BULETINUL Universităţii Petrol – Gaze din Ploieşti	Vol. LVIII No. 2 bis / 2006	175-180	Seria Tehnică

Optimal Design of Electromechanical Systems Propeller Fans

Anca Petrişor, Marius Constantin Popescu

University of Craiova,
e-mail: mrpopescu@em.ucv.ro

Abstract

In this paper it is presented the optimal design problem of an impeller with a view to electromechanical systems optimization. The mathematical model, used for power loss determination in the time unit due to the fluid rotational movement, it is obtained starting from impulse equation [1], [2], [3].
The optimization problem is solved used the calculus of variations method. Finally, the optimal values of the variables which define the impeller are obtained.
In the case of using an induction motor it isn't a big difference between the flow and pressure theoretical values resulted following the optimization process and those are resulted in fact due to an efficient propeller fan drive.

Introduction

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The optimization problem is solved used the calculus of variations method. Finally, the optimal values of the variables which define the impeller are obtained.

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Mathematical description of the propeller fans

The variables of a propeller fan (the axial speed v, the angular speed ω_f induced by the impeller, the fluid circulation Γ through the impeller, the power P_w for rotational movement, the power P_t for translation movement, the rotational torque M, the traction force F, the propeller pitch J_v) related to the nominal values corresponding to the aerodynamical phenomenon evolution, will give the quality dimensionless factors of the propeller fun (by expressing the propeller fan efficiency function of the impeller duty):

$$v_t = \frac{v}{\omega \cdot R}; \quad \mu_t = \frac{F}{2 \cdot \pi \cdot \omega^2 \cdot R^4 \cdot \rho};$$
$$p_t = \frac{P_t}{2 \cdot \pi \cdot v^3 \cdot R^2 \cdot \rho}; \quad \gamma = \frac{N \cdot \Gamma}{4 \cdot \pi \cdot \omega \cdot R^2};$$
$$v_m = \frac{\omega_f}{2 \cdot \omega}; \quad \mu_m = \frac{M}{2 \cdot \pi \cdot \omega^2 \cdot R^5 \cdot \rho};$$
$$p_m = \frac{P_m}{2 \cdot \pi \cdot \omega^3 \cdot R^5 \cdot \rho}; \quad j = \frac{\pi \cdot v}{\omega \cdot R}. \tag{1}$$

where ω is the angular speed of the driving motor, ρ is the fluid density and N represents the number of fan blades.

As single variable for all the other variables, is chosen the distance x defined by the ratio r/R, in relatives coordinates, where R is the impeller radius in absolute coordinates and r is the distance between the center of the impeller and the fluid volume that intersects the disk.

In case of an impeller, the traction force and the rotational torque, for fluid circulation volume that intersect the impeller disk, are:

$$F = 4\pi\rho\int_0^R v^2 \cdot r \cdot dr = 4\pi\rho\omega^2 R^4 \int_0^1 v_t^2 \cdot x \cdot dx = 2\pi\rho\omega^2 R^4 \int_0^1 v_t^2 \cdot d(x^2); \tag{2}$$

$$M = 4\pi\rho\int_0^R \frac{\omega_f}{2} v \cdot r^3 \cdot dr = 4\pi\rho\omega^2 R^5 \int_0^1 v_m \cdot v_t \cdot x^3 \cdot dx = 2\pi\rho\omega^2 R^5 \int_0^1 v_m \cdot v_t \cdot x^2 \cdot d(x^2) \tag{3}$$

For a impeller, relating the traction force and the rotational torque to the nominal value of the force and torque, because of the rotational movement, results

$$\mu_t = \frac{F}{2\pi\rho\omega^2 R^4} = \int_0^1 v_t^2 \cdot d(x^2);$$
$$\mu_m = \frac{M}{2\pi\rho\omega^2 R^5} = \int_0^1 v_m \cdot v_t \cdot x^2 \cdot d(x^2). \tag{4}$$

The general equation for rotational movement, respectively for translation movement (for non-zero speed) for an elementary fluid volume that passes through the disk surface of an impeller, in accordance with the impulse method, is

$$\omega \cdot dM = v \cdot dF + \omega_f \cdot dM / 2 \tag{5}$$

where $\omega \cdot dM$ is the elementary motor output and incoming at the impeller disk; $v \cdot dF$ is the elementary lost power for fluid circulation through the impeller disk; $\omega_f \cdot dM / 2$ is the elementary lost power due to the fluid rotation through the impeller disk, that is generally small but has a big influence on the optimal distribution of the traction force on the impeller disk.

