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A Method of Calculation of the Axial Reactions of the Short Cylinders Subjected to an Uniform Internal Pressure

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Abstract

In the paper, it is presented a methodology that allows the calculation of the axial reaction in a cylinders embodied at the ends and subjected to a uniform internal pressure.
The solution is put under the form of the origin parameters and may be applied for any other types of external loads and limit conditions.
The results obtained are analysed in a calculus example.

Key words: *axial reaction, loads, shell.*

General Equations

It is considered a cylindrical shell subjected to a uniform internal pressure p (fig. 1). The cylinder has the median radius of curvature r, the semilength l and the thickness h.

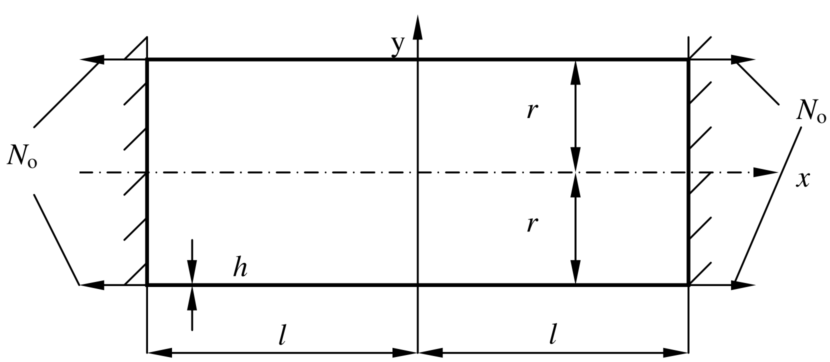

Fig. 1. The cylinder embodied at the ends

The bending of the cylindrical shell is described by the following differential equation [2]:

$$\frac{d^4 w}{dx^4} + 4\beta^4 \cdot w = \frac{1}{D}\left(p - \mu \frac{N_x}{r} \right) \tag{1}$$

where $w(x)$ is the slope of the current section, and the coefficient β is defined by the relation [1]:

$$\beta = \frac{\sqrt[4]{3\left(1-\mu^2\right)}}{\sqrt{rh}} \tag{2}$$

In the relations (1) and (2) μ represents the Poisson coefficient and D the cylindrical rigidity of the shell, $D = \dfrac{Eh^3}{12\left(1-\mu^2\right)}$.

In the bending theory the circumferential strain is expressed by the relation [1]:

$$\varepsilon_\theta = \frac{w}{r} \tag{3}$$

From the generalised Hook's law the circumferential strain can be put under the form:

$$\varepsilon_\theta = \frac{1}{E}\left(\sigma_\theta - \mu \cdot \sigma_x\right) = \frac{1}{Eh}\left(N_\theta - \mu \cdot N_x\right) \tag{4}$$

From relations (3) and (4), the particular solution of the differential equation (1) can be expressed under the form:

$$\overline{w} = \frac{r}{Eh}\left(N_\theta - \mu \cdot N_x\right) = \frac{pr^2}{Eh}\left(1 - \mu\frac{N_o}{pr}\right) \tag{5}$$

Taking into consideration the relation (5) the general solution of the differential equation (1) can be written as:

$$w = e^{-\beta \cdot x}\left(C_1 \cdot \cos\beta \cdot x + C_2 \cdot \sin\beta \cdot x\right) + e^{\beta \cdot x}\left(C_3 \cdot \cos\beta \cdot x + C_4 \cdot \sin\beta \cdot x\right) + \overline{w} \tag{6}$$

Using the differential relations between forces and external loads [1], it can be obtained the expressions of the slope, bending moment and shear force of the shell:

$$\begin{aligned} \varphi = & -\beta \cdot e^{-\beta \cdot x}\left[\left(C_1 - C_2\right)\cos\beta \cdot x + \left(C_1 + C_2\right)\sin\beta \cdot x\right] + \\ & + \beta \cdot e^{\beta \cdot x}\left[\left(C_3 + C_4\right)\cos\beta \cdot x + \left(C_4 - C_3\right)\sin\beta \cdot x\right] \end{aligned} \tag{7}$$

$$\begin{aligned} M_x = D\frac{d^2 w}{dx^2} = & \beta^2 D \cdot e^{-\beta \cdot x}\left(-2C_2 \cos\beta \cdot x + 2C_1 \sin\beta \cdot x\right) + \\ & + \beta^2 D \cdot e^{\beta \cdot x}\left(2C_4 \cos\beta \cdot x - 2C_3 \sin\beta \cdot x\right) \end{aligned} \tag{8}$$

$$\begin{aligned} T = \frac{dM}{dx} = & -\beta^3 D \cdot e^{-\beta \cdot x}\left[\left(-2C_2 - 2C_1\right)\cos\beta \cdot x + \left(2C_1 - 2C_2\right)\sin\beta \cdot x\right] + \\ & + \beta^3 D \cdot e^{\beta \cdot x}\left[\left(2C_4 - 2C_3\right)\cos\beta \cdot x - \left(2C_3 + 2C_4\right)\sin\beta \cdot x\right] \end{aligned} \tag{9}$$

