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The Construction Of Point Projection On The Hyperboloid Surfaces

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Abstract

In this paper, the authors presented the complete construction of a point on the hyperboloid sheet when one of its projections (in this case, the vertical one) is known. We showed the hyperboloid projections by drawing its characteristic elements: the minor axis, the necklace circle, the foci, the centre of the generating hyperbola and the asymptotes. The authors used o few helpful constructions for the purpose of solving this problem. Because of the hyperboloid symmetry in comparison with the minor axis, they found two solutions for the problem. Also, they consider that this procedure can be extended into another case, such as the construction in a lateral projection of points laying on the hyperboloid.

Keywords: *hyperboloid, construction, points.*

Theoretical considerations

The general canonical form of the hyperboloid of one sheet is described by the general equation:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1 \qquad (1).$$

In mathematics, it is a type of surface in three dimensions, known as a quadric. A hyperboloid of one sheet can be obtained by revolving a hyperbola around an axis, known as “the semi-minor axis”.

So, in a plane, a hyperbola can be described by the following reduced equation:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}-1=0 \qquad (2),$$

where a is the semi-major axis length and b is the semi-minor axis length. The $A(a,\ 0)$ and $A`(-a,\ 0)$ are the bend points (vertices) of the hyperbola. In comparison with a Cartesian coordinate system, they can define the focus distance $2c$. A point M(x,y) belongs to a hyperbola if, beside the condition (2), it is also satisfied the following algebraic relation:

$$b^2=c^2-a^2 \qquad (3).$$

The lines, which pass through the coordinate system origin and have $\pm\frac{b}{a}$ as slopes are named the asymptotes of the hyperbola and, at large distance from the foci, the branches of the hyperbola begin to approximate them.

The construction of a hyperboloid of one sheet

This kind of hyperboloid is obtained by rotating a hyperbola *(K)* around a semi-minor *(Δ)* (as it is shown in fig.1). Also, we can say that also a surface of a hyperboloid of one sheet it is obtained if, around a given axis, they revolve a line which is not situated in the same plane with the *(Δ)* axis (skew line). Because of that, this hyperboloid is called also a “doubly ruled surface”.

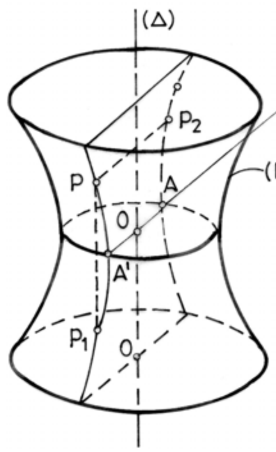

Fig. 1. Points on hyperboloid

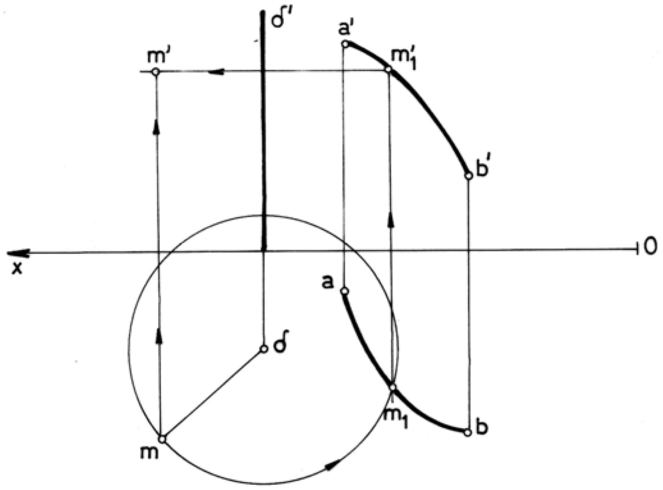

Fig. 2. Generating a revolved surface

In general, in the three dimensional space, we can generate a revolved surface if an arc $\overset{\frown}{AB}\,(\overset{\frown}{ab}, \overset{\frown}{a`b`})$ is revolved around a vertical axis *(Δ)(δ, δ`),* as it is shown in figure 2. By example, if we wish to find the vertical projection of a point *M*, which is situated on the sheet generated by the arc $\overset{\frown}{AB}$ revolving, we must make the following steps:

