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Regarding Some Interactive Criteria Used in Isotropic and Quasi-Isotropic Materials Fracture Mechanics

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Abstract

The technical safety of the mechanical structures is perfectly ensured if in the design phase of these the values of the elastic, thermal and mechanical characteristics are very well known, and set rigorously experimentally. Obviously, in this context, the development technological processes and the allowed treatments are taken into account. Data, during the considered mechanical construction operation, so they are placed in the calculation relationships and in the failure criteria, have an important role. This paper presents the mathematical expressions offered by the most representative fracture criteria of the isotropic or quasi-isotropic materials, as the metal or composite particles (fillers) are.

Key words: *Safety technical, interactive criteria*

Introduction

The metallic materials used in the construction of the industrial mechanic equipments, in general are very well known in terms of the values of the elastic, thermal and mechanical properties and their evolution over time, in different conditions of outdoor use. The accumulated databases are extremely useful in choosing the admissible stresses and safety coefficients, normal standardized. At present the manufacturing technologies, in the context outlined above, are very well controlled, accompanied by high-precision modern equipment, present in all specific operations. In the case of some composite with filler components (particles, chips, etc.) the differences between the values of the mechanical and thermal characteristics can be very different, from many causes (the nature of the material itself, the technology of making the semi-finished or finished products) such that designers attention should be much deeper. The fracture of the composite materials is characterized by a great complexity, subjected to some structural properties and the mechanical processes, too [1]. In this case, the microscopic fracture is preceded by the distribution of the mechanical stresses developed in the process of deformation. In the specialty literature several physical models are exposed, with wide application in the industrial use, materialized in mathematical expressions of the fracture criteria, set both by theoretical and experimental way [2-4].

In the presented paper some of the expressions of the failure criteria used in the analysis of isotropic or quasi-isotropic materials are shown, structure which stands some similarities, but also differences, or nuances determined usually by the individualized nature of the analyzed material, respectively of the performed experiments.

It should not neglect the purposes of the above, and the contribution given of the experimental evolution equipment intake was available.

The interactive criteria, by their content, predict only the fracture moment, not how to carry it (the fracture mechanisms are not described).

Expressions of the Fracture Interactive Criteria

Criterion Huber M. T. (1904), von Mises R. (1913), Hencky H. Z. (1924)

For the isotropic materials this ***(H – M – H)*** criterion is written under the known form for the main normal stresses [5-7]:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\cdot\sigma_c^2 , \tag{1}$$

so, that for the plane state of the (σ_1, σ_2, σ_3) main stresses, is reduced to [5]:

$$3\cdot\left[(\sigma_1 - \sigma_2)/\sigma_c\right]^2 + \left[(\sigma_1 + \sigma_2)/\sigma_c\right]^2 = 4 , \tag{2}$$

representing an ellipse in the plane of the principal stresses (σ_c representing the yield limit of the material).

The specialty literature, in the case of the isotropic metallic materials, mentions the assessment on the integrity state through some functions like [6-11]:

$$f(\sigma_{ij}) = f(\sigma_1, \sigma_2, \sigma_3, \tau_{12}, \tau_{13}, \tau_{31}) \le 1, \tag{3}$$

or, expressed in another way:

$$f(\sigma_{ij}) = \begin{pmatrix} \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\cdot\sigma_2 - \sigma_2\cdot\sigma_3 - \\ - \sigma_3\cdot\sigma_1 + 3\cdot\tau_{12}^2 + 3\cdot\tau_{23}^2 + 3\cdot\tau_{31}^2 \end{pmatrix} \Big/ \sigma_c^2 \le 1. \tag{4}$$

In the case of a reference spatial system, respectively for the main directions (when tangential stresses are void [12]):

$$f(\sigma_{ij}) = \left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\cdot\sigma_2 - \sigma_2\cdot\sigma_3 - \sigma_3\cdot\sigma_1\right)/\sigma_c^2 \le 1, \tag{5}$$

expressions involving stresses developed in structure and the σ_c yield limit of the material (instead this can be used, as appropriate, for crackly materials, the σ_r fracture resistance). The yield limit or the fracture resistance can be changed with the stress/the σ_a admissible resistance, accepted in this case, according to the technical norms in force (by considering the σ_c yield limit, - tenacious material - or the fracture σ_r resistance - crackly materials, respectively the safety coefficients values c_c or c_r [12]) or experimentally determined.

