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Axial Vibrations of Continuous Beams with Concentrated Mass

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Abstract

In the paper is presented a methodology that allows a dynamical analysis of a continuous beam with some supplimentary concentrated mass for a real case of a drilling equipment.
Based on the theory of vibrations of continuous beams in the paper is developed a numerical algorithm that allows the calculation of the proper frequencies for such a structure. There are analysed some different positions of the concentrated mass and the influence of this positions in the proper frequencies is obtained.

Keywords: *axial vibration, proper frequencies, concentrated mass*

General Equations

It is considered a vertical continuous beam with the length l, the aria of the cross section A, density ρ and specific mass $\overline{m}$ (fig. 1a).

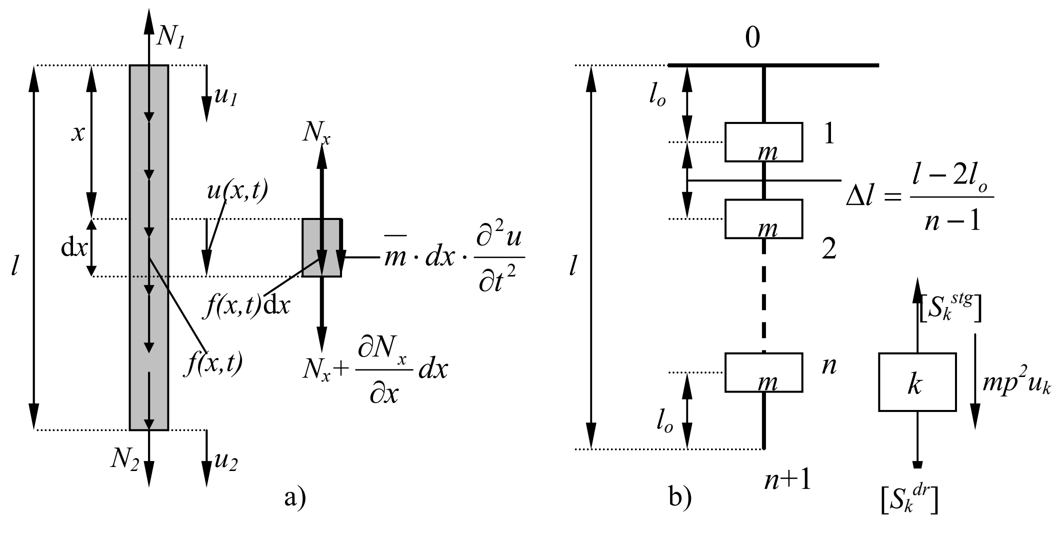

Fig. 1. *Loads and geometry of continuous beam with concentrated mass*

In the hypothesis that the cross sectional aria is constant along the length of the beam, the differential equation of the axial vibrations of the beam is [1] :

$$EA\frac{\partial^2 u}{\partial x^2} - \overline{m}\frac{\partial^2 u}{\partial t^2} = -f(x,t) \tag{1}$$

where *EA* is the axial rigidity of the beam, *u(x,t)* is the displacement in the current section of the beam and *f(x,t)* is the external force that acts along the beam. If the external forces are absent the above equality becomes :

$$EA\frac{\partial^2 u}{\partial x^2} - \overline{m}\frac{\partial^2 u}{\partial t^2} = 0 \tag{2}$$

Using the Fourier method the *u(x,t)* function can be written under the form :

$$u(x,t) = u_x(x) \cdot T(t) \tag{3}$$

and from (2) the following differential equation is obtained :

$$\frac{d^2 u}{dx^2} + a^2 \cdot u_x = 0 \tag{4}$$

where the *a* parameter is defined by the relation :

$$a^2 = \frac{\overline{m} \cdot p^2}{EA} = \frac{\rho}{E} p^2 \tag{5}$$

The solution of the (4) differential equation is put under the form :

$$u_x = C_1 \sin ax + C_2 \cos ax \tag{6}$$

The (4) differential equation has to verify the following limit conditions :

$$x = 0 \Rightarrow u_x = u_1 \qquad \text{a)}$$

$$x = l \Rightarrow N_x = EA\frac{du_x}{dx} = N_1 \qquad \text{b)} \tag{7}$$

Taking into consideration the (7) limit conditions, the expressions for u_x and N_x become :

