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Structural Reliability Approach in Thermal Fatigue Crack Growth by Stochastic Modeling

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Abstract

The problem of thermal fatigue in mixing areas arises in nuclear piping where a turbulent mixing or vortices produce rapid fluid temperature fluctuations with random frequencies. The assessment of fatigue crack growth due to cyclic thermal loads arising from turbulent mixing presents significant challenges, principally due to the difficulty of establishing the actual loading spectrum. To apply the stochastic approach of thermal fatigue, a frequency temperature response function is proposed. For the elastic thermal stresses distribution solutions, the magnitude of the frequency response function is first derived and checked against the prediction by FEA. The connection between SIF's power spectral density (PSD) and temperature's PSD is assured with SIF frequency response function modulus. The frequency of the peaks of each magnitude for K_I is supposed to be a stationary narrow-band Gaussian process. The probabilities of failure are estimated by means of the Monte Carlo methods considering a limit state function.

Key words: *structural reliability, thermal stresses, thermal fatigue, stochastic approach*

Introduction

The assessment of the thermal fatigue damage (crack initiation) and subsequent crack growth due to thermal stresses from turbulent mixing or vortices in light water reactor (LWR) piping systems remains a demanding task and effort continues to be devoted to experimental, FEA and analytical studies. The problem of thermal fatigue in mixing areas arises in pipes where a turbulent mixing or vortices produce rapid fluid temperature fluctuations with random frequencies. Structures exposed to such temperature fluctuations can suffer thermal fatigue damage and, subsequently, cracking phenomena, which can produce through wall cracks. Thermal striping is defined as a random temperature fluctuation produced by incomplete mixing of fluid streams at different temperatures. It can arise in certain light water reactor, but also in certain fast breeder reactor structures, notably those situated above the core, because of the large temperature differences that exist between sodium emerging from the core sub-assemblies and from the breeder sub-assemblies. Other areas of potential occurrence include pressurized water reactor nozzles where stratified flows are encountered. In dry-out zones in steam generators, the fluid/steam boundary can oscillate and induce temperature fluctuations on component surface [1]. The results in temperature fluctuations can be local or global and induce random variations of the local temperature gradients in the structural walls of the pipe, which lead to cyclic thermal stresses and strains. The strain variations result in fatigue damage, cracking and crack growth. In particular, one of the most complex issues is the accurate representation of the load. Transient temperature response in the interior of an infinite slab to a sinusoidal surface-temperature input has been investigated by several researchers [2, 3, 4, 5]. For cylindrical geometry it was used mainly isothermal internal boundary condition and with various type of thermal loading at outer surface [1, 6, 7]. The determination of the influence of such a random process on subsurface temperatures is of great importance in establishing the proper depth at which temperature sensitive becomes a concern. Utilizing the method of random process theory it is possible to determine statistical averages such as the mean and standard deviation of the response from the corresponding statistical description of the input process provided that the governing differential equations are linear. If in addition the applied random process is normally distributed the output process will also be normal. This will be assumed in this case. The effect of spatial incoherence in surface temperature fluctuation can be used to calculate the mean square stresses and the mean square equivalent strain range that may be used as a measure of crack initiation likelihood [8]. Also, this type of incoherence has effect on the stress intensity factor in thermal striping. By assuming a perfect spatially coherence but a temporal incoherence it was developed a method of calculating the crack propagation using linear elastic fracture mechanics and stochastic properties of temperature spectrum [6].Thermal striping remains an important subject in the structural integrity area, also for future fast spectrum reactors [9], with the objective of establishing thermal striping limits or appropriate screening criteria.

The present study proposes a stochastic model to assess thermal fatigue crack growth in mixing tees, based on the power spectral density (PSD) of temperature fluctuation at the inner pipe surface. The results of the stochastic approach to thermal fatigue crack growth in mixing tees, completed with the probabilistic input to account for the variability in the material characteristics, are given as probability of failure as function of time reference period. The Civaux 1 damage case is chosen as application of the model predictions.

Statistical Properties of the Thermal Spectrum and the Temperature Frequency Response Function

The main assumption is that the temperature spectrum at the inner pipe surface can be modeled as a stationary Gaussian narrow-band process [6], and that its power spectral density is known. Firstly, an analytical solution for temperature distribution in the wall-thickness of the pipe is derived under sinusoidal thermal loading at the inner surface. A frequency temperature response function is proposed, in the framework of the single-input single-output approach. In the next step the frequency response function is proposed both for stress and stress intensity factor distributions.

