componentă constantă şi o componentă proporţională cu viteza, în ipoteza momentului de inerţie constant şi a proporţionalităţi cuplului electromagnetic şi curentului, cu ajutorul calcului variaţional clasic, se determină traiectoria extremală şi comanda extremală care să asigure valoarea minimă a pierderii de energie cauzată de curentul de sarcină in procesele de accelerare şi respectiv decelerare. Utilizând calculatorul se obţin reprezentările grafice ale acestor variabile în timp.