The C_1, C_2, C_3 and C_4 constants can be calculated from the limit conditions :

$$x = 0 \Rightarrow w = w_o, \varphi = \varphi_o, T = T_o, M = M_o \tag{10}$$

Using the limit conditions (10), the constants C_1, C_2, C_3, and C_4 allow that the expressions of displacements, bending moment and shear force to be written under the forms:

$$w(x) = w_o \cos\beta x \cdot ch\beta x + \frac{\varphi_o}{2\beta}\left(\cos\beta x \cdot sh\beta x + \sin\beta x \cdot ch\beta x\right) + \\ + \frac{M_o}{2\beta^2 D}\sin\beta x \cdot sh\beta x + \frac{T_o}{4\beta^3 D}\left(\sin\beta x \cdot ch\beta x - \cos\beta x \cdot sh\beta x\right) + \\ + \overline{w}\left(1 - \cos\beta x \cdot ch\beta x\right) \tag{11}$$

$$\varphi(x) = w_o \cdot \beta\left(\cos\beta x \cdot sh\beta x - \sin\beta x \cdot ch\beta x\right) + \varphi_o \cdot \cos\beta x \cdot ch\beta x + \\ + \frac{M_o}{2\beta D}\left(\cos\beta x \cdot sh\beta x + \sin\beta x \cdot ch\beta x\right) + \frac{T_o}{2\beta^2 D}\sin\beta x \cdot sh\beta x + \\ + \overline{w} \cdot \beta \cdot \left(\sin\beta x \cdot ch\beta x - \cos\beta x \cdot sh\beta x\right) \tag{12}$$

$$M_x(x) = -2w_o \cdot \beta^2 D \sin\beta x \cdot sh\beta x + \varphi_o \cdot \beta D\left(\cos\beta x \cdot sh\beta x - \sin\beta x \cdot ch\beta x\right) + \\ + M_o \cdot \cos\beta x \cdot ch\beta x + \frac{T_o}{2\beta}\left(\cos\beta x \cdot sh\beta x + \sin\beta x \cdot ch\beta x\right) + \\ + 2 \cdot \beta^2 \cdot D \cdot \overline{w} \cdot \sin\beta x \cdot sh\beta x \tag{13}$$

$$T(x) = -2 \cdot w_o \beta^3 D\left(\sin\beta x \cdot ch\beta x + \cos\beta x \cdot sh\beta x\right) - 2 \cdot \varphi_o \beta^2 D \cdot \sin\beta x \cdot sh\beta x + \\ + M_o \cdot \beta\left(\cos\beta x \cdot sh\beta x - \sin\beta x \cdot ch\beta x\right) + T_o \cos\beta x \cdot ch\beta x + \\ + 2 \cdot \overline{w} \cdot \beta^3 D\left(\sin\beta x \cdot ch\beta x + \cos\beta x \cdot sh\beta x\right) \tag{14}$$

The relations (11), (12), (13), (14) have the advantage that contain the origin parameters that can be easily used for any tipes of boundary conditions.

For the cylinder from figure 1, it can be noticed that $\varphi_o = 0$ and $T_o = 0$, and at the ends have to be satisfied the limit conditions:

$$x = \pm l \Rightarrow \begin{cases} w(\pm l) = 0 \\ \varphi(\pm l) = 0 \end{cases} \tag{15}$$

Replacing the relations (11) and (12) in (15) allows the calculation of the constants w_o and M_o. After bending the length of the shell can be expressed with the relation:

$$l' = 2 \cdot \int_0^l \sqrt{1 + \varphi^2(x)} \cdot dx \tag{16}$$

The difference between the final and the initial length is:

$$\Delta l = l' - 2l \tag{17}$$

The differences between the final and initial length of the shell can be also calculated taking into consideration the axial deformation that results from the Hook's law, when the cylinder is axially loaded with the force N_o :

$$\Delta l = l' - 2l = \frac{\left(N_o 2 \cdot \pi \cdot r\right) \cdot 2 \cdot l}{EA} \tag{18}$$

where A is the area of the transversal cross section of the cylinder.

The relation (18) represents in fact an equation that contains the unknown N_o, that can be calculated numerically using a specialised calculus programme.

