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The Comparative Study of the Stability of the Vector Control Systems That Contain In the Loop Luenberger and Kalman Type Estimators

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Abstract

The paper deals with speed vector-controlled induction motor drive systems, in which the rotor flux estimation is performed using Luenberger and Kalman estimators. A comparative study of the stability of the estimators and of the control systems which are included in the control loop is presented using both simplified and full discretisation method and modifying the sampling time used for each of the methods. The study is focused especially on the low and high speed regions. The conclusions are based on the comparing of the eigenvalues of the estimators in each studied case.

Key words: *sensorless vector control, induction motor, flux estimator, Luenberger, Kalman, discretisation.*

Introduction

The paper deals with speed vector-controlled induction motor drive systems, in which the rotor flux estimation is performed using Luenberger and Kalman estimators. A comparative study of the stability of the estimators and the control systems which are included in the control loop is presented, taking into account the followings:

- The motor is a squirrel cage induction motor;
- The stability study is performed taking into account various motor speed and sampling time variations, both in the presence and in the absence of the measuring and process noises. The study is also taking into account the degree of discretisation of the considered models;
- The tuning parameters of the controllers in the vector control scheme are computed for a direct flux measuring control scheme oriented on the rotor flux. These parameters are the same both for the control scheme that uses a Luenberger estimator and the one using a Kalman estimator;
- The stability study of the estimator is performed taking into account the control loops.

The Kalman Estimator

The mathematical model of the Kalman estimator is deduced based on the stochastic model of the induction motor, a mathematical model given by the canonic state equations:

$$\begin{cases} x(k+1) = F_k \cdot x(k) + H_k \cdot u(k) + w(k) \\ y(k) = C \cdot x(k) + v(k) \end{cases} \quad (1)$$

in which the noise vector of the process *w(k)* and the measuring noise *v(k)* are considered as Gauss type vectors having the following properties:

$$\begin{aligned} &E[w(k)] = E[v(k)] = 0;\ E\left[w(i) \cdot w^T(j)\right] = \\ &= Q_k \cdot \delta_{ij}\ ;E\left[v(i) \cdot v^T(j)\right] = R_k \cdot \delta_{ij} \end{aligned} \quad (2)$$

where *E* is the statistic average and * is the Kronecker operator.

The matrices F_k and H_k are deduced from the matrices A_k and B of the induction motor model in time domain by discretisation. In practice two of such models are imposed: one deduced by complete discretisation that makes the Kalman estimator more precise but which requires much more computing resources:

$$F_k = I + A_k \cdot T + A_k^2 \cdot \frac{T^2}{2};H_k = B \cdot T + A_k \cdot B \cdot \frac{T^2}{2} \quad (3)$$

And other obtained through simplified discretisation that makes the Kalman estimator to be less precisely, but which does not require high computing resources:

$$F_k = I + A_k \cdot T\,;H_k = B \cdot T \quad (4)$$

In formula (3) and (4) the *T* variable is the sampling time and the *A*, *B* and *C* matrices have the following shape

$$\begin{aligned} A_k &= \begin{bmatrix} a_{11} & 0 & a_{13} & a_{14} \cdot \omega_k \\ 0 & a_{11} & a_{14} \cdot \omega_k & a_{13} \\ a_{31} & 0 & a_{33} & -\omega_k \\ 0 & a_{31} & \omega_k & a_{33} \end{bmatrix}; \\ B &= \begin{bmatrix} b_{11} & 0 & 0 & 0 \\ 0 & b_{11} & 0 & 0 \end{bmatrix}^T; C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{aligned} \quad (5)$$

where:

$$a_{11} = -\left(\frac{1}{T_s \cdot \sigma} + \frac{1-\sigma}{T_r \cdot \sigma}\right);\ a_{13} = \frac{L_m}{L_s \cdot L_r \cdot T_r \cdot \sigma};$$

$$a_{14} = \frac{L_m}{L_s \cdot L_r \cdot \sigma};\ a_{31} = \frac{L_m}{T_r};\ a_{33} = -\frac{1}{T_r};\ b_{11} = \frac{1}{L_s \cdot \sigma};$$

$$T_s = \frac{L_s}{R_s}\ ;\ T_r = \frac{L_r}{R_r}\ ;\ \sigma = 1 - \frac{L_m^2}{L_s \cdot L_r}$$

L_s, L_r, L_m are the stator, rotor and mutual inductances; R_s , R_r are the stator and rotor resistances and σ is the mutual dispersion coefficient.

