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The Torsion of Thin Wall I-Profile Section Beams With Uneven Legs

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Abstract

The paper presents a method for calculating movement and tensions that appear in straight, thin-walled, I-section poles with uneven legs while stressed in torque.
The calculus method considers the real situation, when section movement is blocked, leading to both normal and tangent tensions. The results obtain using this method differ from those using the elementary torque stress theory that considers section movement to be free. Therefore, the method presented in this paper should be used for proper sizing of this pole types.

Keywords: *torque, momentum and bi-momentum of bending-torsion, stress.*

Sectorial Caracteristics For I_Shaped Section With Uneven Legs

The center of gravity O and the center of bending – torsion are on the symmetry axis .
The distance between the P pole and the center of gravity O is:

$$PO = \frac{s_3 \cdot t_3 \cdot s_2 + \left(s_2 - \frac{t_1 + t_3}{2} \right) \cdot t_2 \cdot \frac{s_2}{2}}{s_1 \cdot t_1 + \left(s_2 - \frac{t_1 + t_3}{2} \right) + s_3 \cdot t_3} \tag{1}$$

To determine C the starting point is the arbitrary pole P and the origin radius Py . The diagram of the sectorial coordinates $\omega_P = \int_0^s h_P \cdot ds$,is illustrated in figure $1, b$.

For the D_1 and D_2 points, the following values are obtained:

$$\omega_P(D_1) = -\frac{1}{2} \cdot s_2 \cdot s_3 ; \quad \omega_P(D) = 0 \text{ and } \omega_P(D_2) = \frac{1}{2} \cdot s_2 \cdot s_3$$

The position of C on the Oy axis is determined according to [2],

$$a_y = PC = \frac{\int_{(A)} \omega_P \cdot z \cdot dA}{I_y} \tag{2}$$

We draw the diagram of z (fig. $1, c$) and applying the rule of Vereşceaghin we obtain:

$$\int_{(A)} \omega_P \cdot z \cdot dA = \frac{1}{12} \cdot s_2 \cdot s_3^3 \cdot t_3 \tag{3}$$

We calculate the moment of inertia in respect to the Oy axis:

$$I_y = \frac{1}{12} \cdot \left[s_1^3 \cdot t_1 + \left(s_2 - \frac{t_1 + t_3}{2} \right) \cdot t_2^3 + s_3^3 \cdot t_3 \right] \tag{4}$$