$$u_x = u_1 \cos ax + \frac{N_1}{EAa}\sin ax \qquad \text{a)}$$

$$N_x = -EAa \cdot u_1 \cdot \sin ax + N_1 \cos ax \qquad \text{b)} \tag{8}$$

The (8) relations suggest a matriceal form like :

$$\begin{bmatrix} u_x \\ N_x \end{bmatrix} = \begin{bmatrix} \cos ax & \frac{1}{EAa}\sin ax \\ -EAa\sin ax & \cos ax \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ N_1 \end{bmatrix} \tag{9}$$

or a concentrated form as :

$$[S_x] = [D_x] \cdot [S_1] \tag{9'}$$

The matrixes from (9') are obtained by a direct identification between (9) and (9') equalities.

The (9') relation allows the written of a matriceal expression between the dynamical parameters from the ends of the beam presented in figure 1a :

$$[S_2]=[D_1]\cdot[S_1] \tag{10}$$

When the beam contains supplementary concentrated mass (fig.1b) the passing over such a mass can be expressed by the relation :

$$[S_1^{dr}]=\begin{bmatrix}1 & 0\\ -mp^2 & 1\end{bmatrix}\cdot[S_1^{stg}]=[\overline{D_1}]\cdot[S_1^{stg}] \tag{11}$$

If all the concentrated mass are taken into consideration and the fact that the lengths between to consecutive mass are identically, the (10) and (11) relations can be written successively :

$$
\begin{aligned}
&[S_1^{stg}]=[D_1]\cdot[S_o]\\
&[S_1^{dr}]=[\overline{D_1}]\cdot[S_1^{stg}]=[\overline{D_1}]\cdot[D_1]\cdot[S_o]\\
&[S_2^{stg}]=[D_2]\cdot[S_1^{dr}]=[D_2]\cdot[\overline{D_1}]\cdot[D_1]\cdot[S_o]\\
&[S_2^{dr}]=[\overline{D_2}]\cdot[S_2^{stg}]=[\overline{D_2}]\cdot[D_2]\cdot[\overline{D_1}]\cdot[D_1]\cdot[S_o] \qquad (12)\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&[S_n^{dr}]=[\overline{D_n}]\cdot[D_n]\cdot[\overline{D_{n-1}}]\cdot[D_{n-1}]\cdots[\overline{D_1}]\cdot[D_1]\cdot[S_o]\\
&[S_{n+1}]=[D_{n+1}]\cdot[S_n^{dr}]=[D_{n+1}]\cdot[\overline{D_n}]\cdot[D_n]\cdot[\overline{D_{n-1}}]\cdot[D_{n-1}]\cdots[\overline{D_1}]\cdot[D_1]\cdot[S_o]
\end{aligned}
$$

The dynamical matrixes that appear in (12) have the following forms :

$$[D_1]=[D_{n+1}]=\begin{bmatrix}\cos al_o & \frac{1}{EAa}\sin al_o\\ -EAa\sin al_o & \cos al_o\end{bmatrix}=[M_1] \qquad \text{a)}$$

$$[D_2]=[D_3]=...=[D_n]=\begin{bmatrix}\cos a\Delta l & \frac{1}{EAa}\sin a\Delta l\\ -EAa\sin a\Delta l & \cos a\Delta l\end{bmatrix} \qquad \text{b)} \qquad (13)$$

$$[\overline{D_1}]=[\overline{D_2}]=...=[\overline{D_n}]=\begin{bmatrix}1 & 0\\ -mp^2 & 1\end{bmatrix}=[M_2] \qquad \text{c)}$$