In these conditions, considering the sizes of the input vector, the input stator voltages with respect to the oriented axis system dq:

$$u(k) = \left[u_{ds}(k) \quad u_{dq}(k)\right]^T, \quad (6)$$

the sizes of the state vector, the stator currents and the rotor flux in the oriented axis system dq:

$$\hat{x}(k) = \left[i_{ds}(k) \quad i_{qs}(k) \quad \psi_{dr}(k) \quad \psi_{qr}(k)\right]^T \qquad (7)$$

and the dimensions of the output vector, the stator currents in the oriented axis system dq:

$$y(k) = \left[i_{ds}(k) \quad i_{dq}(k)\right]^T \qquad (8)$$

the Kalman estimator algorithm becomes:

$$\begin{aligned}
&\Gamma(k/k-1) = F_{k-1} \cdot P(k-1/k-1) \cdot F_{k-1}^T + Q_{k-1} \\
&K(k) = \Gamma(k/k-1) \cdot C^T \cdot \left[C \cdot \Gamma(k/k-1) \cdot C^T + R_k\right]^{-1} \\
&\hat{x}(k/k-1) = F_{k-1} \cdot \hat{x}(k-1/k-1) + H_{k-1} \cdot u(k-1) \qquad (9) \\
&\hat{x}(k/k) = \hat{x}(k/k-1) + K(k) \cdot \left[y(k) - C \cdot \hat{x}(k/k-1)\right] \\
&P(k/k) = \left[I_4 - K(k) \cdot C\right] \cdot \Gamma(k/k-1)
\end{aligned}$$

where: *K(k)* is the Kalman matrix;) $\hat{x}(k/k)$ is the estimated state vector at the moment $k \cdot T$; Γ*(k/k-1)* is the aprioric covariant matrix of the extrapolated state $\hat{x}(k/k-1)$ and *P(k/k)* is the aposterioric covariant matrix of the estimated state $\hat{x}(k/k)$.

The estimation error of the Kalman estimator is: $\tilde{x}(k/k) = x(k) - \hat{x}(k/k)$ and the initial conditions are $P(0/0) = P_0$ and $\hat{x}(0/0) = \hat{x}_0$ where $\hat{x}_0$ is considered to be a *0* vector and P_0 is determined as a solution of the Riccati equation of the estimator. The covariance matrices *Q* and *R* are constant and are tuned according to the following formulae [3]:

$$R = \begin{bmatrix} \sigma_u^2 & 0 \\ 0 & \sigma_u^2 \end{bmatrix}$$

$$Q = \begin{bmatrix} \sigma_i^2 & 0 & \rho \cdot \sigma_\psi \cdot \sigma_i & 0 \\ 0 & \sigma_i^2 & 0 & \rho \cdot \sigma_\psi \cdot \sigma_i \\ \rho \cdot \sigma_\psi \cdot \sigma_i & 0 & \sigma_\psi^2 & 0 \\ 0 & \rho \cdot \sigma_\psi \cdot \sigma_i & 0 & \sigma_\psi^2 \end{bmatrix} \qquad (10)$$

where: σ_u is the variant introduced by the input u_s ; σ_i and σ_ψ are the variants introduced by the state vector components (i_s and ψ_r);

The Luenberger Estimator

As in the case of the Kalman estimator, the equations that define the Luenberger estimator are deduced based on the mathematical model of the induction motor, model given by the canonical state equations (1) in which the noise vector of the process *w(k)* and the measuring noise *v(k)* are considered 0 vectors. Based on the above data, the Luenberger estimator model becomes:

$$\hat{x}(k+1) = F_k \cdot \hat{x}(k) + H_k \cdot u(k) + L_k \cdot (y(k) - C \cdot \hat{x}(k)) \qquad (11)$$

where the matrices F_k and H_k are deduced from the matrices *A* and *B* of the induction motor model in time domain by discretisation, using one of the methods given by the relations (3) or (4).