The analytical solution for the time dependent temperature profile for an infinite hollow cylinder has been developed in a previous paper [10]. A short overview will be given as follows. Assuming an infinite hollow cylinder made of a homogeneous isotropic material, with inner and outer radii r_i and r_o, the 1D heat diffusion equation has the form:

$$\frac{\partial^2\Theta}{\partial r^2}+\frac{1}{r}\frac{\partial\Theta}{\partial r}=\frac{1}{k}\frac{\partial\Theta}{\partial t}, \ (r_i \le r \le r_o,\ t \ge 0) \quad (1)$$

where

$$\Theta(r,t)=T(r,t)-T_0 \quad (2)$$

is the temperature change from the reference temperature at any radial position r and at time t. The reference temperature T_0 is the body temperature in the unstrained state or the temperature at the initial state. The thermal diffusivity is defined as

$$k=\frac{\lambda}{\rho c} \quad (3)$$

where λ is the thermal conductivity, ρ is the mass density and c is the specific heat conduction. The solution $\Theta(r,t)$ must satisfy the boundary conditions:

$$\Theta(r_i,t) = q(t), (t \geq 0) \tag{4}$$

$$\Theta(r_0,t) = 0, (t \geq 0) \tag{5}$$

and the initial condition

$$\Theta(r,0) = 0, (r_i \leq r \leq r_o) \tag{6}$$

The function $q(t)$ is a known function of time representing the thermal boundary condition applied at the inner surface of cylinder. The analytical solution for the arbitrary boundary condition, $q(t)$, is given in [10] by using the finite Hankel transform, and it is expressed by:

$$\Theta(r,t) = k\pi \sum_{n=1}^{\infty} \frac{s_n^2 J_0^2(s_n r_o)}{J_0^2(s_n r_o) - J_0^2(s_n r_i)} \times \left[J_0(s_n r) Y_0(s_n r_i) - J_0(s_n r_i) Y_0(s_n r) \right] \times \left[e^{-k s_n^2 t} \int_0^t e^{k s_n^2 \tau} q(\tau) d\tau \right]. \tag{7}$$

Here $J_0(z)$, $Y_0(z)$ are Bessel functions of first and second kind of order 0 and s_n are the positive roots of the transcendental equation:

$$Y_0(s_n r_i) J_0(s_n r_o) - J_0(s_n r_i) Y_0(s_n r_o) = 0. \tag{8}$$

A sinusoidal thermal loading at the inner pipe surface is assumed:

$$q(t) = \Theta_0 \cdot \sin(\omega t) = \Theta_0 \cdot \sin(2\pi f t) \tag{9}$$

where Θ_0 is the amplitude of temperature wave, ω and f correspond to angular frequency in radians/second and cycles/second (Hz), respectively, and t is time. Then the final form of the analytical solution for the temperature distribution through the wall-thickness of the pipe is

$$\Theta(r,\omega,t) = k\pi \sum_{n=1}^{\infty} \frac{s_n^2 J_0^2(s_n r_o)}{J_0^2(s_n r_o) - J_0^2(s_n r_i)} \times \left[J_0(s_n r) Y_0(s_n r_i) - J_0(s_n r_i) Y_0(s_n r) \right] \times \left[\Theta_0 \frac{\omega e^{-k s_n^2 t} + (k s_n^2) \sin(\omega t) - \omega \sin(\omega t)}{(k s_n^2)^2 + \omega^2} \right]. \tag{10}$$

In the article [10] the predictions of the analytical solution, given by equation (10), have been checked by a comparison with the finite element analyses performed with ABAQUS computer code (fig. 1), with good agreement [10, 11].

The steady-state response of a linear single-input, single-output system (SISO) [12] to a real sinusoidal input of the form:

$$u(t) = A \sin(\omega t + \psi) \tag{11}$$

is a sinusoidal function. Here A is the amplitude of the input and ψ is an arbitrary phase angle. For sake of simplicity we consider $\psi = 0$. This function has the same angular frequency ω as the input, but modified in its amplitude by the factor $|H(\omega)|$, and shifted in phase by the quantity $\phi(i\omega)$

$$y_s(t) = A |H(\omega)| \sin(\omega t + \psi + \varphi(i\omega)) \tag{12}$$

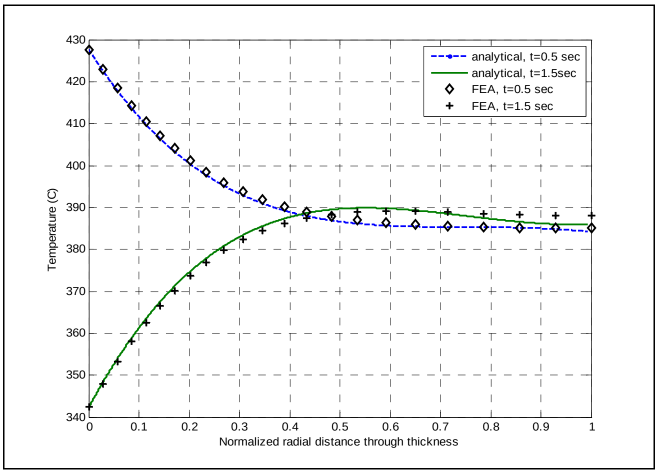

Fig. 1. Comparison between analytical predictions and FEA temperature profiles for a thermal loading at frequency f=0.5Hz [10]

Thus, in general, the steady state response of a linear single-input, single-output system to a sinusoidal input $u(t)=A \sin\omega t$ can be characterized in terms of the magnitude of the frequency response function (FRF), $|H(\omega)|$, and the phase shift $\phi(i\omega)=\angle H(\omega)$. The magnitude of the frequency response function represents the ratio of the output amplitude to the input amplitude as a function of frequency.