Criterion Tresca H. E. (1865)

For the **isotropic materials** and for the main directions of the normal stresses, it is written as [5, 6, 7, 13]:

$$max\left\{\left|\sigma_1-\sigma_2\right|,\left|\sigma_2-\sigma_3\right|,\left|\sigma_3-\sigma_1\right|\right\}=\sigma_c, \tag{6}$$

as for the plane state of main normal stresses $\left(\sigma_3=0\right)$ is converted to [5]:

$$\left(\sigma_1-\sigma_2\right)^2=\sigma_c^2, \tag{7}$$

respectively, in the case of a general solicitation [5, 6, 11]:

$$\left[\left(\sigma_1-\sigma_2\right)/\sigma_c\right]^2+4\cdot\left(\tau_{12}/\sigma_c\right)^2=1. \tag{8}$$

Criterion Burzyński W. T. (1928 – 1929)

The case where there is a considerable difference between the yield limits determined at the σ_{ct} stretching/traction, respectively of σ_{cc} compression is considered, recorded of the report [14]:

$$k_c=\sigma_{cc}/\sigma_{ct}, \tag{9}$$

so we can write the function[14]:

$$\left\{\begin{aligned}&0,5\cdot\left[\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_3-\sigma_1\right)^2+\left(\sigma_1-\sigma_2\right)^2\right]+\\&\quad+\left(\sigma_{cc}-\sigma_{ct}\right)\cdot\left(\sigma_1+\sigma_2+\sigma_3\right)\end{aligned}\right\}\Big/\left(\sigma_{cc}\cdot\sigma_{ct}\right)=1. \tag{10}$$

Note: In the paper [14], based on some appropriate experiments, the corresponding values for the sizes involved in the study are obtained: for **PLA / PBAT** [poly (lactic - acid)] / [poly (butylene adipose / terephalate)]: $k_c=1,7\ ....\ 3,5$; for a mixture of maize: $k_c=1,46$, for example.

For an isotropic material, elastic symmetrical ($\sigma_c=\sigma_{cc}=\sigma_{ct};\ \ k_c=1$) according to ***Huber, M. T. - von Mises R.*** [14] we can write the following correlation between the normal yield limit and the shear limit:

$$\tau_c^H=\sigma_c/\sqrt{3}, \tag{11}$$

so in the case of an elastic asymmetric material ($\sigma_{cc}\neq\sigma_{ct};\ k_c\neq 1$) is written:

$$\tau_c^B=\sqrt{\left(\sigma_{cc}\cdot\sigma_{ct}\right)/3}. \tag{12}$$

Therefore, the (10) equality changes, resulting [14]:

$$\left[\begin{aligned}&\left(1-\lambda_B\right)\cdot\left(\sigma_2-\sigma_3\right)^2+\lambda_B\cdot\left(\sigma_3-\sigma_1\right)^2+\\&+\left(1-\lambda_B\right)\cdot\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_{cc}-\sigma_{ct}\right)\cdot\left(\sigma_1+\sigma_2+\sigma_3\right)\end{aligned}\right]\Big/\left(\sigma_{cc}\cdot\sigma_{ct}\right)=1, \tag{13}$$

where the parameter interfere:

$$\lambda_B=\left(\sigma_{cc}\cdot\sigma_{ct}\right)\Big/\left[2\cdot\left(\tau_c^B\right)^2\right]-1. \tag{14}$$