The most common method for the on-line computing of the Luenberger matrix is the using of the formula that ensures the proportionality between the motor poles and the estimator ones. These formulas are deduced in the continuous case, obtaining the matrix L, from which, by discretisation, the L_k matrix is deduced.
By ensuring the proportionality, the result is the stability of the estimator, without influence of the speed ω_k. Obviously the proportionality will not be maintained after the discretisation, but the stability will be. The mathematical expression of L matrix is:

$$L = \begin{bmatrix} k_{11} & -k_{12} \\ k_{12} & k_{11} \\ k_{21} & -k_{22} \\ k_{22} & k_{21} \end{bmatrix} \qquad (11)$$

where:

$$k_{11} = (a_1 + a_2) \cdot (1-k) \quad ; \quad k_{12} = \omega_k \cdot (1-k);$$
$$k_{21} = (a_3 + \gamma \cdot a_1) \cdot (1-k^2) - \gamma \cdot k_{11}; \quad k_{22} = \gamma \cdot k_{12}$$

and

$$a_1 = a_{11}; \; a_2 = a_{33}; \; a_3 = a_{31}; \; \gamma = \frac{1}{a_{14}}.$$

From (11) by discretisation one can get:

$$L_k = L \cdot T + A_k \cdot L \cdot \frac{T^2}{2} \qquad (12)$$

the Luenberger matrix deduced after the complete discretisation, respectively

$$L_k = L \cdot T \qquad (13)$$

the Luenberger matrix obtained after the simplified discretisation.

The estimation error of the Luenberger estimator is:

$$\tilde{x}(k) = x(k) - \hat{x}(k).$$

The Stability Study of the Control Systems

The algorithm used in the stability study of the control systems having in the control loop Kalman and Luenberger type estimators is the one described in paper [4]. The stability study is based on the determination of the eigenvalues of an increased matrix. The matrix on which the study is performed is:

$$F_\Delta = \begin{bmatrix} F_{k-1} - H_{k-1} \cdot M_a(k-1) & H_{k-1} \cdot M_a(k-1) \\ 0 & (I - K_a(k) \cdot C) \cdot F_{k-1} \end{bmatrix} \quad (14)$$

where: M_a is the command matrix that ensures the connection between the input values vector and the estimated state values vector and K_a is equal to the Kalman matrix $K(k)$ the case of the stability study of the control system that contains a Kalman estimator in the control loop, respectively with the Luenberger matrix L_k in the case when the control system

contains a Luenberger estimator in the control loop.

The relation that the command matrix M_a must verify is:

$$u(k) = -M_a(k) \cdot \hat{x}(k) \qquad (15)$$

The matrix M_a that verifies the relation (15) has the following structure:

$$M_a(k) = \begin{bmatrix} 0 & 0 & -\frac{u_{ds}(k)}{|\psi_r(k)|} & \frac{u_{qs}(k)}{|\psi_r(k)|} \\ 0 & 0 & -\frac{u_{qs}(k)}{|\psi_r(k)|} & -\frac{u_{ds}(k)}{|\psi_r(k)|} \end{bmatrix} \quad (16)$$

Application

As an example for the above theoretical issues a squirrel cage induction motor whit is considered, with the following parameters: $PN = 500$ [W]; $U_N = 127$ [V]; $I_N = 2{,}9$ [A]; $n_N = 1400$ [rot/min]; $z_p=2$; $M_N=3{,}41$ [Nm] ; $R_s = 4.495$ [Ω]; $R_r =5.365$[Ω]; $L_s = 165$ [mH]; $L_r = 162$ [mH] ; $L_m = 149$ [mH]; $J = 0.00095$ [Kgm^2].

The stability study has been performed using real-time simulation in Matlab/Simulink environment using S-Function blocks. It has to be mentioned from the start that for several reference speed tahograms and for several controller tuning parameters, the eigenvalues of the control systems that contain in the loop one of the two estimators will be different during the transient states. For this reason the tuning parameters of the controllers used in the speed control system will be identical in both Kalman and Luenberger estimator based drives. The tuning parameters of the PI controllers were computed for a control structure with direct airgap flux measurement and orientation on the rotor flux.

