# The Servovalves Model Used in the Dynamic Sensitivity Analysis

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#### Abstract

In this paper, there is presented a variant of the linear model for the type of electrohydraulic servovalve with two amplification stages from the nonlinear status equation; the coefficients of the linear model are described in terms of parameters of the hydraulic valve.

Key words: dynamic model, electrohydraulic servovalve, the coefficients of the linear model.

# **Dynamic Models of the Electrohydraulic Actuators**

The electrohydraulic servo system shown in fig. 1 consists of a two stage flow control servovalve and a double–end actuator. The servovalve has a symmetrical double nozzle and a torque motor driven flapper for the first stage, and a closed centre four–way sliding spool for the second stage. Figure 1 displays two types of feedback spring commonly used: a cantilever spring connecting the flapper and spool, and a spring directly acting on the spool.

The system nonlinear model and linearised model are presented for each type of feedback spring. The nonlinear dynamic model presented below is a compilation of results from Merritt Watton, and Lin and Akers. The system has ten state variables and two input variables as depicted in fig.1.

 $\theta = \Omega$ 

The torque motor stage dynamics are given by [3]:

(1)

$$\Omega = 1/J\{K_1 i - (K_a - K_m)\theta - K[x_s + (R + B)\theta] - B_v \Omega - (p_{s1} - p_{s2})XA_nR - (2)\}$$

$$-4\pi C^{2}_{qn} R[(X_{tm} - R\theta)^{2} p_{s1} - (X_{tm} + R\theta)^{2} p_{s2}]\}$$

where  $\Omega$  and  $\theta$  both represent the rotation angle of the palette; J – the inertia moment of the mobile equipment; K – the amplification in current; R, B – rigidity coefficients,  $K_a = K_m$  – the coefficient which aims at the elasticity of the lamella;  $p_{S1}$  and  $p_{S2}$  - the pressures from the two control rooms.

The first term from the equation of the dynamic model of the couple engine (2) is the driving moment of the torque motor. The last five terms are restoring moments on the armature from the net rotational stiffness, the cantilever feedback spring stiffness while this spring exists, the

damping in the torsion moment, the pressure difference across the nozzle, and the dynamic flow force on the flap (palette), respectively. The flapper nozzle stage dynamics are given by:

$$p_{S1} = \beta_1 / V_{S1} \{ C_{qn} A_0 \sqrt{\frac{1(P_S - p_{S1})}{p}} - C_{qn} a_{01} - \sqrt{\frac{1p_{S1}}{p}} - A_{sVs} \}$$
(3)

$$p_{S2} = \beta_1 / V_{S2} \{ C_{qn} A_0 \sqrt{\frac{2(P_s - p_{S2})}{p}} - C_{qn} a_{02} \sqrt{\frac{2p_{S2}}{p}} - A_{sVs} \}$$
(4)

where:  $P_s$  is the liquid pressure from the source;  $V_{s1}$  and  $V_{s2}$  – the fluid volume from the left/right control room;  $C_{gn}$  – the flow coefficient; p – the fluid density; l – constant flow rate;  $A_0$  – the variable surface;  $a_{01} = (X_{tm} - R\theta)\pi D_0$  – curtain aria (cylindrical surface of energy);  $a_{02} = (X_{tm} - R\theta)\pi D_0$  – curtain aria;  $D_0$  – the diameter of the nozzle orifice;  $V_{s1} = V_{s0} + Ax_s$ ;  $V_{s2} = V_{s0} - Ax_s$ .



Fig. 1. Electrohydraulic servo-actuator

The two terms in equation (3) refer to the volume flows on the  $p_1$  side, through the first orifice and through the nozzle, respectively. The first two terms from equation (4) refer to the volume flow on the  $p_2$  side, through the second orifice, and through the nozzle respectively. Lin and Akers included a leakage volume flow term:  $q_1=K_1(p_1 - p_2)$  which is omitted in this paper. The force balance on the spool and the movement equation of the mobile armature are given by the following two relationships[4]:  $x_s$ 

$$=v_s$$
 (5)

$$v_s = 1/M_s \{ (p_{s1} - p_{s2})A_s - B_s v_s - f - [\rho L_2 q_1 - \rho L_1 q_2] - F_{sp} \}$$
(6)

$$q_1 = C_q W x \sqrt{\frac{2(P_s - p_{a1})}{p}}, \quad q_2 = C_q W x \sqrt{\frac{2p_{a2}}{p}}$$
(7)

$$v_s > 0, \ f = 0, \ q_1 = 0, \ q_2 = 0$$
 (8)

$$v_s > 0 \quad f = 2C_q W x \cos \theta_t (P_s - p_{a2} + p_{a1})$$
 (9)

$$q_1 = C_q W x \sqrt{\frac{2p_{a1}}{p}}, \quad q_2 = C_q W x \sqrt{\frac{2(P_s - p_{a2})}{p}}$$
 (10)

where  $q_1$  and  $q_2$  are the flow rate coefficients (depending on the sign of  $v_s$ ); W – the form coefficient of the nozzle;  $p_{a1}$  and  $p_{a2}$  – the pressures in the two control rooms; f – the friction force;  $\theta$  – the reaction angle of the fluids nozzle;  $C_q$  – flow rate coefficient; x – the palette movement according to the direction of the nozzle axle.