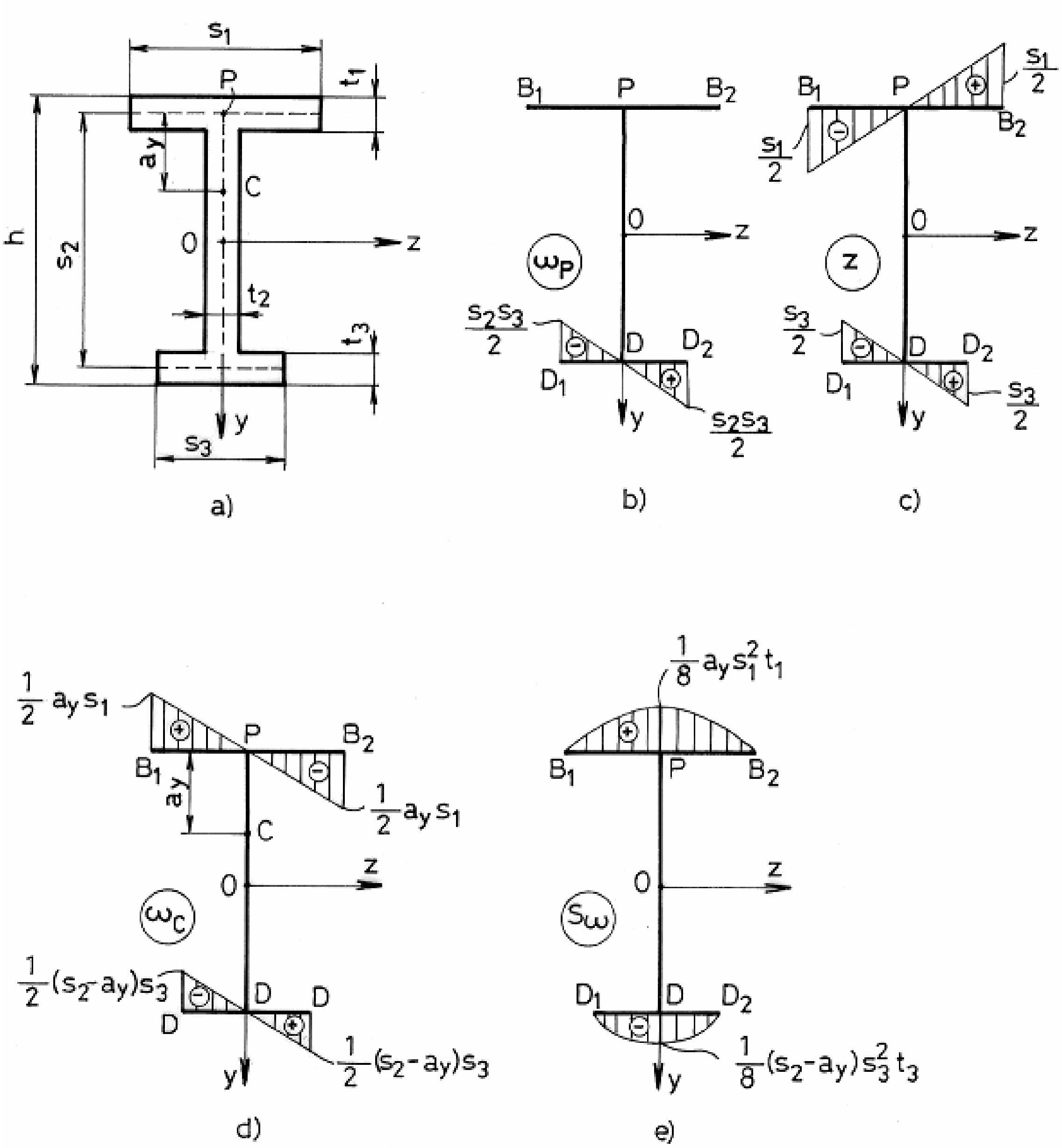

Fig. 1

With the bending torsion pole C we determine the new diagram $\omega = \omega_C$ (fig. $1, d$). The origin radius remains the symmetry axis Oy.

As a result, on the upper leg in B_1 and on the lower leg in D_1:

$$\omega(B_1) = \frac{1}{2} \cdot a_y \cdot s_1 \; ; \; \omega(D_1) = -\frac{1}{2} \cdot (s_2 - a_y) \cdot s_3$$

The static sectorial moment (5) has the diagram illustrated in figure $1, e$.

$$S_\omega = \int_{(A_1)} \omega \cdot dA, \tag{5}$$

On the heart S_ω is null. On the upper leg from B_1 to B_2, S_ω has a parabolic variation.

$$S_{\omega\max} = \frac{1}{8} \cdot a_y \cdot s_1^2 \cdot t_1 \tag{6}$$

On the lower leg from D_1 to D_2 S_ω has a parabolic variation.

$$S_{\omega,D} = -\frac{1}{8} \cdot \left(s_2 - a_y\right) \cdot s_3^2 \cdot t_3 \tag{7}$$

The sectorial moment of inertia is calculated by integrating the ω diagram (fig. 1,d) with itself:

$$I_\omega = \int_{(A)} \omega^2 \cdot dA = \frac{1}{3} \cdot s_1 \cdot \left(\frac{a_y \cdot s_1}{s}\right)^2 \cdot t_1 + \frac{1}{3} \cdot s_3 \cdot \left[\frac{\left(s_2 - a_y\right) \cdot s_3}{2}\right]^2 \cdot t_3 \tag{8}$$

The torsion moment of inertia with free movement:

$$I_t = \frac{1}{3} \cdot \left[s_1 \cdot t_1^3 + \left(s_2 - \frac{t_1 + t_3}{2}\right) \cdot t_2^3 + s_3 \cdot t_3^3\right] \tag{9}$$

Application

Let us consider the AB beam (fig. $2, a$) perfectly encased in A and free in B, having a length of $l = 2m$, and a cross-section as illustrated in fig. $2, b$. The pole is under uniform distributed moments of torsion (fig. $2, a$) of $\mathcal{M}_t$ intensity.