The values are:

- For the speed regulator: the proportional component $k_\omega = 10$ and the time component is $T_\omega = 9000$ [sec] ;
- For the torque regulator: the proportional component $k_M = 10.1988$ and the time component is $T_M = 1020$ [sec];
- For the current regulators: the proportional component $k_I = 5.9881$ and the time component is $T_I = 754.4176$ [sec] ;
- For the flux regulator: the proportional component $3834\ .\ 501 = k_\psi$ and the time component is $T_\psi = 2374.7$ [sec].

Similar to the case of the tuning parameters of the regulators, the tahogram of the control system reference speed will be identical in all cases of stability study, with respect to the increase time of the speed to the reference speed value. In all cases the increase time is one second without influence from the imposed reference speed value.

Taking all these into consideration, the first studied case is the Luenberger and Kalman estimators implementation using full discretisation method. The stability study in this case is performed at very high speeds and a very small sampling time, neglecting the measurement noise. The simulation results presented in Figure 1 and Figure 2 have been obtained at the reference rotation speed of 30000 [rot/min] and in the case of the Luenberger estimator at the value 1.3 for the k coefficient. In these diagrams the open loop eigenvalues of the estimator are presented in black, the motor eigenvalues in blue and the eigenvalues of the control system that contains one of the two estimators in the loop are presented in red.

The sampling time used is T= 53.3 [μ sec], specific to the TMS320F2812 processor, when a frequency of 18.75 [kHz] is required for the inverters IGBT commutation. Comparing the two diagrams one can see that the Luenberger estimator in open loop approaches the stability limit becoming unstable from the rotation speed of 22800 [rot/min] while the Kalman estimator in open loop is behaving very well even at the imposed reference speed. On the other hand the control system that contains the Luenberger estimator in closed-loop as well as the control system that contains the Kalman estimator in closed-loop will be internally stable. It can be seen that at the starting moment of the motor the control system becomes unstable and as the speed of the motor shaft approaches the reference speed it becomes internally stable. From the point of view of the dynamic performances it is noticeable the fact that when the eigenvalues of the motor are on the right side of the eigenvalues of the estimator the dynamic performances of the control system are very good. Comparing the two diagrams from this point of view one can state that the Luenberger estimator is behaving very well from the stability point of view, as well as from the dynamic performances ones, up to the speed of 12000 [rot/min],while the Kalman estimator is behaving very well on the entire range of speed variation. At speeds higher than 40000 [rot/min], both Kalman estimator and the system that contains the Kalman or Luenberger estimator becomes unstable.

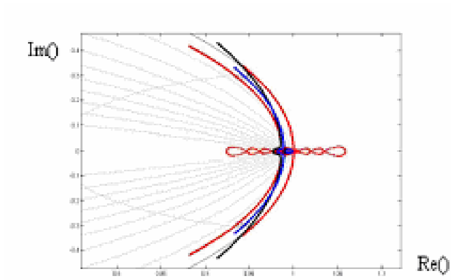

Fig. 1. The eigenvalues for the case with the Luenberger estimator used for the first case

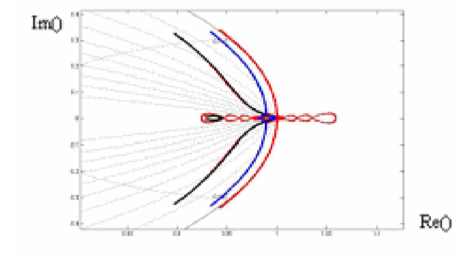

Fig. 2. The eigenvalues for the case with the Kalman estimator used for the first case

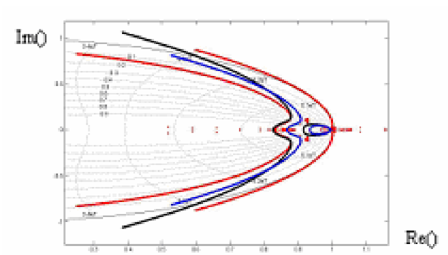

Fig. 3. The eigenvalues for the case with the Luenberger estimator used for the second case