Another formulation of the ***Burzyński W. T.*** criterion takes the form [14]:

$$\left[\sigma_1^2-R_B\cdot\sigma_3\cdot\sigma_1+\sigma_2^2+\left(\sigma_{cc}-\sigma_{ct}\right)\cdot\left(\sigma_1+\sigma_3\right)\right]\Big/\left(\sigma_{cc}\cdot\sigma_{ct}\right)=1, \tag{15}$$

where the R_B factor interfere:

$$R_B = 2 - \left[\sigma_{cc} \cdot \sigma_{ct} + 2 \cdot \left(\sigma_{cc} - \sigma_{ct} \right) \cdot \sigma^*_{cc} \right] \Big/ \left(\sigma^*_{cc} \right)^2 , \quad (16)$$

where the presence of the yield stress is remarkable for a bi-axial solicitation. When the experimental value of the σ^*_{cc} stress is not known, can be used the [14] equality:

$$\sigma^*_{cc} = \frac{\left(\sigma_{cc} - \sigma_{ct} \right) \cdot \tau_c^2 + \sqrt{\left(\sigma_{cc} - \sigma_{ct} \right)^2 \cdot \tau_c^4 + \sigma_{cc} \cdot \sigma_{ct} \cdot \tau_c^2 \cdot \left(- \sigma_{cc} \cdot \sigma_{ct} + 4 \cdot \tau_c^2 \right)}}{- \sigma_{cc} \cdot \sigma_{ct} + 4 \cdot \tau_c^2} . \quad (17)$$

Criterion Hersey V. A. (1954)

The expression of the Journal of Applied Mechanics time, Transactions ASME 21 (1954, p. 241-249), took into account the results of the papers: ***Norton F. H.***, The creep of steel at high temperatures, Mc. Graw - Hill, 1929, New York and ***Bailey R.W***., Creep of steel under simple and compound stresses and the use of high initial temperature in steam power plants, Transmission in Tokyo Section Meeting world Power Conference, Konai - kai Publishing, 1929, Tokyo [5].

For the main directions of the normal stresses, in case of a spatial solicitation in an **isotropic material**, the criterion renders the following general form of the ***Huber - Mises - Hencky*** criterion [5, 7, 15-18]:

$$\left(\sigma_1 - \sigma_2 \right)^a + \left(\sigma_2 - \sigma_3 \right)^a + \left(\sigma_3 - \sigma_1 \right)^a = 2 \cdot \sigma_c^a , \quad (18)$$

so for the actual state of stresses we can write:

$$\left\{ \left[\begin{array}{c} \left(\sigma_1 - \sigma_2 \right)^a + \left(\sigma_2 - \sigma_3 \right)^a + \\ + \left(\sigma_3 - \sigma_1 \right)^a \end{array} \right] \Big/ 2 \right\}^{1/a} = \sigma_c , \quad (19)$$

where σ_c represent the yield limit at the uni-axial solicitation, and a - the exponent dependent on the crystallographic structure of the material [5].

The $f\left(\sigma_i \right)$ function - (5) - this time takes the form:

$$f\left(\sigma_{ij} \right) = \left\{ \left[\left(\sigma_1 - \sigma_2 \right)^a + \left(\sigma_2 - \sigma_3 \right)^a + \left(\sigma_3 - \sigma_1 \right)^a \right] \Big/ 2 \right\}^{1/a} \Big/ \sigma_c \le 1. \quad (20)$$

or, using the module of the stresses differences:

$$f\left(\sigma_{ij} \right) = \left\{ \left[\left| \sigma_1 - \sigma_2 \right|^a + \left| \sigma_2 - \sigma_3 \right|^a + \left| \sigma_3 - \sigma_1 \right|^a \right] \Big/ 2 \right\}^{1/a} \Big/ \sigma_c \le 1, \quad (21)$$

according to the options expressed in the paper [17].

Remark: For $a = 2$ the known form of the ***Huber M. T.– v. Mises R. – Hencky M. Z*** criterion results. For $a \to \infty$ the results are more appropriate of those given of the ***Tresca H. E.*** criterion [5, 15-17]. For: $1 \langle a \langle 2$ and $a \rangle 4$ the curve defined by the (18) equality is between ***Tresca*** hexagon ($a = 1$) and the ***v. Mises*** ellipse ($a = 2$) and for $2 \langle a \langle 4$ the resulted curve lies outside the ***v. Mises*** ellipse [5]. Figure 1, for some finite values, shows the direction of the previous observation [7, 16].