The temperature fluctuations in the pipe-wall is then given by

$$\Theta(r,\omega,t) \approx \Theta_0 \left|H_T(r,\omega)\right| \times \sin\left[\omega t - \varphi\right] \tag{13}$$

where $|H_T(r,\omega)|$ is the magnitude of temperature frequency response function.

In a conservative way, if the lag phase is approximate as $\varphi \approx 0$, a comparison between temperature profiles through thickness predicted by equations (10) and (13) are displayed in Figure 2. With the same temperature range at the inner pipe surface and with a slightly deeper penetration through the wall, the prediction of temperature response with assumed magnitude of frequency temperature response may be reasonable accounted. Figure 3 shows the influence of the loading frequency, in case of sinusoidal input, on the temperature frequency response magnitude, for several points inside across the wall of the pipe, considering the geometry and parameters from Civaux case [13]. The highest value of response is obtained at $x/l=4/9=0.44$, while for deeper points in the pipe-wall its values decrease fast for whole range of frequencies.

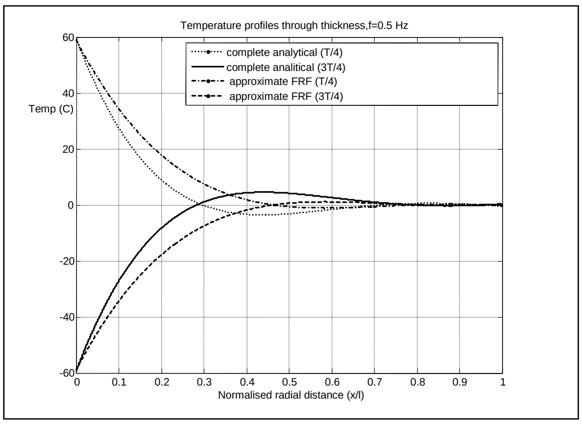

Fig. 2. Comparison between predictions of temperature profile from complete analytical solution and those obtained by means of the analytical temperature frequency response function in the pipe wall

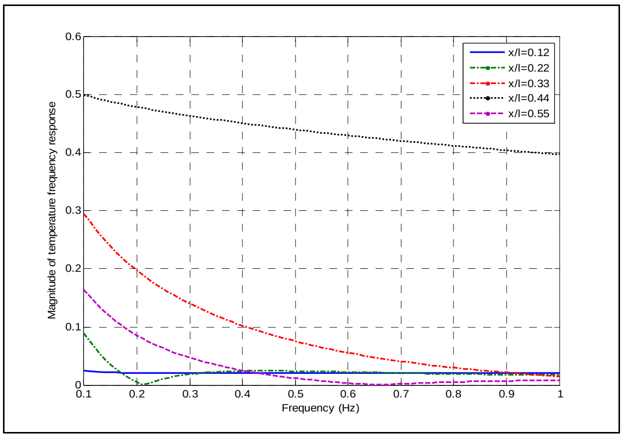

Fig. 3. Dependence of temperature frequency response magnitude on the loading frequency for various depths through the thickness (*l* is the wall-thickness and *x* originates at inner pipe surface).

Modeling of the Stress Response to Random Thermal Input

To obtain the stress frequency response function a similar approach is used. The general solution of elastic thermal stress components (hoop, radial and axial) related to a sinusoidal loading at the inner surface was derived in [10]. Subsequently, the stress frequency response is obtained by means of temperature frequency response function, which will be used in the corresponding analytical solution for stress distribution. A comparison with FEA prediction is made together with a sensitivity analysis versus frequency range.The hoop stress distribution at each time increment is given by the following relationships in the case of plane strain [14]:

$$\sigma_\theta(r,\omega,t) = \frac{\alpha \cdot E}{1-\nu}\left[\frac{1}{r^2}\cdot I_1(r,\omega,t) + \frac{r^2+a^2}{r^2\cdot(b^2-a^2)}\cdot I_2(\omega,t) - \Theta(r,\omega,t) \right] \tag{14}$$

The mathematical relationships for the integrals I1(r,ω,t) and I2(ω,t) are given in [10, 11, 15].

In the present application only hoop stress component, σθ, is considered, to derive its frequency response function, but with a similar approach the frequency response forms for radial (σ_r) and axial (σ_z) stresses can be obtained. The general approach is to substitute the temperature frequency response function in the solution of thermal stress components to make possible obtaining the stress frequency response function [7]. Both integrals may be written in the form similar to equation (13), such as:

$$I_1(r,\omega,t) \approx \Theta_0 \cdot \left[\sum_{n=1}^{\infty} \left| H_{I_1,n}(r,\omega,s_n) \right| \right] \cdot \sin(\omega t - \varphi) \tag{15}$$

and

$$I_2(\omega,t) \approx \Theta_0 \cdot \left[\sum_{n=1}^{\infty} \left| H_{I_2,n}(\omega,s_n) \right| \right] \cdot \sin(\omega t - \varphi) \tag{16}$$

The hoop stress from equation (14) becomes:

$$\sigma_\theta(r,\omega,t) \approx \Theta_0 \cdot \left| H_{\sigma_\theta}(r,\omega) \right| \cdot \sin(\omega t - \varphi) \tag{17}$$

with $\left| H_{\sigma_\theta}(r,\omega) \right|$ is the magnitude for stress frequency response function.