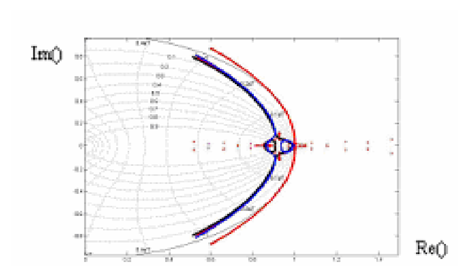

Fig. 4. The eigenvalues for the case with the Kalman estimator used for the second case

The second studied case is the one when the complete discretisation method is used for the implementation of the Kalman and the Luenberger estimators but the sampling time used is higher. In this case the process and measurement noises are also neglected. After the simulation, using a sampling time T = 426.7 [μ sec] corresponding to the above stated processor, for an inverter IGBT with commutation frequency of 2.34 [kHz] the following diagrams, corresponding to the Luenberger and Kalman estimators, are obtained (Figure 3 and Figure 4).

The reference speed imposed in this case is 10000 [rot/min]. It can be pointed out that the instability of the Luenberger estimator and of the control system that contains a Luenberger estimator in the control loop appears at a speed much lower that in the case when a smaller sampling time is used. As in the previous case, a phenomenon appears that imposes the eigenvaluesof the estimator to remain behind the eigenvalues of the motor, phenomenon that produces the decrease of the dynamic performances. This is happening at the moment when the speed increases over 5500 [rot/min] in the case of the Luenberger estimator and over 9000 [rot/min] in the case of the Kalman estimator. As in the previous case, at the starting point, the control system in unstable tending to become internally stable while the motor shaft speed is getting closer to the reference value. The control systems become unstable at speeds close tothe reference speed immediately after the loss of the dynamic performances of the estimators.

The third case is when the simplified discretisation method is used for the Luenberger and Kalman estimator implementation. Here, the sampling time used is the same with the one used in the first case. The stability study is done neglecting the process and measurement noises at a speed of 10000 [rot/min]. After the simulation results are presented in Figure 5 and Figure 6.

Comparing the two diagrams it can be observed that the effect of the simplified discretisation resembles the one of the utilization of a higher sampling time. The speed at which the estimator instability appears is higher in this case that in the previous one. In this case the control system enters the instability state at medium speeds, that makes this discretisation method impossible to be used in the control systems at high speeds. In all the presented cases the stability analysis of the Luenberger estimator was done for a *k* coefficient equal to 1.3.

Finally, the eigenvalues of the estimator and the control systems that includes a Luenberger estimator in the loop are presented, using *k=20*. The real time simulation results for the above case are presented in Figure 7.

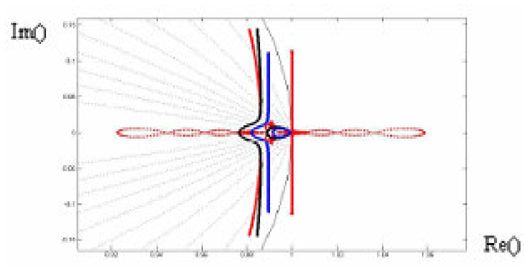

Fig. 5. The eigenvalues for the case with the Luenberger estimator used for the third case

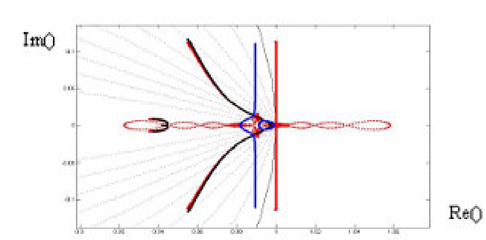

Fig. 6. The eigenvalues for the case with the Kalman estimator used for the third case

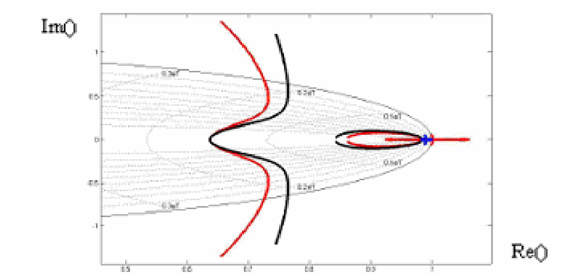

Fig. 7. The eigenvalues for the using the Luenberger estimator with k = 20

The stability study in this case was performed for a complete discretisation using the same sampling time used in the first case. The study was also performed neglecting the process and measuring noises, for a reference speed of 10000 [rot/min]. It can be noticed that the estimator and the control system enter the instability state immediately after the rotation speed of 5129 [rot/min] is overcome. It can be stated that as *k* is greater the rotation speed where instability appears is lower.