Calculus Example

It is considered a cylinder with the medium radius r = 500 mm, semilength l = 500 mm, thickness h and longitudinal elasticity modulus $E = 2.1.10^5$ N/mm^2. The cylinder is loaded with a uniform internal pressure p =1 N/mm^2.

In order to determine the axial reaction N_o, a specialised calculus programme has been made. The algorithm has the following steps:

- the coefficients of the system of equation (15) are calculated and the constants w_o and M_o are determined;
- it is calculated the final length of the elastic curve of the shell with the relation (16);
- it is calculated the difference between the final and the initial length of the shell using the relation (17);
- the final equation (18) is solved numerically and the axial reaction N_o is calculated. For the above example, the axial reaction has a small value ($N_o = 0.27$ N/mm), that can be neglected.

The functions of the displacement, slope, bending moment and shear force are represented respectively in the figures 2, 3, 4 and 5.

Analysing the bending moment (fig. 4), it can be noticed that the maximum value is reached at the ends: $M_x^{max} = 1500$ N.mm/mm . The strength condition can be verified in the same places:

$$\sigma_x^{max} = \frac{6 \cdot M_x^{max}}{h^2} + \frac{N_o}{h} = \frac{6 \cdot 1500}{100} + \frac{0.27}{10} = 90.027 \text{ N/mm}^2 < \sigma_a$$

Analysing the above result, it can be noticed that the axial reaction has (in this example) a small effect and can be neglected. However cases may exist in which the effect of the axial reactions is important.