The rotation φ of the y-axis section x, around the Cx axis function of the original parameters from the paper [1] has the expression:

$$\varphi = \varphi_0 + \varphi_0' \frac{shkx}{k} + \frac{B_{w,0}}{G \cdot I_t} \cdot \left(1 - chkx\right) + \frac{M_{x,0}}{k \cdot G \cdot I_t} \cdot \left(kx - shkx\right) + \overline{\varphi} \tag{10}$$

Function of $\varphi(x)$ from the paper [1] the expression is:

- The bending-torsion bimomentum

$$B_\omega = -E \cdot I_\omega \cdot \frac{d^2\varphi}{dx^2} = -E \cdot I_\omega \cdot \varphi'' \; ; \tag{11}$$

- The bending-torsion momentum

$$M_\omega = -E \cdot I_\omega \cdot \frac{d^3\varphi}{dx^3} = -E \cdot I_\omega \cdot \varphi''' \; ; \tag{12}$$

- The moment of torsion corresponding to the free movement

$$M_t = M_x - M_w = -G \cdot I_t \cdot \frac{d\varphi}{dx} = -G \cdot I_t \cdot \varphi' \; . \tag{13}$$

In this case, the particular solution, given by [1], is:

$$\overline{\varphi} = -\frac{\mathcal{M}_t}{G \cdot I_t} \cdot \left[\frac{1}{2} \cdot (kx)^2 + \left(1 - chkx\right)\right]. \tag{14}$$

Choosing the origin O to be the encasement in A, $O \equiv A$, we know: $\varphi_0 = 0$ and $\varphi_0' = 0$, because the u on the Ox axis is null.

From the equation of moments in respect to the Ox axis (fig. $2, a$) we obtain $M_{x,A} = \mathcal{M}_t \cdot l$, so $M_{x,0} = \mathcal{M}_t \cdot l$.

With these values the expression (10) becomes:

$$\varphi=\frac{B_{\omega,0}}{G\cdot I_t}\cdot(1-chkx)+\frac{\mathcal{M}_t\cdot l}{k\cdot G\cdot I_t}(kx-shkx)-\frac{\mathcal{M}_t}{k^2\cdot G\cdot I_t}\left[\frac{1}{2}\cdot(kx)^2+(1-chkx)\right] \tag{15}$$

At the free end B $(x=l)$, since $\sigma_\omega=0$, the condition $B_\omega(l)=0$ must be fulfilled, which using (11), becomes:

$$B_\omega(l)=-E\cdot I_\omega\cdot\varphi''(l)=B_{\omega,0}\cdot chkl+\frac{1}{k}\cdot\mathcal{M}_t\cdot l\cdot shkl+\frac{\mathcal{M}_t}{k^2}(1-chkl)=0 \tag{16}$$

From this we obtain the value:

$$B_{\omega,0}=-\frac{\mathcal{M}_t\cdot l}{k}\cdot\Phi(kl) \tag{17}$$

where:

$$\Phi(kl)=\frac{1}{kl\cdot chkl}\cdot(kl\cdot shkl+1-chkl) \tag{18}$$

Introducing (17) in (15), we obtain the function of rotation φ (fig. $2,c$):

$$\varphi=\frac{\mathcal{M}_t\cdot l}{k\cdot G\cdot I_t}\cdot\left[-\left(\Phi+\frac{1}{k\cdot l}\right)\cdot(1-chkx)+kx-shkx-\frac{1}{2}\cdot\frac{k\cdot x^2}{l}\right]. \tag{19}$$

The derivate of this (fig. $2,d$) is

$$\varphi'=\frac{\mathcal{M}_t\cdot l}{G\cdot I_t}\cdot\left[\left(\Phi+\frac{1}{k\cdot l}\right)\cdot shkx-chkx+\frac{l-x}{l}\right]. \tag{20}$$

With the expression (11) we determine the bimomentum of bending – torsion (fig. $2,e$):

$$B_\omega=-E\cdot I_\omega\cdot\varphi''=-\frac{\mathcal{M}_t\cdot l^2}{k\cdot l}\cdot\left[\left(\Phi+\frac{1}{k\cdot l}\right)\cdot chkx-shkx-\frac{1}{k\cdot l}\right]. \tag{21}$$

Applying the (12) expression we get the bending – torsion momentum (fig. $2,f$):

$$M_\omega=E\cdot I_\omega\cdot\varphi'''=\mathcal{M}_t\cdot l\cdot\left[\left(\Phi+\frac{1}{k\cdot l}\right)\cdot shkx-chkx\right]. \tag{22}$$

With (14) we determine the torsion momentum corresponding to the free movement:

$$M_t=-G\cdot I_t\cdot\varphi'=-\mathcal{M}_t\cdot l\cdot\left[\left(\Phi+\frac{1}{k\cdot l}\right)\cdot shkx-chkx+\frac{l-x}{l}\right]. \tag{23}$$

as illustrated in fig $2,g$.