When the stability study is performed in the presence of the process and measurement noises, the eigenvalues diagrams are almost identical with the exception that the speed limit at which the dynamic performances are satisfactory decreases. This is happening more obviously in the case of the estimators and the control systems with Luenberger estimator in the control loop.

Conclusions

In the case of the estimator and the control system that includes a Luenberger estimator in the control loop it can be pointed out that the estimator becomes unstable for high speeds. When the simplified discretisation is used the stability appears faster, also influenced by the *k* amplification factor. This is also valid for the control system, because it becomes unstable as soon as the dynamic performances of the estimator are worsening, otherwise stated when the eigenvalues of the estimator remain behind the eigenvalues of the motor.

In the case of the Kalman estimator the conclusions are identical with the difference that the rotation values at which the estimator and the control system instability appears are much greater that in the case of Luenberger estimator. It can be noticed that when using a high sampling time, the eigenvalues of the motor, the estimator and some of the eigenvalues of the control system are almost identical, the only difference being athigh speeds where the dynamic performances of the estimator decrease.

By comparing the two estimators and control systems we can admit without any doubt that the estimator and the control system that includes a Kalman estimator in the loop are superior to the ones with a Luenberger estimator. The only disadvantage is the greater computing effort as well as the tuning of the covariance matrices Q and R.

Based on the presented algorithm, a real time simulation with hardware included in the control loop can be performed and it will allow the obtaining of the eigenvalues of the estimator, motor and control loop in various functioning regimes of the motor.

References

1. C. Houpis, G. Lamont, "Digital control systems – theory, hardware, soft ware", *McGraw-Hill,Inc*. (1992)
2. C.Ilas, V.Bostan, "Tehnici adaptive de control a motorului asincron: comanda vectoriala fara masurarea vitezei" , *Litografia U.P.B.* (2001) I.S. Jacobs and C.P. Bean, "Fine particles, thin films and exchange anisotropy," in *Magnetism*, vol. III, G.T. Rado and H. Suhl, Eds. New York: Academic, 1963, pp. 271-350.
3. M. Hilairet, C. Darengosse, F. Auger, P. Chevrel, "Synthesis and analysis of robust flux observers for induction machines", *IFAC Symp. on Robust Control Design*, Prague. (2000) R. Nicole, "Title of paper with only first word capitalized," *J. Name Stand. Abbrev.*, in press.
4. T. Pana , O. Stoicuta "The stability study of the Kalman filter used in the vectorial control system for the speed of an induction motor", CNAE *2004, Cluj – Napoca* (2004)
5. T. Pana, "Controlul sistemelor de actionare vectoriala cu motoare de inductie", *Ed. Medimaira Cluj-Napoca*. (2001)
6. K. Ohyama, G.M. Asher, M. Sumner "Comparative Testing of High Performance Sensorless Induction Motor Drives", *International Power Electronics Conference IPEC-Tokyo 2000*
7. H. Sugimoto "One Improving Measures of Stability in Regeneration Operation of Speed Sensroless Vector Control Induction Motor System using Adaptive Observer of Secondary Magnetic Flux", *International Power Electronics Conference IPEC-Tokyo 2000*
8. G. Griva, E. Profumo, M. Abrate, R. Bojoi "Improved Luenberger Stator Flux Observer for High Speed Sensorless Induction Motor Drives", *InternationalPower Electronics Conference IPEC-Tokyo 2000*

Studiul comparativ al sistemelor de control vectorial cu ajutorul estimatoarelor de flux Luenberger şi Kalman

Rezumat

Articolul se ocupă de studiul comparativ al sistemelor de control vectorial al motoarelor asincrone utilizând, ca estimatoare de flux rotoric, estimatoarele Luenberger sau Kalman. Studiul are în vedere domeniile vitezelor joase şi înalte. Se compară valorile proprii ale estimatoarelor pentru fiecare caz studiat.