Note: In case of the composites, the function characteristic for the state to avoid their deterioration takes the form [8]:

$$f\begin{pmatrix} \sigma_1, \sigma_2, \sigma_3, \tau_{12}, \tau_{23}, \tau_{31}, X_T, \\ X_C, Y_T, Y_C, Z_T, Z_C, S, Q, R \end{pmatrix} \le 1. \tag{22}$$

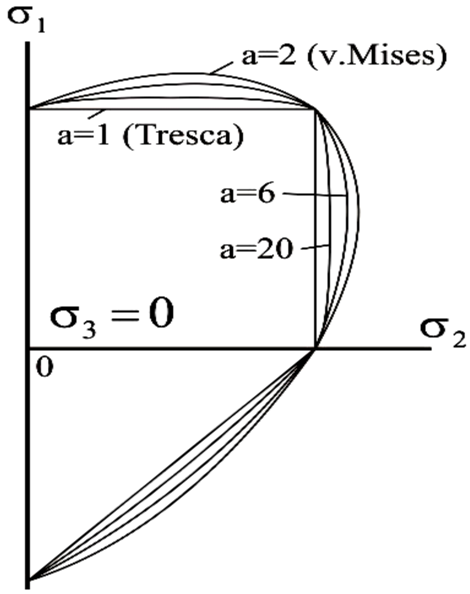

Fig. 1. Graphic variation concordant with the (18) expression, for the plane state of stresses [16, 17]

Remark: In the criteria set out below, the fracture of the composite material in longitudinal or transversal direction is considered, maintaining the reference system mentioned above.

Criterion Marin J. (1957)

In this case, based on the expression of the total energy of deformation applied to an isotropic material, proposing the following expression [2]:

$$\begin{gathered}(\sigma_1 - a)^2 + (\sigma_2 - b)^2 + (\sigma_3 - c)^2 + \\ q\cdot\left[(\sigma_1 - a)\cdot(\sigma_2 - b) + (\sigma_2 - b)\cdot(\sigma_3 - c) + (\sigma_3 - c)\cdot(\sigma_1 - a)\right] = \sigma_{max}^2,\end{gathered} \tag{23}$$

where the a, b, c, q sizes are dependent on the type of the state of stresses. After the corresponding developments the equality will be obtained:

$$\begin{gathered}\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + K_{1M}\cdot\sigma_1 + K_{2M}\cdot\sigma_2 + K_{3M}\cdot\sigma_3 + \\ + K_{4M}\cdot(\sigma_1\cdot\sigma_2 + \sigma_2\cdot\sigma_3 + \sigma_3\cdot\sigma_1) = K_{5M},\end{gathered} \tag{24}$$

where:

$$K_{1M} = -(2\cdot a + b + c);\; K_{2M} = -(a + 2\cdot b + c);\, K_{3M} = -(a + b + 2\cdot c); \tag{25}$$

$$K_{4M} = q;\; K_{5M} = \sigma_{max}^2 - a^2 - b^2 - c^2 - a\cdot b - b\cdot c - c\cdot a. \tag{26}$$

For example, in the case of the $(\sigma_3 = c = 0)$ biaxial state, the (61) equality can be written in the form [2]:

$$\sigma_1^2 + \sigma_2^2 + K_{4M}\cdot\sigma_1\cdot\sigma_2 + K_{1M}^*\cdot\sigma_1 + K_{2M}^*\cdot\sigma_2 = K_{5M}^*, \tag{27}$$

with the appropriate notations:

$$K^*_{1M} = -2\cdot a - q\cdot b; \quad K^*_{2M} = -2\cdot b - q\cdot a; \tag{28}$$

$$K^*_{5M} = \sigma^2_{max} - a^2 - b^2 - a\cdot b\ . \tag{29}$$

It appears that in the previous equalities the main stresses are present, showing an advantage for the isotropic materials, because the shear phenomenon is removed. There is a big disadvantage for the anisotropic materials, being very rarely applied to the composite materials, given that the main directions do not always coincide with the orthotropic axes.