Figure 4 illustrates the comparison between the hoop stress from equation (17) with the complete analytical solution and FEA analysis from [15]. As can be seen from the comparison, it can be concluded that the magnitude of stress frequency response is reasonable described by equation (17), in a conservative way. The magnitude of the stress frequency response has an interesting dependence on loading frequency across the wall thickness, as it is illustrated in Figure 5. Note that frequency responses for the temperature and those for the hoop stress have not the same dependence from frequency and positioning the thickness. Moreover, moving into the pipe wall, for each locations, the frequencies for which the maximum values of respective response functions are reached are not the same as well. Figure 6 displays the profiles of the function $\left|H_{\sigma_\theta}(r,\omega)\right|$, for several frequencies in the range 0.1 Hz to 1.0 Hz. This figure shows a complementary feature to those from Figure 5 and illustrates the sensitivity of this frequency response function to the loading frequency, across the wall-thickness.

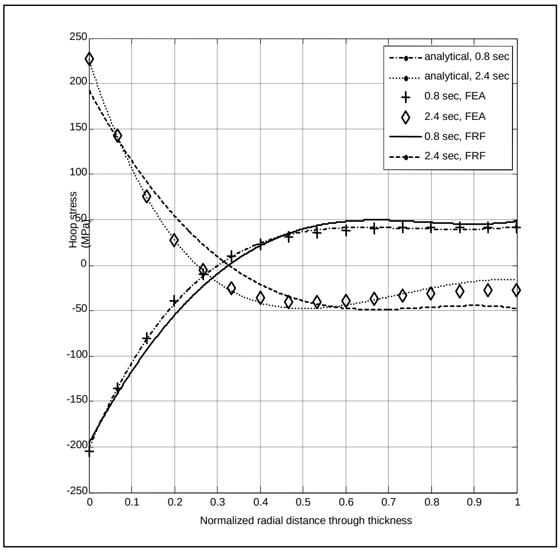

Fig. 4. Comparison between predictions for hoop stress: complete analytical solution, FEA, and by means of stress frequency response function (frequency of sinusoidal thermal loading f=0.3 Hz).

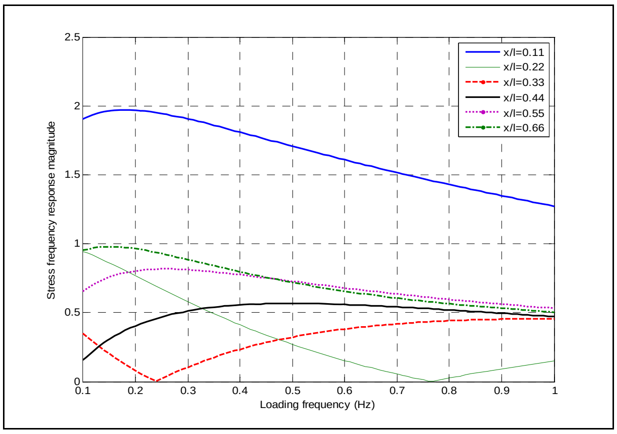

Fig. 5. Magnitude of the stress frequency response function versus loading frequency for various depths through the thickness.

The Stress Intensity Factor Frequency Response

Let assume that there is a shallow crack of infinite length on the inner surface and parallel to the tube axis. The approach to derive the stress intensity factors is based on the polynomial representation of stress components through the wall-thickness of the pipe [16].

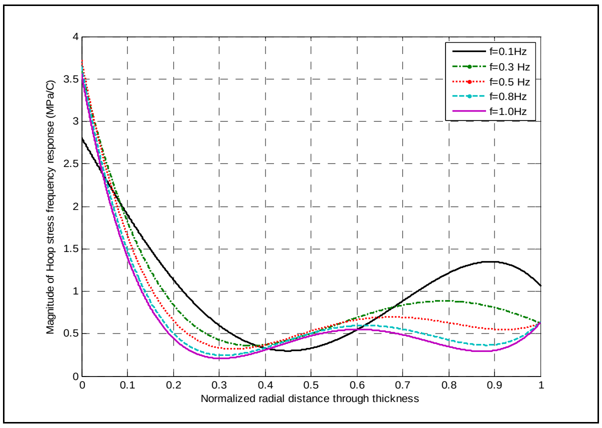

Fig. 6. Magnitude of the stress frequency response function through the wall thickness for various loading frequency.