The values from the diagrams are calculated for the section present in fig. $2,b$, which has:

$$\int_{(A)}\omega_P\cdot z\cdot dA=\frac{1}{12}\cdot s_3^3\cdot s_2\cdot t_3=43{,}2\cdot10^7\,mm^5;\ I_y=42{,}653\cdot10^6\,mm^4;$$

$$a_y=\frac{\int_{(A)}\omega_P\cdot z\cdot dA}{I_y}=101{,}3mm;\ PO=141mm;\ I_t=14{,}12\cdot10^4\,mm^4.$$

$$\sigma_\omega=\frac{B_\omega}{I_\omega}\cdot\omega\ ;\ \tau_\omega=\frac{M_\omega\cdot S_\omega}{t\cdot I_\omega}\cdot\omega\ ;\ \tau_t=\frac{M_t}{I_t}\cdot t \tag{24}$$

The diagram of normal stresses σ_ω, fig $2,h$.

From the diagram $\omega=\omega_c$ (fig. $1,d$), we get:

$$\omega(D_2) = -\omega(D_1) = 11{,}922 \cdot 10^3\, mm^2;\quad \omega(B_1) = -\omega(B_2) = 7{,}5975 \cdot 10^3\, mm^2 .$$

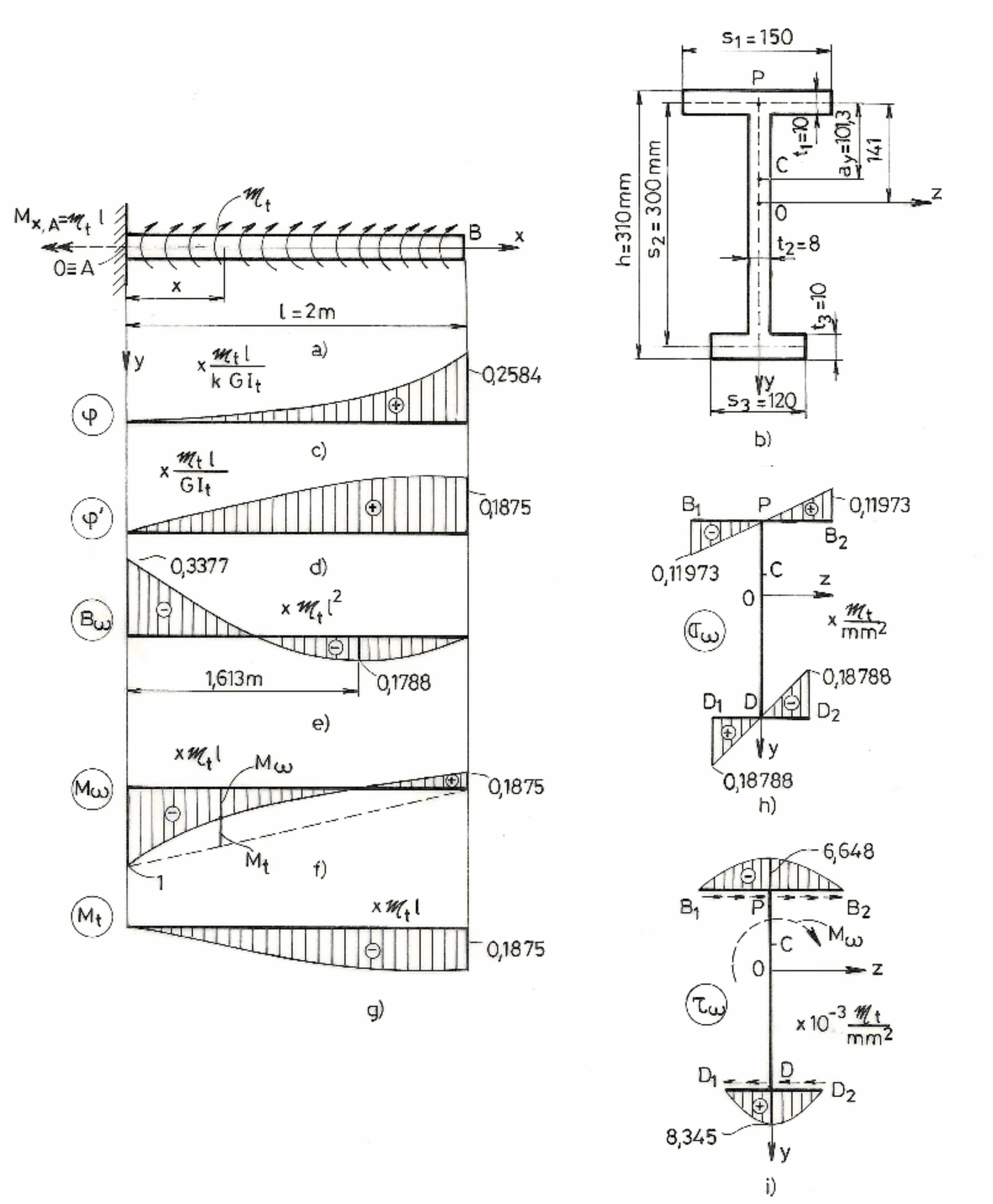

Fig. 2.

In the encasing section $A(x=0)$, $B_{\omega,A} = -0{,}3377 \cdot \mathcal{M}_t l^2$:

$$\sigma_\omega(D_1) = -\sigma_\omega(D_2) = 0{,}18788\frac{\mathcal{M}_t}{mm^2}\ ;\ \sigma_\omega(B_2) = -\sigma_\omega(B_1) = 0{,}11973\frac{\mathcal{M}_t}{mm^2}.$$

The diagram of tangential stresses τ_ω, fig $2, i$ .From the S_ω diagram (fig. $1, e$) we get:

$$S_\omega(D) = 357{,}66 \cdot 10^4\, mm^4;\ S_\omega(P) = 284{,}906 \cdot 10^4\, mm^4 .$$

In the encasing section $A(x=0)$, where $M_{\omega,A} = -\mathcal{M}_t \cdot l$ (fig 2, f), we get:

$$\tau_\omega(D) = 8{,}345 \cdot 10^{-3} \frac{\mathcal{M}_t}{mm^2} \;;\; \tau_\omega(P) = 6{,}648 \cdot 10^{-3} \frac{\mathcal{M}_t}{mm^2}.$$

