

# The Selection of the Optimal Variant of the Robot Using Multicriterial Decisions Procedures

Bogdan Mirodotescu

Universitatea Petrol-Gaze din Ploiești, Bd. București, 39, Ploiești  
e-mail: office@formafit.ro

## Abstract

*In this paper, it is briefly presented a new original method of selecting the optimal variant of the type of industrial robot necessary for the execution of a given operation, based on the multicriterial decisions theory.*

**Key words:** *industrial robot, expert system, decision, criterion.*

## Introduction

Being given an application (by means of its technological route) and a multitude of industrial robots  $R = \{R_i, i = 1, n\}$  made by various companies and specialized for the tasks required by the proposed application (that means adequate to the indicated technological process, for the functional point of view) it is taken into discussion the selection of the robot  $R_i \in R$ , that would realize the imposed application.

This choice is, in its essence, a decisional act because it consists of the decision of selecting one of the existing robots  $R_i$ .

Logically the decision consists of the selection of the “best” variant of the robot on a certain criterion, which means an optimal choice. As it is known, (see [1]), the selection of the “best” (optimal) variant is a matter of optimizing and it is solved by various methods of the operational research of optimized programming or of the optimal decision theory, which supposes to define a purpose function which will indicate the optimal variant when, according to the situation, it will take the maximum value (maximization) or the minimum value (minimization).

## An Optimization Method Based on Multicriterial Decisions

Within the mathematical decisions theory [1], it was developed a general theory of multicriterial decisions, according to which the selection (the decision of such selection) of a precise variant from more possible variants can be made taking into account simultaneously more criteria,  $m$ , by which the performances of the  $n$  variants are evaluated, so that the selected variant respect as strictly as possible the qualities of performance wanted by the decision maker, being in this way an optimal variant.

For the selection of an optimal variant of industrial robot based on the multicriterial decisions theory the next algorithm has been proposed (and applied by the author in his doctor degree thesis: “Expert system for leading an autonomic robot”, U.P.G. Ploiesti, 2007).

- i) A list is drawn up:  $R_1, R_2, \dots, R_n$ , with  $n$  robots possibly to be used for the realization of the given application (based on the functional and constructive data given by the robot builder in the technical books and the prospectus of the robots that can be delivered).

These robots ( $R_1, R_2, \dots, R_n$ ) represent the  $n$  variants of the problem from which, through the method proposed here – the multicriterial decision – the robot considered optimal will be selected.

- ii) As it was shown in [3], it is determined  $m$  evaluation parameters for the functional performances of the  $n$  industrial robots considered apriori, that will form the  $m$  selection criteria of an optimal variant:  $C_1, C_2, \dots, C_m$ . The value of these  $m$  parameters, determined for all the  $n$  robots (considered as variants) form a matrix **A** (called the consequences matrix) which has the form indicated in table 1.

**Table 1.** The consequences matrix  $\mathbf{A} = [A(i, j)]$ ,  $i = 1, 2, \dots, m$ ;  $j = 1, 2, \dots, n$

C Selection Criterion (performances)				
V Variant (types of robots)	$C_1$	$C_2$	...	$C_m$
$V_1$	$A(1, 1)$	$A(2, 1)$	...	$A(m, 1)$
$V_2$	$A(1, 2)$	$A(2, 2)$	...	$A(m, 2)$
...	...	...	...	...
$V_n$	$A(1, n)$	$A(2, n)$	...	$A(m, n)$

The name given to the matrix **A** result from the fact that its elements,  $A(i, j)$ , express the consequences for every possible variant,  $V_i$ ,  $i = 1, 2, \dots, n$ , for every adopted decision criterion,  $C_j$ ,  $j = 1, 2, \dots, m$ .

The elements of this matrix  $A(i, j)$  represent evaluations (under the form of numerical data or fuzzy evaluations) of the performances  $C_1, C_2, \dots, C_m$  that present every robot, considered as variant  $V_1, V_2, \dots, V_n$ , that may be the optimal solution. For example [2], for a robot  $V_i = R_i = \text{PUMA}$ , the selection criteria of the optimal variant (as parameter of evaluation of the performances of the robot PUMA) could be the followings:

- o  $A(i, 1) = \text{THE COST OF THE ROBOT} = 2000\$$ ;
- o  $A(i, 2) = \text{THE MASS OF THE ROBOT} = 20 \text{ kg}$ ;
- o  $A(i, 3) = \text{THE RELIABILITY OF THE ROBOT} = \text{medium}$ ;
- o  $A(i, 4) = \text{HANDLING} = \text{very big}$ ;
- o  $A(i, m) = \text{THE DURATION OF A TECHNOLOGICAL CYCLE} = 71.01 \text{ seconds}$ .

- iii) In the third phase of the algorithm, the consequences matrix **A** (from table 1) transform in a normalized matrix **R**, with the elements  $r(i, j)$ ,  $i = 1, 2, \dots, n$ ;  $j = 1, 2, \dots, m$ , having an under unity numerical value (obtained as the ratio between the values  $A(i, j)$  and a certain value  $A^*$ ).