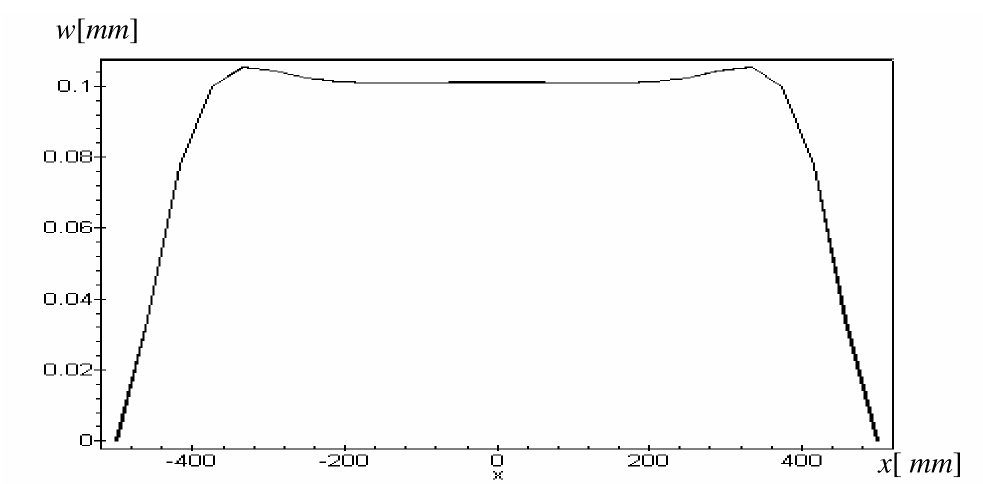

Fig. 2. The displacement of the elastic curve

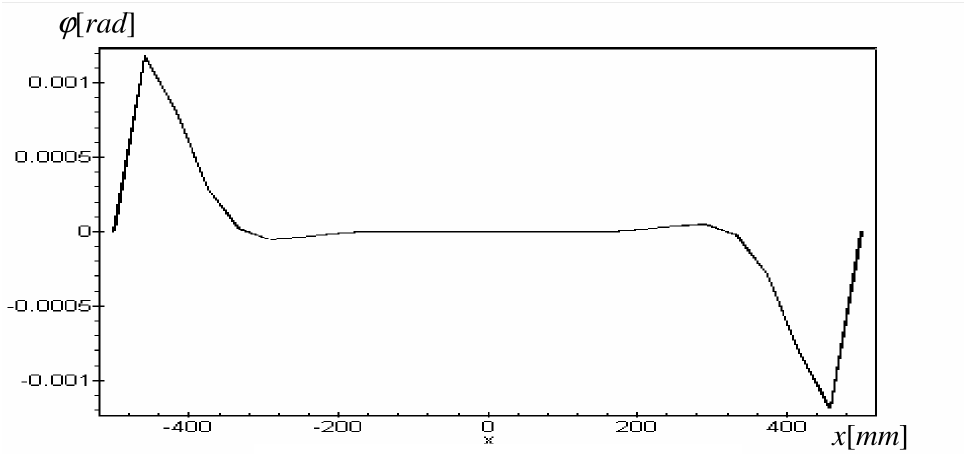

Fig. 3. The slope of the elastic curve

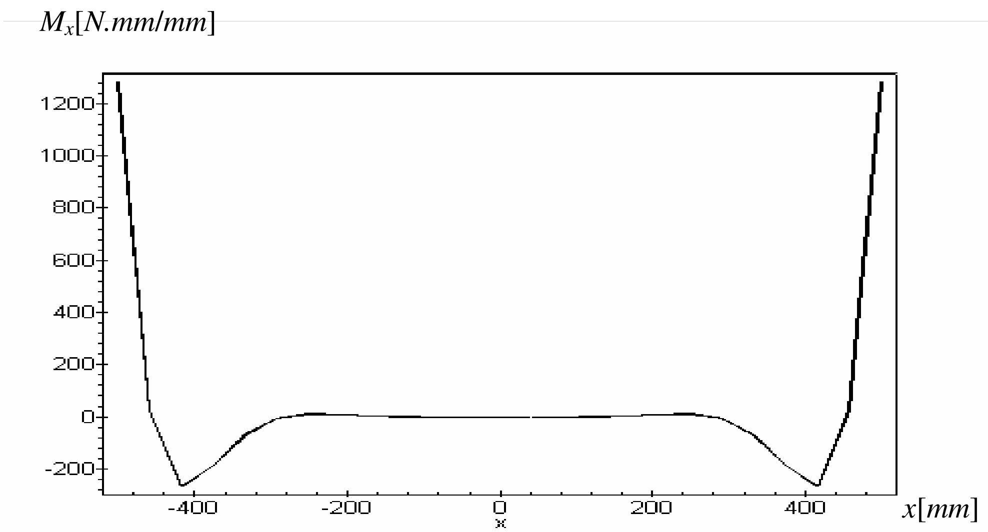

Fig. 4. The bending moment M_x

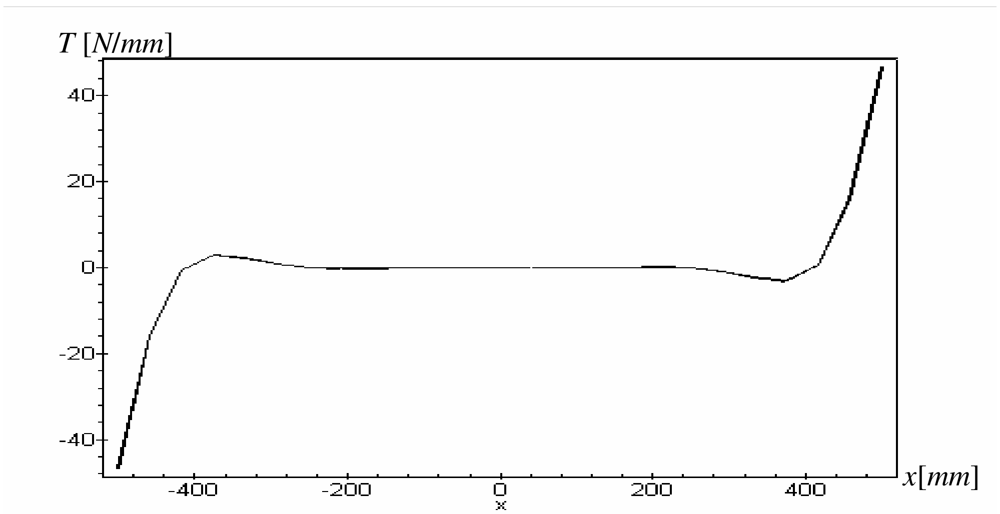

Fig. 5. The shear force T

Conclusions

In the paper, it is presented a methodology of determination of the axial reactions from a short cylindrical shell embodied at the ends and loaded with internal pressure.

The above methodology can be used also for some other types of limit conditions and the axial forces that results have to be added with those produced by the thermal effects.

References

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O metodă de calcul al reacţiunilor axiale ale cilindrilor scurţi presurizaţi interior

Rezumat

În lucrare se prezintă o metodologie de determinare a reacţiunii axiale dintr-un cilindru scurt încastrat la ambele capete şi presurizat interior cu presiune constantă. Metodologia este importantă deoarece acest efect era neglijat până în prezent şi evidenţierea sa poate avea efecte care, suprapuse peste cele termice să conducă la solicitări importante. Metodologia prezentată este aplicabilă oricăror tipuri de legături şi oricăror sarcini transversale. Soluţia ecuaţiei diferenţiale obţinute este pusă sub forma parametrilor în origine, ceea ce facilitează determinarea constantelor din condiţii la limită. Rezultatele obţinute sunt analizate pe un exemplu de calcul.