To evaluate the Mode I stress intensity factor, K_I, for surface crack under thermal stresses, the procedure from [16] was followed, which uses the following relation:

$$K_I\left(\frac{a}{l}\right)=\sqrt{\frac{\pi a}{Q}}\cdot\left[\quad G_0\sigma_0+G_1\sigma_1\cdot\left(\frac{a}{l}\right)+G_2\sigma_2\cdot\left(\frac{a}{l}\right)^2+G_3\sigma_3\cdot\left(\frac{a}{l}\right)^3+G_4\sigma_4\cdot\left(\frac{a}{l}\right)^4\quad\right] \quad (18)$$

where G_0 to G_4 are the influence coefficients (or magnification factors) and σ_i (i=0,…,4) are the coefficients for polynomial stress distribution. In the case of a long axial crack and also fully circumferential crack on inner pipe surface the Q parameter is considered as Q=1. The calculation of the SIF from the surface temperature variation can be regarded as a frequency response calculation with absolute value (magnitude) $\left|H_K\left(\frac{a}{l},\omega\right)\right|$. The methodology was derived elsewhere [4, 6], and the weight function method is the most used. The magnitude of frequency transfer function for SIF may be written in terms of the stress frequency response [4]. To do this, the function $\left|H_{\sigma_\theta}\left(r,\omega\right)\right|$ is written as through-thickness profile

$$\left|H_{\sigma_\theta}\left(\frac{x}{l},\omega\right)\right|=h_0(\omega)+h_1(\omega)\cdot\left(\frac{x}{l}\right)+h_2(\omega)\cdot\left(\frac{x}{l}\right)^2+h_3(\omega)\cdot\left(\frac{x}{l}\right)^3+h_4(\omega)\cdot\left(\frac{x}{l}\right)^4 \quad . \quad (19)$$

From the dependencies displayed in Figure 6, it seen that the fitting coefficients h_j (j=0,…,4) depend on the loading frequency ω, ($\omega=2\pi f$). The magnitude of SIF frequency response function (or amplitude of the frequency transfer function for SIF) is assumed to be given by

$$\left|H_K\left(\frac{a}{l},\omega\right)\right|=\sqrt{\pi a}\cdot\left|G_K\left(\frac{a}{l},\omega\right)\right| \quad (20)$$

with

$$\left|G_K\left(\frac{a}{l},\omega\right)\right|=\left|\sum_{i=0}^{4}h_i(\omega)\cdot G_i\left(\frac{a}{l}\right)\cdot\left(\frac{a}{l}\right)^i\right|. \quad (21)$$

Figure 7 shows the dependence of magnitude of SIF frequency response function, $\left|H_K\left(\frac{a}{l},\omega\right)\right|$ on loading frequency for various crack depth. As crack is growing into the thickness, the magnitude response is higher. Note that for small crack depth the magnitude of SIF response is almost the same for whole of frequency range, and for deeper cracks the maximum of response is reached for 0.2-0.3 Hz. For the reference geometry considered in the work [13], the frequency response function can be used to obtain the stress intensity factor, K_I. Its dependence on loading frequency for various crack depth is given by (see fig. 8):

$$K\left(\frac{a}{l},\omega,t\right)=\Theta_0\cdot\sqrt{\pi a}\cdot\left|G_K\left(\frac{a}{l},\omega\right)\right|\cdot\sin(\omega t-\phi)\,. \qquad (22)$$

The examination of this behavior of K_I, which is calculated for the instant of time $t=T/4$ (with T= time period of loading), suggests a highest value for frequency f=0.3 Hz, which is in a good agreement with previous study [15].

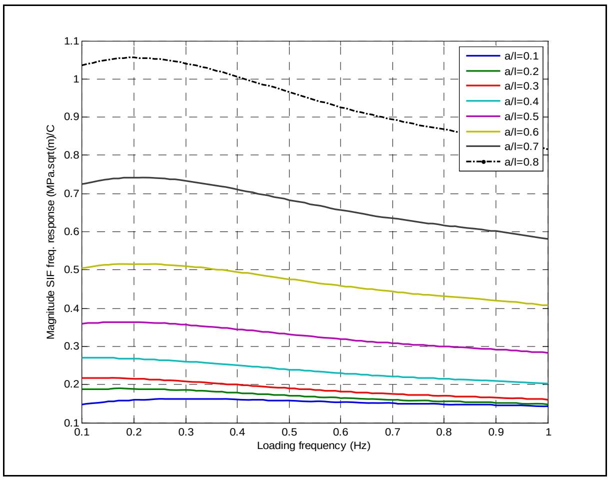

Fig. 7. The dependence of SIF frequency response magnitude as function on loading frequency (Hz) on crack depth

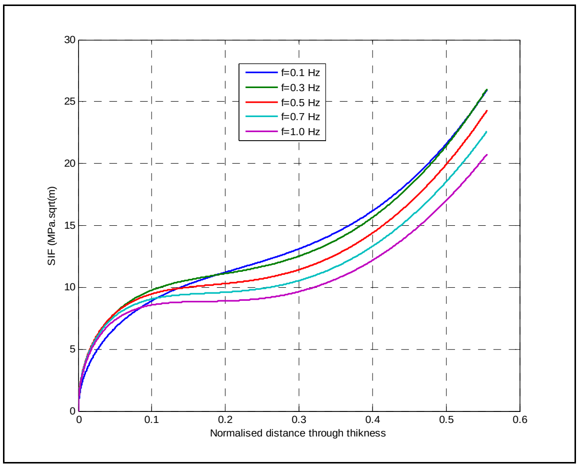

Fig. 8. Stress intensity factor (instant $T/4$) using SIF frequency response function versus crack depth.