This normalization can be made in such way that in the new matrix **R** all the criteria will be, for every variant, the ratio between the difference from a maximum and that maximum, so that  $r(i, j) \leq 1$  for every  $i = 1, 2, \dots, n$  and  $j = 1, 2, \dots, m$ .

The normalization is done in the following way:

- o for the criteria that follows the *maximum* the following transformation is used:

$$r(i, j) = A(i, j) / A_j^* \quad (1)$$

where  $A_j^* = \max \{A(1, j), A(2, j), \dots, A(n, j)\}$  is the biggest value of the consequences on the column  $C_j$ ;

o for the criteria that follows the *minimum* the following transformation is used:

$$r(i, j) = [1 / A(1, j)] / [\max(1 / A_j^*)] ; \quad (2)$$

o the matrix  $\mathbf{R}$ , of the normalized consequences from table 2 has all the elements  $r(i, j) \leq 1$  and they are of maximum type.

**Table 2.** The normalized matrix of the consequences  $\mathbf{R}$

The criterion $\mathbf{C}$	$C_1$	$C_2$	...	$C_m$
Variant $\mathbf{V}$				
$V_1$	$r(1, 1)$	$r(2, 1)$	...	$r(m, 1)$
$V_2$	$r(1, 2)$	$r(2, 2)$	...	$r(m, 2)$
...	...	...	...	...
$V_n$	$r(1, n)$	$r(2, n)$	...	$r(m, n)$

iu) Using table 2, that means the normalized matrix of the consequences  $\mathbf{R}$ , it can be determined the variant  $V_j$ , considered as optimal by using several few methods:

k. *The method MaxMin*

Using table 2 (with the matrix  $\mathbf{R}$ ), the method MaxMin consists of determining the optimal variant  $V^*$  as being:

$$V^* = \max \min \{r(i, j)\} \quad (3)$$

The determination of the optimal variant  $V^*$  by using the relation (3) is done as follows:

o it is selected, for each column of table 2 (of  $\mathbf{R}$ ) and for each of the variants  $V_1, V_2, \dots, V_n$ , the one that has the smallest value, obtaining  $n$  minimum values:

$$V_{\min} = \{r(V_1), r(V_2), \dots, r(V_n)\} \quad (4)$$

where:  $r(V_j) = \min \{r(1, j), r(2, j), \dots, r(m, j)\}$ ,  $j = 1, 2, \dots, n$ ;

o from the multitude of  $n$  values obtained by using (4) it is chosen a single value  $V^*$ , as being the biggest from the range  $r(V_j)$  given by (4), that means:

$$V^* = \max \{r(V_1), r(V_2), \dots, r(V_n)\} \quad (5)$$

which is, therefore, the optimal variant of the robot.

kk. *The method of the graphs*

It is an original method, more efficient, that consists of following the next algorithm (taking as basis the matrix  $\mathbf{R}$  from table 2):

1<sup>0</sup>. On the basis of the values  $r(i, j)$ , are built the so-called graphs of preorder  $G_j = (V, U_i)$  associated to every criterion of decision  $C_j$ , every graph representing the relation of preorder defined on the multitude  $V$ , of all the possible alternatives (variants) by the criterion  $C_j$ . The elements of the multitudes  $V$  constitute the nodes of these oriented graphs, their arcs being defined as follows:

If  $V_k$  overlimits  $V_l$  in ranging based on the criterion  $C_j$ , an oriented arc is outlined from  $V_l$  to  $V_k$ , but if in this ranging  $V_l$  and  $V_k$  are equivalent, it will be outlined among these nodes two arcs, oriented in contrary directions;

2<sup>0</sup>. For the next step, the synthesis – graph  $G_0 = (V, U_0)$  is built, using the graphs  $G_j$ ,  $1 \leq j \leq m$  (determined at step 1<sup>0</sup>). The arcs of the graph  $G_0$  are defined as follows: the arc  $(V_k, V_l) \in V_0$  if and only if  $(V_k, V_l) \in U_j$  for every value of  $j = 1, 2, \dots, m$ . If the multitude of arcs of the synthesis – graph is not null and it is determined that a node  $V_l$  of the graph has the characteristic that every other node of the graph is the

origin of an arc with the border on the node  $V_l$ , the alternative (variant) corresponding to the node  $V_l$  is the optimal one; contrarily, we will pass at the next step, 3<sup>0</sup>;