- we draw the circle having the *δm* radius;
- we determine the intersection m_1 between the circle and the horizontal projection of the arc, $\overset{\frown}{ab}$;
- by drawing the perpendicular line from m_1 on *(ox)* axis, we determine the vertical projection $m_1`$ situated on the same projection, meaning the vertical one of the arc $\overset{\frown}{a`b`}$;
- by drawing the perpendicular line from *m* on *(ox)* axis and the parallel from $m_1`$ to *(ox)* axis, we can find, at their intersection, the searched vertical projection of the point *M*, which is $m`$.

The construction of points on a hyperboloid surface using the triple orthographic projection

In figure 3 it is shown the construction of points on a hyperboloid surface, having their projection on the vertical plane *[V]* as known data.

The authors considered that the hyperboloid of one sheet is obtained by revolving the hyperbola *(K)* around a line *(Δ) (δ, δ`),* perpendicular on the *[V]* plane. The *(Δ)* line (or axis) is a minor axis for the hyperbola *(K)*. The point *A (a, a`*, where *a* is the horizontal projection and *a`* the vertical one, is the bend point of the hyperbola and shows the limit of the "necklace circle" of the hyperboloid. One of the two hyperbola foci is given by the *F (f, f`)* point and the hyperbola centre by *K (k, k`)* point. It is obvious that the minor axis *(Δ)* passes through the *K* point. Knowing the vertical projection *p`* of a given point *P* and also that this one belongs to the hyperboloid surface, we put the problem to determinate the horizontal projection of this point and, also, to find its number of solutions. To solve this given problem, we start from the construction principle shown in figure 2.