The first two terms from equation (6) are the forces from the spool pressures and the spool damping respectively. The next two terms are the flow reaction force, which depends on the actuator pressure, and the transient flow force which depends on the actuator flow rates. These two terms show the dependence of the servovalve upon the actuator dynamic given below. The last term is the restoring force from the feedback spring [3]:

$$F_{sp} = K[x_s - (R+B)\theta]/(R+B) + 2K$$
(11)

where K is the elastic constant of the arc main spring. Note that K = 0 for a servovalve with a cantilever feedback spring and K = 0 with a direct feedback spring.

The continuous flow through the actuator leads to the following relations for the control pressures (the pressures in the two control rooms):

$$p_{a1} = \beta_0 / V_{a1}(q_1 - A_1 - v_a), \quad p_{a2} = \beta_0 / V_{a2}(A_1 v_a - q_2)$$
(12)

where:  $V_{a1} = V_{a0} + A_1 x_a$  and  $V_{a2} = V_{a0} - A_1 x_a$  are the fluid volumes from the two rooms.

Finally, the force balance on actuator is given by:

$$x_a = v_a \tag{13}$$

$$c_a = 1/M_t [(p_{a1} - p_{a2}) - A_1 - B_a v_a - f_d]$$
(14)

where  $M_t$  represents the engine weight.

The linearised equations are derived for the two cases of -0 and +0 and are evaluated at the equilibrium states. It is found that the two cases converge to the same linearized equation. It is also found that the servovalve dynamics do not depend on the actuator pressures and flow rates as they do in the nonlinear model. Therefore the linearized model can be represented as a servovalve model in cascade with an actuator model through the spool position. This property may not be preserved if the linearization is conducted around nonzero inputs.

The linearised actuator model derived herein is identical to the existing results [1], [4]:

$$x_a(s)/x_s(s) = -c_1(s^3 + c_2s^2 + c_3s)$$
(15)

where  $(s^3 + c_2 s^2 + c_3 s)$  represents the transfer function, the constants  $c_1, c_2, c_3 > 0$ , and  $x_a(s)$  is the movement of the engine piston (actuator).

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This third order model has one less state than expected from the four state equations (13-15). This is because only the actuator pressure difference  $p_{a1}$ - $p_{a2}$  is used instead of the individual pressures. The transfer function between the actuator position and the perturbing force is:

$$x_a(s) / f_d(s) = -c_4(s^2 + c_2s + c_3)$$
(16)

where  $f_d(s)$  is the perturbing force.

The coefficients  $c_1, ..., c_4$  are expressed in terms of system parameters. The derived linearised model of the two-stage servovalve has the form of a transfer function:

$$x_s(s)/1(s) = (-b_0s + b_1)/(s^5 - a_1s^4 + a_2s^3 + a_3s^2 + a_4s + a_5)$$
(17)

where s is a Laplace variable, and  $a_i$  the coefficients that appear in the transfer function.

The presented servovalve suggests a model of relative structure of the order 4 or 5 depending on the spring type. The coefficients  $a_i$ , i = 1, 2, ..., 5, of the model explicitly expressed in terms of parameters of the physical system can be used for the analysis of the sensibility or of the servovalves dynamic design in different ways. For a servovalve with a feedback spring on the spool  $b_0 = 0$ . The transfer function coefficients are expressed in terms of the servovalve parameters. This fifth order model has one less state than expected from the six state equations (1-10). This is because only the spool pressure difference is used instead of the individual pressures. The derived servovalve model of the equation (17) is different from the models used in previous literature. Some literature [2-4] has assumed a transfer function of the second order of the form:

$$x_{s}(s)/l(s) = k_{2}/(s^{2} - 2\xi\omega_{n}s - \omega_{n}^{2})$$
(18)

Merit [4] derived a transfer function of the third order having the form:

$$x_{s}(s)/l(s) = K_{3}/(s^{3} + h_{1}s^{2} + h_{2}s - h_{3})$$
<sup>(19)</sup>

by neglecting the spool valve resonance, pressure feedback on the flapper and the flow forces on the spool. The servovalve derived model of the equation (19) suggests a model structure of relative order of four or five, depending on the type of spring feedback. The model coefficients expressed explicitly in terms of the system's physical parameters can be used for sensitivity analysis or design of the servovalve dynamics in various ways. For example, if only some servovalve parameters are known, this model can identify the unknown parameters from experimental data. As another example, the new model can test the effect on the servovalve dynamics of a given parameter, such as the cantilever spring constant K.