The analysis above has been performed by considering a sinusoidal thermal loading as surface temperature fluctuations. For mixing tees the surface temperature variation is a random process. The input of surface temperature fluctuations can always be characterized by its power spectral density (PSD), which is the Fourier transform of the autocorrelation function. This may be obtained from experimental measurements. Moreover, it is also necessary to postulate a probability distribution functional for temperature. This will be taken to be Gaussian, implying a Gaussian probability density function for temperature at any instant [6], which is completely described by its PSD.

The approach followed is to consider the temperature fluctuation and its spectrum as a Gaussian stationary narrow-band process. In this case, the magnitude of SIF frequency response function, $\left|H_K\left(x_a,\omega\right)\right|$, relates the PSD of SIF, $S_K\left(x_a,\omega\right)$, and PSD of surface temperature $S_T(\omega)$, respectively, as

$$S_K\left(x_a,\omega\right)=\left|H_K\left(x_a,\omega\right)\right|^2 S_T\left(\omega\right) \tag{23}$$

with $x_a=a/l$ crack depth to thickness ratio.

The mean square (variance) of the SIF is given by

$$K_{rms}^2\left(x_a\right)=\int_{-\infty}^{\infty} S_K\left(x_a,\omega\right)d\omega \quad . \tag{24}$$

Moreover, from practical point of view it is considered the one-sided PSD with frequency expressed in Hertz (cycles/second)

$$W_T\left(f\right)=4\pi\cdot S_T\left(\omega\right) \tag{25}$$

with $W_T(f)$ expressed in (° C)^2/Hz, and $f=\dfrac{\omega}{2\pi}$ in Hz and the PSD of SIF is given by

$$W_K\left(x_a,f\right)=\left|H_K\left(x_a,f\right)\right|^2 W_T\left(f\right) \tag{26}$$

In the following we consider the one-sided PSD of temperature with $W_T\left(f\right)=W_{T0}=const.$, for a range of frequencies is considered $f\in\left[f_1,f_2\right]$, Figure 9.

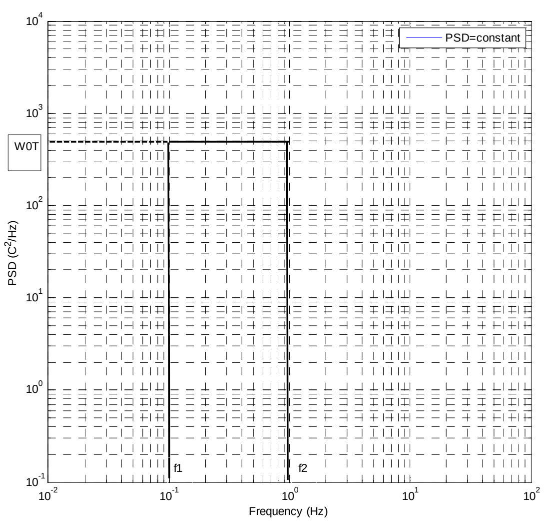

Fig. 9. One-sided PSD for temperature fluctuations

The frequency of peaks of any magnitude for K_I, which is supposed to be a stationary narrow-band Gaussian process with τ delay time is characterized by Rayleigh distribution:

$$f_{K_I}\left(x_a,\tau=0\right)=\frac{K_I}{K_{rms}^2\left(x_a\right)}\exp\left[-\frac{1}{2}\frac{K_I}{K_{rms}^2\left(x_a\right)}\right] \tag{27}$$

Thermal Fatigue Crack Growth Lifetime Estimate

The present analysis assumes the Paris law for crack growth per cycle as

$$\frac{da}{dN} = C \cdot \left(\Delta K\right)^n \tag{28}$$

where N is the number of maxima and ΔK is the range between the maximum and next minimum; here the range between maximum and next zero is considered:

$$\Delta K = K \,. \tag{29}$$

The stochastic model for thermal fatigue crack growth developed includes a first part incorporating stochastic loads (derived into stochastic behavior of K) and a second one, that deals with Monte Carlo simulation, to accommodate statistical characteristics of crack growth under constant amplitude.