By integrating in respect with the variable x, the equation (4.1) becomes:

$$2\pi\rho\omega^3 R^5 \int_0^1 v_m \cdot v_t \cdot x^2 \cdot d(x^2) = 2\pi\omega^3 R^5 \int_0^1 v_t^3 \cdot d(x^2) + 2\pi\omega^3 R^5 \int_0^1 v_m^2 \cdot v_t \cdot x^2 \cdot d(x^2). \tag{6}$$

Relating to the power nominal value due to the rotational movement, it is obtained

$$\int_0^1 v_m \cdot v_t \cdot x^2 \cdot d(x^2) = \int_0^1 v_t^3 \cdot d(x^2) + \int_0^1 v_m^2 \cdot v_t \cdot x^2 \cdot d(x^2) \tag{7}$$

that for an elementary fluid volume is

$$v_t^2 = x^2 \cdot v_m \cdot (1 - v_m). \tag{8}$$

For a propeller fan, the ideal efficiency of an impeller disk elementary surface (induced local efficiency) in relative and absolute coordinates is [4]

$$\eta_{li} = \frac{v \cdot dF}{\omega \cdot dM} = \frac{v_t^2}{v_m \cdot x^2 = 1 - v_m}. \tag{9}$$

The power loss in the propeller fan, due to the rotation, in absolute and relative coordinates, is

$$\Delta P = P_m - P_t, \quad \Delta p = p_m - p_t, \tag{10}$$

where driving motor output (using x, v_t and v_m variables) it is written as

$$P_m = 2 \cdot \pi \cdot \rho \cdot \omega^3 \cdot R^5 \cdot \int_0^1 v_m \cdot v_t \cdot x^2 dx \tag{11}$$

that related to the nominal power value due to the rotational movement will be

$$p_m = \int_0^1 v_m \cdot v_t \cdot x^2 d(x^2) \tag{12}$$

and necessary power for fluid circulation (v=ct. on the impeller disk, in the traction case) is

$$P_t = v \cdot F = 4 \cdot \pi \cdot \rho \cdot v \cdot \int_0^R v^2 dr = 4 \cdot \pi \cdot \rho \cdot v^3 \int_0^R dr = 2 \cdot \pi \cdot \rho \cdot v^3 \cdot R^2 1 \int_0^R d(x^2) \tag{13}$$

that related to the nominal power value due to the traction will be

$$p_t = \int_0^1 d(x^2). \tag{14}$$

The propeller fan mathematical description is completed by the following dependencies between the traction force μ_t, the rotational torque μ_m and the energy w

$$w_t = \int \mu_t dx; \quad w_m = \int \mu_m dx. \tag{15}$$

The optimal design using propeller fans power loss criterion that must realized a traction force

This problem appears in the case of propeller fan design such as for a given fan traction force, the driving power or the power loss due to the fluid rotation to be minimum or the induced efficiency to be maximum.

The mathematical model used for power loss determination in time unit due to the fluid rotational movement, is given by the general movement equation of a fluid elementary volume.

The optimization can also be extended in the case of existing additional power loss due to the friction on impeller blades (fluid viscozity) since the expressions for the elementary traction force and movement are the same (the frictions force don’t change the pressure field, but only the efficiency η_l.

The optimization criterion

The problem can be solved using the calculus of variations method [5]. Considering the power loss as optimization criterion, due to the fluid rotational movement, then power loss in time unit for a fluid volume that pass through the propeller fan, in relatives coordinates, is given by the following integral

$$\Delta w = \int_0^1 (v_t \cdot v_m \cdot x^2 - 1) d(x^2)\,. \tag{16}$$

The statement of optimization problem

The optimization problem consists in finding the v_m, v_t extremals trajectories that assure the minimum power loss due to the rotational movement estimated by the integral

$$J = \int_0^1 v_t \cdot v_m \cdot x^2 - 1) d(x^2) = \min \tag{17}$$

observe that constraint required to the fan traction force μ_t

$$\int_0^1 v_t^2 \cdot d(x^2) = ct \tag{18}$$

and the restriction for the fluid angular speed ($v_m < 0.5$).

This is a problem of conditioned izoperimetrical extreme. An auxiliary function is built for changing in a simple extreme problem using Lagrange constants multipliers [6], [7]

$$F(v_t, v_m) = v_t \cdot v_m \cdot x^2 - 1 + \lambda_0 \cdot v_t^2\,, \tag{19}$$

where λ_0 is Lagrange multiplier that is going to be established from the condition (18)

In this way will be established the performance number extremals

$$J = \int_0^1 F \cdot d(x^2) \tag{20}$$

that will be the same with those corresponding to the performance number given by the integral (17) .