Criterion Hosford W. F. – 1 (1966)

The formulation given in Metals Engineering Quarterly, 6, June 1966, p. 13-19, bring an amendment of the ***Hill R.*** criterion (1948), for isotropic materials:

$$\begin{aligned} &A\cdot\sigma_1 + B\cdot\sigma_2 + (-B-A)\cdot\sigma_3 + F\cdot(\sigma_2-\sigma_3)^2 + \\ &+ G\cdot(\sigma_3-\sigma_1)^2 + H\cdot(\sigma_1-\sigma_2)^2 = 1, \end{aligned} \tag{30}$$

where $A,\ B,\ F,\ G,\ H$ are the material factors [19].

Criterion Hosford W. F. – 2 (1972)

(to see the author's paper published in Journal of Applied Mechanics, 36, 1972, p. 607-609) proposes a ***Hill R.*** – 1 (1948) criterion generalization, for isotropic materials, written as [5, 6]:

$$f(\sigma_{ij}) = \left[F\cdot(\sigma_2-\sigma_3)^a + G\cdot(\sigma_3-\sigma_1)^a + H\cdot(\sigma_1-\sigma_2)^a\right] / \sigma_c \le 1. \tag{31}$$

The difference in relation with the ***Hill R – 1*** criterion is expressed in the way of experimentation on the a exponent value [5]. It is noted, however, that the previous expression does not contain the influence of shear stress characteristic to the material.

Criterion Hosford W. F. – 3 (1979)

Based on the author's paper published in Proc. 7t h North American Metalworking Conf. (NMRC), SME Deaborn, MI, 1979, p. 101 – 197 [13], for the solicitation spatial state, without being considered the effect of the shear stresses, the following form is take into account [6] :

$$f(\sigma_{ij}) = F\cdot|\sigma_2-\sigma_3|^n + G\cdot|\sigma_3-\sigma_1|^n + H\cdot|\sigma_1-\sigma_2|^n \le 1, \tag{32}$$

where the exponent is dependent on the crystal structure of the material (for example, $n = 6$- is recommended for bcc materials (body-centered cubic; cubic network with centered volume), respectively $n = 8$- for the fcc materials (face centered cubic, cubic network with centered face) (see ***Hosford W. F.***, Materials Science and Engineering, A 257, 1998, p. 1-8). Taking into account the ***Lankford*** coefficients, we can write for the plane state of solicitation:

$$f(\sigma_{ij}) = \left[R_T\cdot|\sigma_1|^n + R_L\cdot|\sigma_2|^n + R_L\cdot R_T\cdot|\sigma_1-\sigma_2|^n\right] / \left[R_T\cdot(R_L+1)\cdot\sigma_c^n\right] \le 1, \tag{33}$$

in accordance with the specification of the work: ***Banabic D., Bunge H. J., Pöklandt K., Tekkaya A.***, Formability of Mettalic, Springer – Verlag Berlin, 2000.

Criterion Raghava R. - Cadell M. R. - Yeh Y. S. G. (1973)

For the polymeric materials (polycarbonate and PVC) solicited in hydrostatic condition, as stated previously mentioned authors, it can use the following adaptation of the criterion ***Huber – Mises – Hencky*** [7, 20-23]:

$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2+6\cdot(C-T)\cdot\sigma_m=2\cdot C\cdot T, \qquad (34)$$

where, outside the σ_1, σ_2, σ_3 normal stresses, are taken into consideration:

$$\sigma_m=(\sigma_1+\sigma_2+\sigma_3)/3; \qquad (35)$$

C, T represent the material yield stress under the application of compression, respectively stretching, and atmospheric environment; P – the testing hydrostatic pressure; S – the yield stress determined under hydrostatic condition (in general); S_c, S_t – the yield stress set under hydrostatic compression with the P pressure, respectively at stretching, under the same conditions; R – the normalized yield stress, under the action of the P hydrostatic pressure; R_c, R_t – the normalized yield stress under the hydrostatic compression solicitation, respectively P hydrostatic streching ($R_c=S_c/T$; $R_t=S_t/T$).