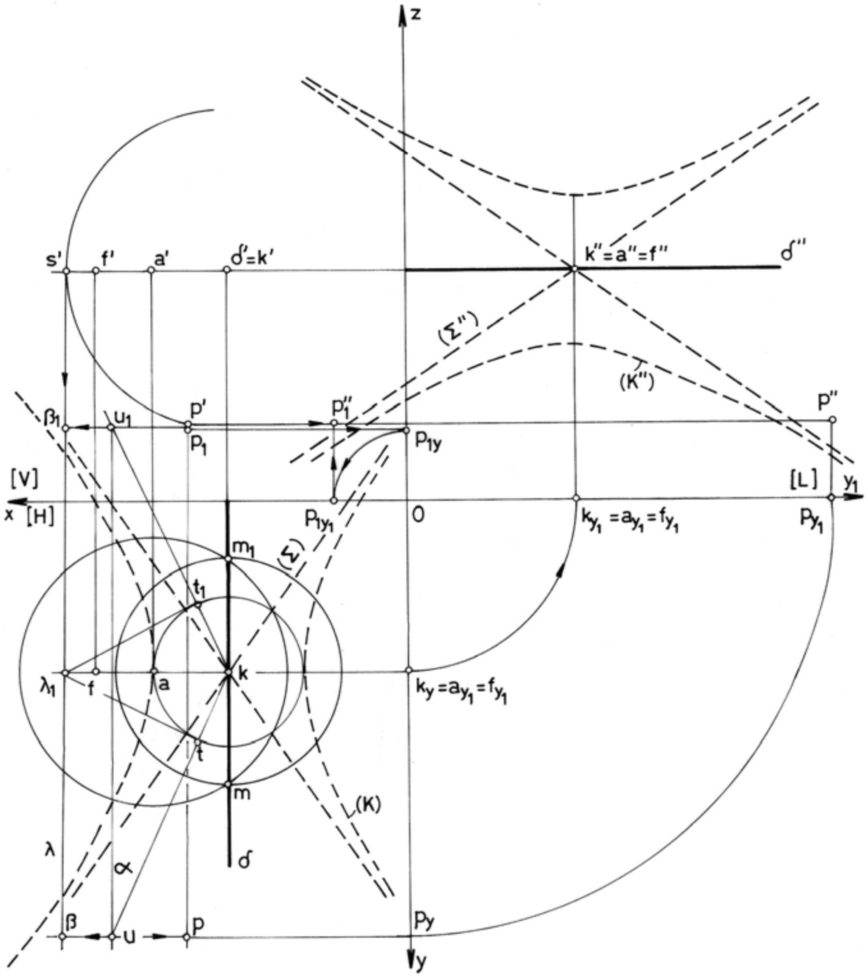

Fig. 3. Points construction on a hyperboloid surface

Firstly, we draw the asymptotes of the *(K)* hyperbola, starting from the relation (4) and using the relations (1), (2), (3). The projections of these asymptotes are drawn using dotted lines, in horizontal projection being *(Σ)* and in lateral projection *(Σ``)*. Using the hyperbola equation (2), we drew the horizontal projection *(K)* and lateral projection *(K``)* of the hyperbola that generates the hyperboloid, which was not limited in the figure (3) by any perpendicular plane on the *(Δ)* line.

So, the circle having the centre in *k`* and the *k`p`* radius intersects the parallel drawn through *k`* to the *(ox)* axis in *s`*. The corresponding line that goes through s` intersects the *(K)* hyperbola in β and β_1 points, of which drawing will be shown as it follows.

The circle having the centre in *a* point and the *kf* radius intersects the horizontal projection of the minor axis δ in the *m* and m_1 points. We draw the tangent line α to the circle having the k centre and the mm_1 diameter. The tangent line α is parallel with the horizontal projection δ and intersects the *ka* line in the λ_1 point. We draw $\lambda_1 t$ and $\lambda_1 t_1$, which are the tangent lines through λ_1 point to the circle of *ka* radius from the horizontal projection of the hyperbola (corresponding to the necklace circle). The lines *kt* and kt_1 intersect α in the *u* and u_1 points. We draw the parallels through *u* and u_1 points to *(ka)*, which intersect the λ line in the β and β_1 points and, also, the corresponding line of *p`* point in *p* and p_1. And so, we obtain two solutions for this problem, as we expected, because of the symmetry of the hyperboloid in comparison with the *(Δ)* line.

Then, using the known procedures, we draw the lateral projections *p``* and $p_1$`` on the hyperboloid sheet lateral projection, (*K``)*. By obvious motives, the *(*K) and (K``) hyperbolas weren't strictly limited, with the aim of solving this theoretical problem. In vertical projection, the hyperbola is visible only through the necklace circle having the *k`* centre and *k`a`* as radius. In the same way, we can find also the vertical projection (projections) of point laying on the hyperboloid sheet, for which we know only the horizontal projection.

Conclusions

In this paper, the authors presented the complete construction of a point on the hyperboloid sheet when one of its projections (in this case, the vertical one) is known. We showed the hyperboloid projections by drawing its characteristic elements: the minor axis, the necklace circle, the foci, the centre of the generating hyperbola and the asymptotes. The authors used o few helpful constructions for the purpose of solving this problem. Because of the hyperboloid symmetry in comparison with the minor axis, they found two solutions for the problem. Also, they consider that this procedure can be extended into another case, such as the construction in a lateral projection of points laying on the hyperboloid.

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Construcţia proiecţiilor punctelor pe suprafeţe hiperboloidale

Rezumat

În cadrul acestui articol se prezintă o metodă teoretica de determinare a proiecţiilor unor puncte pe o suprafaţă hiperboloidală, atunci când se cunoaşte una dintre proiecţii, în acest caz cea verticală. După trasarea elementelor definitorii ax netransvers, cerc colier, focar, s-au folosit construcţii ajutătoare pentru soluţionarea problemei. Au rezultat două soluţii datorită simetriei suprafeţei considerate, acestea fiind reflectate şi în construcţia laterală.