From the resistance condition, after T_τ, in the point D_1 from the $A(x=0)$ section,

$$\sigma_{ech} = \sigma_\omega(D_1) = 0{,}18788 \frac{\mathcal{M}_t}{mm^2} = \sigma_a = 150 \frac{N}{mm^2}, \text{ we get: } \mathcal{M}_t = 798{,}4 \frac{N \cdot mm}{mm}$$

In all other points of the pole the resistance condition is met.
So, from fig. $2, i$, in the D point of the $A(x=0)$ section,

$$\tau_\omega(D) = 8{,}345 \cdot 10^{-3} \cdot 794{,}4 = 6{,}7 \frac{N}{mm^2} \;;\; \sigma_\omega(D) = 0 \;;\; \tau_t = 0,$$

the resistance condition, after T_τ,

$$\sigma_{ech} = \sqrt{\sigma^2 + \tau^2} = 2 \cdot \tau_{\max} = 13{,}4 \frac{N}{mm^2} < \sigma_a = 150 \frac{N}{mm^2}, \text{ is met.}$$

Conclusions

The paper presents a method to calculate stresses and displacements that appear in thin-walled I-shaped poles with uneven legs. The calculus method considers transversal section displacement to be impaired, real cases generating both normal and tangential stress. For thin sections, this stresses become very high thus breaking the poles. So a resistance check is needed, using the above-presented method.

References

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Torsiunea barelor cu pereti subţiri, profil I cu aripi neegale

Rezumat

Lucrarea prezintă o metodă pentru calculul tensiunilor şi deplasărilor ce apar în barele drepte cu pereţi subţiri având secţiuni în formă de I, cu tălpile neegale, solicitate la torsiune.