Note: The criterion is assigned to the ***Codell R. M., Raghava S. R., Atkins G. A.*** staff – Materials Science and Engineering, vol. 13, 1974, p. 113 – 120, respectively to the ***Pae K. D., Bhateja K. S.*** collective – Journal of Macromolecular Science, Part C, Reviews in Macromolecular Chemistry, vol. C 13, 1975, p. 1 – 75 [23]).

The proposed criterion takes into account the difference in value of the yield stresses at the solicitation of stretching and compression.

Accepting the conditions: $\sigma_2=\sigma_3=-P$; $\sigma_1=S-P$, the (36) equality can be written as:

$$S^2+3\cdot\sigma_m\cdot(C-T)=C\cdot T \quad \text{or} \quad (S/T)^2+(3\cdot\sigma_m/T)\cdot(C/T-1)=C/T. \qquad (36)$$

In the specified conditions the correlations can be written [20]:

$$R_c=-\left\{(Y-1)+\left[(Y+1)^2+12\cdot H\cdot(Y-1)\right]^{1/2}\right\}\Big/2; \qquad (37)$$

$$R_t=-\left\{(Y-1)-\left[(Y+1)^2+12\cdot H\cdot(Y-1)\right]^{1/2}\right\}\Big/2; \qquad (38)$$

$$R=\pm\left[Y-3\cdot M\cdot(Y-1)\right]^{1/2};\ R=S/T;\ Y=C/T;\ M=\sigma_m/T;\ H=P/T. \qquad (39)$$

Criterion Sternstein S. S. - Ongchin L. (1969) [21, 23]

In the same adaptation configuration of the ***Huber – Mises – Hencky*** criterion, for polymeric materials, for the hydrostatic solicitation, the following expression is proposed (accepted by ***Bauwens J. C.,*** Journal Polymer Science, part A – 2, 8, 1970, p. 893 – 901 and ***Asp E. L***. [22], too):

$$\begin{aligned}&\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]^{1/2}+\\&+\left[3\cdot\sqrt{2}(C-T)\cdot\sigma_m\right]\Big/(C+T)=2\cdot\sqrt{2}\cdot C\cdot T\big/(C+T),\end{aligned} \qquad (40)$$

and, in parallel,

$$\pm S + (C - T)\cdot(S - 3\cdot P)/(C + T) = 2\cdot C\cdot T/(C + T). \tag{41}$$

For $\sigma_2 = \sigma_3 = -P;\ \sigma_1 = S - P$, in the (36) equality the $S - 3\cdot P = 3\cdot\sigma_m$ replacement takes place.
In this case, the following equalities can be written:

$$R_c = -(3/2)\cdot H\cdot(Y - 1) - Y;\ R_t = (3/2)\cdot H\cdot[(Y - 1)/Y] + 1; \tag{42}$$

$$R = \pm[2\cdot Y/(Y + 1) - 3\cdot M\cdot(Y - 1)/(Y + 1)], \tag{43}$$

with the appropriate notations reflected of the (39) equalities.

Criterion Barlat F. – Richmond O. (1987)

The paper of the authors published in Materials Science and Engineering, 91, 1987, p. 15-29 [13] is envisaged, so that for the plane state of stresses, taking into account the effect of shear stress is written as [13]:

$$f(\sigma_{ij}) = [|k_1 + k_2|^m + |k_1 - k_2|^m + |2\cdot k_2|^m]/(2\cdot\sigma_c^m) \le 1, \tag{44}$$

where the notations was used [13]:

$$k_1 = (\sigma_1 + \sigma_2)/2;\ k_2 = \sqrt{[(\sigma_1 - \sigma_2)/2]^2 + \tau_{12}^2}. \tag{45}$$