The time-dependent fluctuation of the temperature is correlated with the time dependent fluctuation of crack growth from Paris law. Because the number of loading cycles is a discrete variable with respect to time variable, the number of loading cycles is modified into a continuous variable by introducing an average cyclic rate. So, when time-dependent stochastic analysis is conducted, the crack growth rate of a random flaw size, a, should be written in the following form:

$$\frac{da}{dt} = \frac{da}{dN}\frac{dN}{dt} = \nu_p \frac{da}{dN} \tag{30}$$

where ν_p is the mean rate of maxima, that is constant for a stationary stochastic Gaussian process. For this kind of process, ν_p may be identified with the expected rate of up-crossing rate (equal to the expected rate of peak crossing) [17]

$$\nu_p = \dot{N}_{K,0} = \dot{N}_{K,p} = \frac{\sqrt{\int_{f_1}^{f_2} f^2 W_K\left(x_a, f\right) df}}{\sqrt{\int_{f_1}^{f_2} W_K\left(x_a, f\right) df}} \,. \tag{31}$$

For the one-sided PSD case, as it is displayed in Figure 9, this is equivalent to

$$\nu_p = \frac{\sqrt{\int_{f_1}^{f_2} f^2 \left|H_K\left(x_a, f\right)\right|^2 df}}{\sqrt{\int_{f_1}^{f_2} \left|H_K\left(x_a, f\right)\right|^2 df}} \,. \tag{32}$$

If a linear summation of damage, ignoring the effect of positive minima [6], is assumed, the expected rate of crack growth in respect to cycles is

$$E\left[\frac{da}{dN}\right] = \int_0^\infty C \cdot \left(K\right)^n f_{K_I}\left(x_a, \tau = 0\right) dK \,. \tag{33}$$

By re-arranging the equation (33) and considering the expression of the n^{th} moment of the Rayleigh distribution, the final form of stochastic crack growth rate is given by:

$$\frac{dx_a}{dt}=\frac{C}{l}\cdot\left(\int_{f_1}^{f_2} f^2\left|H_K\left(x_a,f\right)\right|^2 df\right)^{\frac{1}{2}}\times\left(\int_{f_1}^{f_2}\left|H_K\left(x_a,f\right)\right|^2 df\right)^{-\frac{1}{2}}\times$$
$$\times\left[W_{0T}\int_{f_1}^{f_2}\left|H_K\left(x_a,f\right)\right|^2 df\right]^n\times\times 2^{\frac{n}{2}}\Gamma\left(1+\frac{1}{n}\right). \tag{34}$$

where Γ is the Gamma function. This equation must be numerically integrated to obtain the normalized crack length, x_a, as a function of time, when C and n are given deterministically.

Application

A prospective study for the probabilistic approach of thermal fatigue in mixing tees by means of limit state function and Monte Carlo simulation, based on sinusoidal approach, has been done in a previous work [18]. In the present work the limit state function will be based on equation (34) together with a probabilistic input to account for the variability in the initial crack depth and in C scaling parameter.

A crack penetration depth of 80% of the wall thickness has been considered as the limit state of the thermal fatigue damage failure. To combine the stochastic behavior of K with statistical characteristics of crack growth under constant amplitude (C and n Paris law parameters), and also with initial crack depth distribution, the limit state function is defined in the following form:

$$g(t_{ref})=1-\frac{t_{stoch}}{t_{ref}} \tag{35}$$

where: t_{ref} is the reference time period for the thermal fatigue crack growth under thermal spectrum, t_{stoch} is the estimated values of lifetime for stochastic crack growth derived from equation (34).

During the Monte Carlo simulation (MCS), the trials which satisfy the condition

$$g\left(t_{ref}\right)\leq 0 \tag{36}$$

are accounted as n_{fail} and the probability of failure for a certain period of time (t_{ref}) is given by

$$P_f=\frac{n_{fail}}{N_{trials}} \tag{37}$$

where N_{trials} is the total number of trials of the MC simulation. The initial crack size distribution has a very strong influence on the deterministic and also probabilistic assessment of the component lifetime.

The present approach considers only axial long cracks at inner surface, characterized by an exponential distribution for the initial crack depth [18]. Slopes (n) and intercepts (C) for all fatigue data represented by equation (28) are usually highly correlated. Ignoring this correlation can give misleading results in the simulation. An alternate method to account for this correlation is to use a constant slope and put all of variability into the intercept [19]. For a constant slope, the variability in fatigue lives will be directly related to the variability in the material constant C, usually by a lognormal distribution [19]. The geometry and parameters from Civaux case will be considered during this application [13, 20]. For a thermal spectrum assumed to be stationary Gaussian stochastic process we use the one-sided temperature PSD (fig. 9):

$$W_T(f)=\begin{cases} W_{0T}=500C^2/Hz, f\in[0.1,1.0] \quad Hz \\ 0, f\in[0,0.1Hz)\cup(1.0Hz,\infty] \end{cases} \tag{38}$$

By re-conversion using RSA method (Random Spectral Amplitudes) [21], we extract a sample function for temperature that is displayed in Figure 10, with an imposed non-zero mean value.