#### **Experimental System**

The experimental system consists of a Moog servovalve utilising a cantilever feedback spring and a 0.9 kg double–end actuator with an effective area of 3.613 e<sup>4</sup> m<sup>2</sup> (0.56m<sup>2</sup>). The supply pressure is 18.6 MPa. Mobil DTE Light oil is used at an average temperature of  $32^{0}$ C. The actuator and spool are connected to linear variable differential transducers (LVDT) with  $\pm$  2.5 mV noise level. While the servovalve physical parameters are not available for model verification, the experimental frequency responses for the servovalve and the actuator are measured with a signal analyzer, using a "swept sine" method that generates fixed amplitude sine waves of varying frequencies. This is then used to validate the model order and structure. It is desired to measure this data near the equilibrium actuator position  $X_{nr} = 0$ . Since the actuator is of type 1 as the equation (22) indicates, it is difficult to keep the actuator near the equilibrium position in open loop. Therefore a proportional control loop (gain = 1) is used to create a stabilized plant, as shown in fig.2. With different input amplitude levels (15 mV, 30 mV) and 50 mV the frequency responses of the servovalve and actuator area measured are shown in fig.3. Using the frequency responses from the three input amplitudes, an averaged frequency response is computed for the servovalve and the actuator and nominal models are fitted through the use of an equation – error method.



Fig. 3. Frequency responses of the servovalve

We recall that K = 0 for a servovalve with a cantilever feedback spring and K = 0 for a servovalve with a direct spool feedback spring.

The equations contain three unknowns:  $p_{x2}(s) - p_{x2}(x)$ ,  $\theta(s)$  and  $Y_x(s)$ . These unknowns are solved as a function of the input current. The transfer function of the entire servovalve and the transfer function of the couple engine are given by the following two equations respectively:

$$x_{s}(s)/l(s) = (-b_{0}s + b_{1})/(s^{5} - a_{1}s^{4} + a_{2}s^{3} + a_{3}s^{2} + a_{4}s + a_{5})$$
(20)

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$$\theta_s(s)/i(s) = (f_0s^3 + f_1s^2 + f_2s + f_3)/(s^5 + a_1s^4 + a_2s^3 + a_3s^2 + a_4s + a_5)$$
(21)

where [3]:

$$f_0 = D_3, f_1 = D_3 \{D_2 + D_{12}\}, f_2 = D_3 \{D_5 + D_2 D_6 + 2D_3 D_4\}$$
  
 
$$f_3 = D_3 D_2 D_5, g_0 = 2D_3 D_1, g_1 = 2D_3 \{D_1 D_6 + D_3 D_1\}, g_2 = 2D_3 D_1 D_5$$

$$D_1 = K / M_s, D_2 = A_s / M_s, D_3 = KA_s / M, D_4 = A_s / M_s$$
$$D_5 = [K / (R + B) + 2C_q W \cos \theta_f P_s + 2K_s] / M_s D_6 = [B_s + (L_2 - L_1)C_q W \sqrt{P_s q}] / M_s$$

#### Conclusions

The presented servovalve suggests a model structure of relative order of four or five depending on the type of spring feedback. The coefficients of the model expressed explicitly in terms of the system's physical parameters can be used for sensitivity analysis or design of the servovalve dynamics in various ways.

### References

- 1. Dean, H. Kim, Tsu-Chin, Tsao Journal of Dynamic Systems, Measurement, and Control, Vol.122, March 2000.
- 2. Oprean, A., Ispas, C., Ciobanu, E., Dorin, A., Meda, S., Olaru, A., Prodan, D. -Acționari și automatizări hidraulice, Editura Tehnică, București, 1989.
- 3. Călinoiu, C., Vasiliu, N., Vasiliu, D., Catană, I. Modelarea, simularea și identificarea experimentală a servomecanismelor hidraulice, Editura Tehnică, București, 1998.
- 4. Choi, Y. Vibration characteristics of a composite beam containing an electrorheological fluid, *Journal of Intelligent Material Systems and Structures*, 1990, pp.91-104.
- 5. Bobasu, E. Conducerea sistemelor electrohidraulice, Editura Avrameanca, Craiova, 1997.
- 6. Vasiliu, N., Catana, I. Transmisii hidraulice și electrohidraulice, Mașini hidraulice volumice, Editura Tehnică, Bucuresti, 1988,
- 7. Florea, J. Mecanica fluidelor, Editura Didactică și Pedagogică, București, 1979.
- 8. Mazilu, I., Marin, V. Sisteme hidraulice automate, Editura Academiei Române, București, 1982.
- 9. Bonnecaze, R.T. Dynamic simulation of an electrorheological fluid, *Journal of Chemical Physics*, **96**, 1992, pp.2183-2202.

# Model al servovalvei utilizat pentru analiza de senzitivitate dinamică

# Rezumat

În acest articol, este prezentată o variantă a modelului linear pentru tipul de servovalvă electrohidraulică cu două trepte de amplificare, obținută din ecuația de stare neliniară; coeficienții modelului linear sunt prezentați în termenii parametrilor valvei hidraulice.