Barlat F. – Richmond O. criterion develops the ***Hosford W. F.*** expression (1972) for isotropic materials.
For anisotropic materials can write [6, 13]:

$$f(\sigma_{ij}) = [a\cdot|k_1 + k_2|^m + b\cdot|k_1 - k_2|^m + c\cdot|2\cdot k_2|^m]/(2\cdot\sigma_c^m) \le 1, \tag{46}$$

where: $a = b = 2 - c = 2/(1 + R), R$, represent ***Lankford*** coefficient, if normal anisotropy materials.

Criterion Barlat F. and Lian J. (1989)

For materials with planar isotropy (see paper published in the Journal of Plasticity, 5, 1989, p. 51-56) bring some changes in the (45) expressions, written as [5, 13]:

$$k_1^* = (\sigma_1 + h\cdot\sigma_2)/2;\quad k_2^* = \sqrt{[(\sigma_1 - h\cdot\sigma_2)/2]^2 + (p\cdot\tau_{12})^2}, \tag{47}$$

where:

$$a = 2 - c = \begin{bmatrix} 2\cdot(\sigma_c/\tau_{s2})^m - \\ -\ 2\cdot(1 + \sigma_c/\sigma_{cT})^m \end{bmatrix} \Big/ \begin{bmatrix} 1 + (\sigma_c/\sigma_{cT})^m - \\ -(1 + \sigma_c/\sigma_{cT})^m \end{bmatrix}; \tag{48}$$

$$h = \sigma_c/\sigma_{cT};\quad p = (\sigma_c/\tau_{s1})\cdot[2/(2\cdot a + 2^m\cdot c)]^{1/m}, \tag{49}$$

where τ_{s1} and τ_{s2} represent the yield limits determined on two different experimental routes, under the following conditions: $\tau_{s1} = \tau_{12}$ for $\sigma_1 = \sigma_2 = 0$, but $\tau_{s2} = \sigma_2 = -\sigma_1$ and $\tau_{12} = 0$. In terms of the ***Lankford coefficient*** values, the equalities may be capitalized [5, 13]:

$$c = 2 - a = 2 \cdot \sqrt{\left[R_L / \left(1 + R_L\right)\right] \cdot \left[R_T / \left(1 + R_T\right)\right]}; \tag{50}$$

$$h = \sqrt{\left[R_L / \left(1 + R_L\right)\right] \cdot \left[\left(1 + R_T\right) / R_T \right]}. \tag{51}$$

Conclusions

The content of the paper highlights some expressions of some known formulas, and used effectively in practical applications, of the interactive criteria regarding the fracture of the isotropic or quasi-isotropic materials (such as metal or filling composite consisting of particles - metallic or non-metallic - respectively chips etc.). Some differences or adaptations, illustrated of the shown bibliography are distinguished, and draws attention to their use, in various practical cases. The researchers can turn account the expressions listed in individualized experiments, so be observed, from case to case, from material to material, which is the criterion that can certify its safety in use. It is very clear that those interested will find a study that can be used conveniently exploited.

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Cu privire la unele criterii interactive utilizate în mecanica ruperii materialelor izotrope şi cvasi-izotrope

Rezumat

Siguranţa tehnică a structurilor mecanice este perfect asigurată dacă în faza de proiectare a acestora sunt foarte bine cunoscute valorile caracteristicilor elastice, termice şi mecanice, stabilite riguros pe cale experimentală. Evident că în acest context se au în vedere procesele tehnologice de elaborare şi tratamentele acceptate. Un rol important îl au şi datele culese pe parcursul exploatării construcţiilor mecanice considerate, astfel încât acestea să fie introduse în relaţiile de calcul şi în criteriile de rupere. Lucrarea de faţă prezintă exprimările matematice oferite de cele mai reprezentative criterii de rupere a materialelor izotrope sau cvasi-izotrope, aşa cum sunt cele metalice sau compozitele cu particule (umplutură).