- 3<sup>0</sup>. To make a complete ranging based on the synthesis – graph  $G_0$ , it is necessary to introduce the concepts of *concordance* and *discordance* which finally permit the completing of  $G_0$  with new arcs (especially when the arcs of the graph are insufficient), until the selection of a variant – optimal from all the points of view – will be possible. In this way, the following indicators are defined and calculated, for every ranged pair,  $(V_g, V_h)$ , of possible alternatives (variants):

o *The indicator of concordance* (consequences) between two variants  $V_g$  and  $V_h$ :

$$c(V_g, V_h) = \frac{1}{k_1 + k_2 + \dots + k_m} \sum_j k_j + \sum_i \frac{k_i}{2} \quad (6)$$

where  $k_j, j = 1, 2, \dots, m$ , are coefficients of importance of the considered criteria (selected sometimes so that their sum is equal to the unity). This indicator,  $c$ , will have values between 0 and 1, because the first sum from the relation (6) can be calculated only using those indexes  $j$  for which the condition  $r_{gi} > r_{hj}$  is fulfilled, and the second using the indexes  $i$  for which  $r_{gi} = r_{hj}$ , the  $r$  values being taken from the table 2.

o *The discordance indicator* between the same two variants,  $V_g$  and  $V_h$  are calculated as follows:

$$d(V_g, V_h) = \begin{cases} 0, & \text{if } a_{hj} < a_{gj}, \text{ whatever the value of } j=1, 2, \dots, m \\ \frac{1}{d} \max |a_{gj} - a_{hj}| & \text{considering the indexes } j \text{ for which } a_{hj} \geq a_{gj} \end{cases}, \quad (7)$$

where  $d$  is the maximum “distance” between the “marks”  $r_{ij}$  given to the variants. Also this indication will have values ranging between 0 and 1.

- 4<sup>0</sup>. Using the indicators calculated based on the relations (6) and (7), it is introduced in the multitude  $V$  a relationship of over-classing defined as it follows: an alternative (variant)  $V_g$  over-classes a different alternative  $V_h$ , if the following conditions are fulfilled:

$$c(V_g, V_h) \geq p \quad \text{and} \quad d(V_g, V_h) \leq q \quad (8)$$

where  $p$  and  $q$  are two thresholds – values chosen by the decision-maker, ranging between 0 and 1, with the value of  $p$  close to 1 and the value of  $q$  close to 0.

To the relation of over-classing defined by (8), it is associated – for the values given to  $p$  and  $q$  – a graph  $G(p, q) = [V, U(p, q)]$  whose arcs are defined as it follows:

$$(V_g, V_h) \in G(p, q) \Leftrightarrow c(V_g, V_h) \geq p \quad \text{and} \quad d(V_g, V_h) \leq q \quad (9)$$

From the definition of this graph, it results that if  $p \leq p'$  and  $q \leq q'$ , then  $G(p', q')$  is a partial under-graph of  $G(p, q)$ , therefore by decreasing the concordance threshold (from  $p$  to  $p'$ ) and increasing the discordance threshold (from  $q$  to  $q'$ ) new arcs are introduced in the graph  $G(p, q)$ .

Practically, we can start from  $p = 1$  and  $q = 0$ , after which the value of  $p$  is reduced and the value of  $q$  is increased until it is observed that one of the variants over-classes all the others. A variant  $V_0$  of the multitude  $V$  will over-class all the others if  $c(V_0, V_i) \geq p'$  and  $d(V_0, V_i) \leq q'$  for  $1 \leq i \leq n$ , the over-classing being more powerful as  $p'$  is closer to 1 and  $q'$  is closer to 0.

A powerful over-classing shows that the variant  $V_0$ , considered as optimal, is the most preferred for taking the decision of selecting it.

## Conclusions

The more and decisive evaluation criteria of performances have the types of robots from the range where the selection is made, the better chosen will be the robot necessary for the realization of an application through the criteria decision method (especially if using procedures based on the graphs theory) exposed here.

In fact, according to the probabilistic nature or the absence of reliability of the data about the realization of the consequences **C**, the decision rule is adopted as it follows:

- in conditions of certainty, the alternative for which a maximum utility corresponds is selected (see [1]);
- in risk conditions, it is selected an alternative for which a maximum average utility corresponds (according to all the considered criteria);
- in uncertainly conditions, schemes specific to the utility theory will be applied, such as: Hurwicz, Savage, Bayes-Laplace, Wald (see [1]).

## References

1. Dumitrescu, A. – *Bazele ingineriei sistemelor*, Editura Universității din Ploiești, 2005.
2. Lechtman, H., North, S. – Analysis of Industrial Robot Work by the RTM Method, *Industrial Engineering*, April 2004, pp. 38-48.
3. Mirodotescu, B. – Evaluation parameters for the functional performances of the industrial robots, *Buletinul U.P.G. Ploiești, Seria Tehnică*, Vol. LIX, Nr. 2/2007, pp. 33-36.

## Alegerea variantei optime de robot prin proceduri ale deciziilor multicriteriale

### Rezumat

*În lucrare, este prezentată, pe scurt, o nouă metodă originală de alegere optimală a tipului de robot industrial, necesar executării unor operații date, bazată pe teoria deciziilor multicriteriale.*