Taking into consideration (13) the products between the matrixes $[\overline{D_n}]\cdot[D_n]$ become :

$$[\overline{D_1}]\cdot[D_1]=\begin{bmatrix}1 & 0\\ -mp^2 & 1\end{bmatrix}\cdot\begin{bmatrix}\cos al_o & \frac{1}{EAa}\sin al_o\\ -EAa\sin al_o & \cos al_o\end{bmatrix}=$$

$$\begin{bmatrix}\cos al_o & \frac{1}{EAa}\sin al_o\\ -EAa\sin al_o-mp^2\cos al_o & \cos al_o-\frac{ma}{\rho A}\sin al_o\end{bmatrix}=[M_3] \qquad \text{a)}$$

$$\left[\overline{D_2}\right]\cdot\left[D_2\right]=\ldots=\left[\overline{D_n}\right]\cdot\left[D_n\right]=$$

$$\begin{bmatrix} \cos a\Delta l & \frac{1}{EAa}\sin a\Delta l \\ -EAa\sin a\Delta l - mp^2\cos a\Delta l & \cos a\Delta l - \frac{ma}{\rho A}\sin a\Delta l \end{bmatrix} = \left[M_4\right] \quad \text{b)} \qquad (14)$$

Taking into consideration the (13) and (14) relations, the last equality from (12) can be written under the general form :

$$\left[S_{n+1}\right]=\left[M_1\right]\cdot\left[M_4\right]^{n-1}\cdot\left[M_3\right]\cdot\left[S_o\right]=\left[M\right]\cdot\left[S_o\right] \qquad (15)$$

Because all the matrixes from (15) have two lines and two columns, the above relation can be put under the concentrated form :

$$\left[S_{n+1}\right]=\begin{bmatrix} u_{n+1} \\ N_{n+1} \end{bmatrix}=\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}\cdot\begin{bmatrix} u_o \\ N_o \end{bmatrix} \qquad (16)$$

For a beam embedded at the superior end and free at the inferior one (fig.1b), the limit conditions are $u_o = 0$ and $N_{n+1} = 0$. From (16) the following equation is obtained :

$$d_{22} = 0 \qquad (17)$$

If the beam is embedded at the superior end and axially supported at the inferior one, the limit conditions are $u_o = 0$ and $u_{n+1} = 0$. From (16) the following equation is obtained :

$$d_{12} = 0 \qquad (18)$$

Calculus Example

It is considered a beam with l = 1000 *m* made from steel with ρ = 7850 kg/m^3. The continuous beam contains also n = 10 concentrated mass with m = 100 kg, placed at a variable distance l_o. The dynamical analysis of the free vibrations of the beam has been made in two hypothesis : when the beem is embedded at the superior end and free at the inferior one(equation (17) is valid) and when the beam is embedded at the superior and axially simply supported at the inferior one (equation (18) is valid). Using (17) and (18) equations the first free proper frequencies have been calculated for different values for l_o distance in the interval (20 m..450 m).

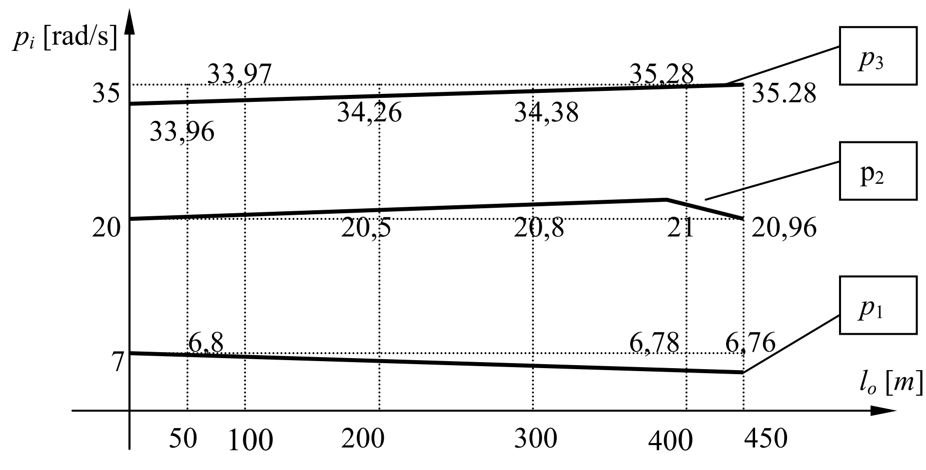

Fig. 2

For the first case (embedded at the superior end and free at the inferior one) the variations of the first free proper frequencies are presented in the figure 2.