The necessary optimal condition is expressed by Euler equation [8]

$$\frac{\partial F}{\partial v_t} = 0 \;\Rightarrow\; v_t \cdot x^2 \frac{\partial v_m}{\partial v_t} + x^2 \cdot v_m - 2\lambda_0 \cdot v_t = 0\,. \tag{21}$$

By derivating with respect to v_1 the equation (8)

$$x^2 \frac{\partial v_m}{\partial v_t} = \frac{2 \cdot v_t}{1 - 2 \cdot v_m} \tag{22}$$

and that replaced in (21) will give

$$\frac{2 \cdot v_t^2}{1 - 2 \cdot v_m} + x^2 \cdot v_m + 2 \cdot \lambda_0 \cdot v_t = 0\,. \tag{23}$$

The equation that illustrates the v_m angular speeds optimal distribution (24) is obtained by replacing the term x^2 in equation (23) with its expression resulted from equation (8) and then divided with v_t

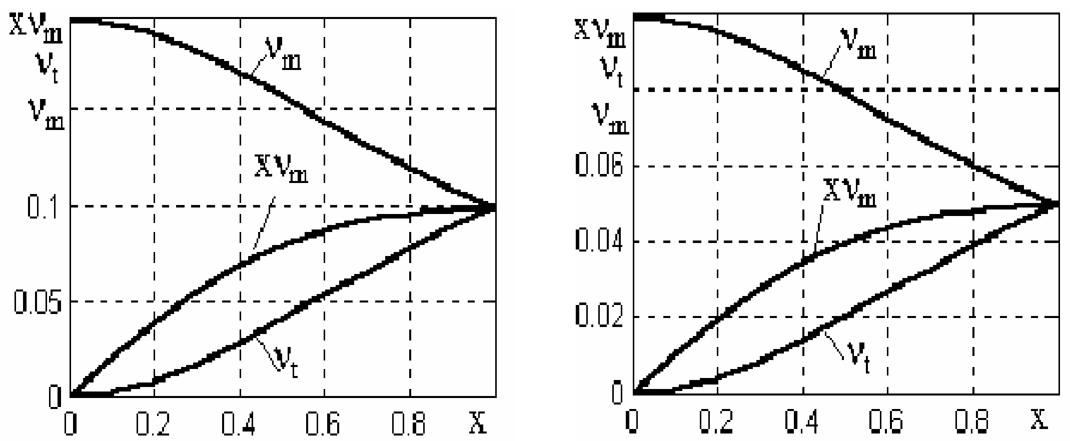
$$\frac{v_t}{1-v_m}+\frac{2\cdot v_t}{1-2\cdot v_m}=-2\cdot\lambda_0. \tag{24}$$

The optimal solution

The analitical solutions is obtained if instead the equation (24) is considering the approximate relation [9], [10]

$$\frac{v_t}{1-v_m}\cong-\lambda_0. \tag{25}$$

This approximation is good enough for practical solutions. Particular case, when local efficiency η_{li} is constantly it is also obtained a constantly efficiency along the blade of the impeller [11].



a) b)

Fig. 1. Optimal speed distribution induced along the propeller fan radius
a) $\eta_{li}=0.8$; b) $\eta_{li}=0.9$.

4. Conclusions

In aerodynamical theory, the curve $x\cdot v_m$ represents the blade width that for an impeller must increase from rotor hub to the center of the propeller fan blade.
Knowing these local speeds on the disk and using aerodynamical theory of the blade element it can be realized the calculus for powers and forces and then the propeller fan rotor design for the required traction force.

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Proiectarea unui ventilator pentru optimizarea sistemelor electromecanice

Rezumat

In lucrare este prezentată problema proiectării optimale a unui ventilator pentru optimizarea sistemelor electromecanice. Modelul matematic utilizat pentru determinarea pierderilor în unitatea de timp, datorită mişcării de rotaţie a fluidului, se obţine pornind de la ecuaţia impulsului [1], [2], [3].

Problema de optimizare se rezolvă folosind metoda de calcul variaţional. In final se obţin valorile optime ale variabilelor care definesc ventilatorul.

În cazul utilizării unui motor de inducţie, nu există deosebiri semnificative între valorile teoretice ale debitului şi presiunii rezultate în urma procesului de optimizare şi cele rezultate de fapt datorită antrenării eficiente a ventilatorului.