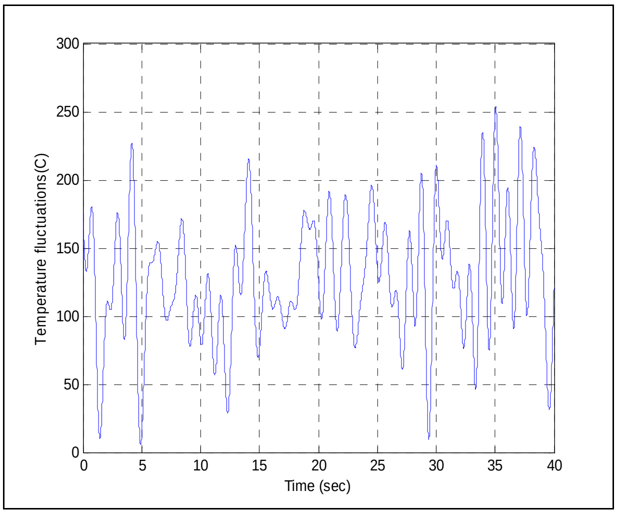

Fig. 10 Sample function of the temperature variation from PSD of a stationary Gaussian narrow-band process (PSD=500 C2/Hz, f =0.1-1.0 Hz, with non-zero mean value)

The Monte Carlo analyses were performed by implementing in the MATLAB environment specific scripts and described function, using a number of trials of 10^4-10^5 order.

The results are displayed in Figure 11. The probabilities of failure, defined by limit state function from equation (35), are given as function of the reference time period. The same graph displays the lifetime for crack penetration through the wall as it has been reported for Civaux case [13]. One can see that the time of 1500 hours, corresponds to a probability of failure of about 80%.

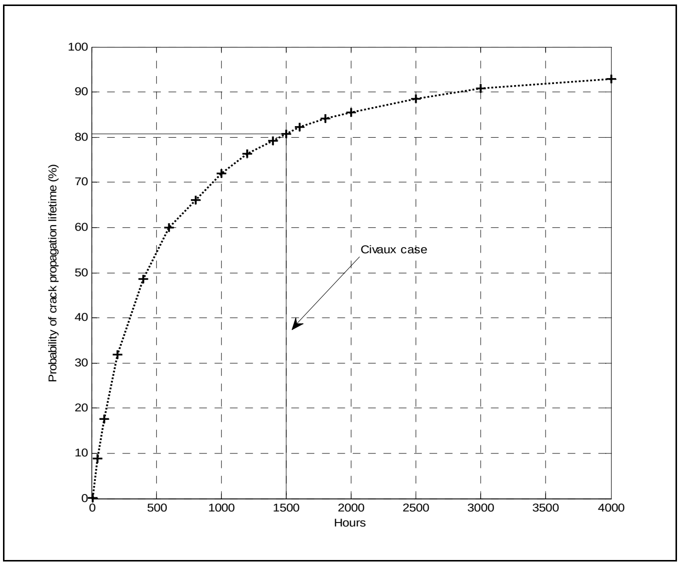

Fig. 11. Probabilities of failure: the stochastic modeling results of fatigue crack growth coupled with probabilistic input for Monte Carlo simulation

Conclusions

The study proposes a stochastic model focused only to assess thermal fatigue crack growth in mixing tees of NPP with the temperature spectrum assumed to be a Gaussian stationary narrow-band stochastic process. The stochastic fatigue crack growth model used includes a main part for incorporating the randomness of the in-service thermal loads, and a second one which includes the description of the statistical characteristics for the crack growth under constant amplitude loadings. Based on the analytical solution of the temperature response (Hankel transform) within the SIN-methodology developed in previous work, a temperature frequency response function through the pipe thickness is developed. By considering the analytical solution for the thermal stresses developed in previous works, a stress frequency response function for the thermal hoop stress is derived and a SIF frequency response magnitude is obtained. With hypothesis of one-sided PSD model for the temperature fluctuation, the PSD of SIF is obtained by means of FRF methodology and, consequently, the expected value of crack growth rate in HCF domain can be assessed using the Rayleigh distribution moments. The variability of the Paris law parameters and of the initial crack size distribution is accounted for within the probabilistic approach and the probabilities of failure are obtained by MCS. The present methodology based on the stochastic modeling of thermal fatigue crack growth can be used to analyze and improve the screening criteria proposed to avoid cracking damage in nuclear piping, especially in tee connection where turbulent mixing of flows with different temperature can occur.

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Abordarea fiabilității structurale a creşterii fisurii de oboseală termică prin modelare stohastică

Rezumat

Problematica oboselii termice este relevantă pentru tubulatura centralelor nucleare unde se realizează amestecul a două lichide aflate inițial la temperaturi diferite şi unde se produc fluctuații rapide de temperatură ale fluidului de amestec. Evaluarea creşterii fisurilor datorită fluctuației sarcinilor termice in aceste zone, prezintă dificultăți serioase de abordare datorită imposibilității determinării spectrului real de frecvențe. In vederea abordării stohastice a propagării unei fisuri de oboseală termică, lucrarea propune o funcție de răspuns in frecvență pentru temperatură. Ulterior sunt obținute soluțiile distribuției de tensiuni elastice şi amplitudinea funcției corespunzătoare de răspuns in frecvență. Conexiunea dintre densitatea spectrală de putere a factorului K_I şi cea a temperaturii este realizată prin intermediul modulului funcției de răspuns in frecvență pentru factorul K_I. Spectrul de frecventă a maximelor pentru K_I se presupune că face parte dintr-un proces staționar Gaussian de banda îngustă. Probabilitățile de rupere sunt estimate pe baza unei funcții corespunzătoare de stare limită.