For the second case (embedded at the superior end and axially simply supported at the other one the first free frequencies are presented in the figure 3.

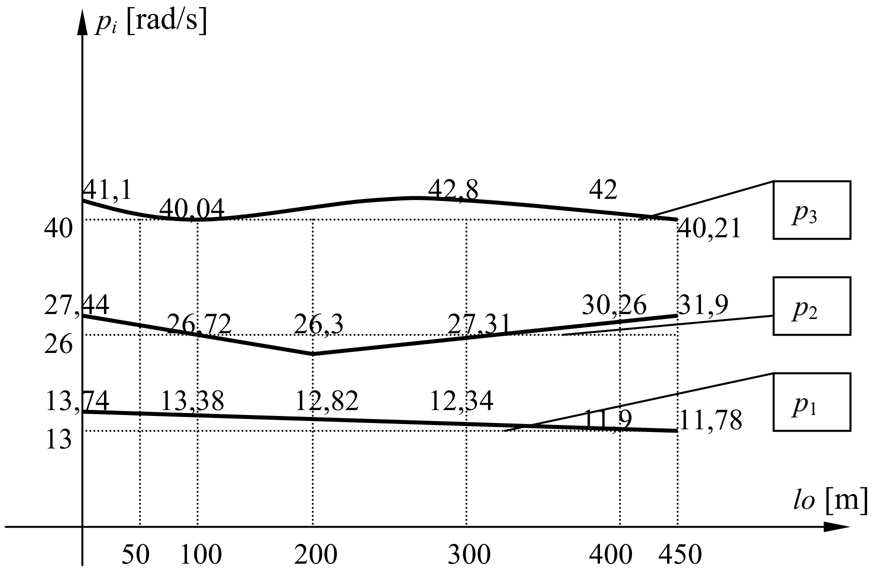

Fig. 3

In order to solve numerically the (17) and (18) equations algorithm and calculus programmes have been developed. The first three roots of the above mentioned equations are the first three proper frequencies of the structure. Analysing the results presented in figure 2 it can be noticed that :

- the first proper frequency (the fundamental one) is nearly constant and does not depend of the positions of the concentrated mass;
- the second proper frequency has a very small variation (nearly 3%);
- the third proper frequency has also a small variation, but higher that the second one (nearly 6%).

When the beam is embedded at the superior and axially simply supported at the inferior one, the results are presented in figure 3. Analysing these results it can be noticed that :

- all the proper frequencies have higher variation than those presented in the above case;
- the first proper frequency has a variation of 16%;
- the second frequency has a variation of 21%;
- the third frequency has a variation of 7%.

From the diagrams presented in figures 2 and 3 it can be noticed that the position of the concentrated mass has an important influence of the proper circular frequencies, reaching the absolute variation of 21%.

The exact calculation of the proper circular frequencies is important because the position of the concentrated mass may be a technological method of avoiding the dangerous dynamical phenomena.

For example, it can be establish a distance between the concentrated mass in order to obtain the values of the proper circular frequencies far away from the external frequencies of the perturbations.

Conclusions

In the paper are presented a methodology of determination of the axial proper circular frequencies for a continuous beam loaded with some concentrated mass. The results obtained allow to obtain the influence of the distance between the concentrated mass in the values of the proper frequencies. The algorithm has been analysed in a calculus example.
The above methodology can be used also for some other types of limit conditions.

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Vibrații axiale ale barelor cu masa continuă încărcate cu mase concentrate

Rezumat

În lucrare se prezintă o metodologie de determinare a pulsațiilor proprii axiale ale barelor cu masa continuă, încărcate cu mase concentrate suplimentare. Metodologia de calcul este bazată pe teoria vibrațiilor barelor cu masa continuă şi este dezvoltat un program de calcul, care furnizează frecvențele proprii ale structurii. Rezultatele obținute sunt abnalizate pe un exemplu de calcul, în două condiții de rezemare. Este evidențiată înfluența pe care distanța dintre mase o induce în valorile pulsațiilor proprii şi este sugerată ideea că, această poziționare a maselor se poate constitui într-o metoda eficientă prin care pot fi evitate fenomenele dinamice periculoase.