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Theoretical Aspects Regarding the Friction Resulted in Simply Supported Circular or Annular Plates

Radu I. Iatan[*], Ruxandra Gh. Mengher[**]

[*] Universitatea POLITEHNICA Bucureşti, Splaiul Independenţei 313, 060042 Bucureşti, Sector 6
e-mail: r_iatan@yahoo.com

[**] LUDAN ENGINEERING S.R.L., Bucureşti

Abstract

This paper presents the influence, on deformation and stress state, of the friction between a circular or annular plate and the simple support. Selection coefficients are introduced to separate the exterior loadings and their partial influence.

Key words: *friction, simply supported, circular and annular plate*

Introduction

Industrial equipment generally has a complex structure in which there are constructive elements such as annular or circular plates, among others. To properly evaluate the stress state when different exterior loadings are acting this paper proposes to also take into consideration the fiction between plate and support in the case of simply supporting the outer and/or inner edges and the inferior face of the plate (the case of vessel bottoms positioned on rigid foundations [1-4]). In this supposition, a circular or annular plate can be deformed so that on the support there is a slide of the contact surface, but the length of the median axis stays the same. The effect, the friction force, can be added to other exterior loadings which can act on the tested plate, the loadings combined lead to the evaluation of the displacements and stress which influence the final decision. In the same context the weight of the plate has to be taken into consideration - as a concentrated force or uniformly distributed, among other forces frequently included in the known literature [5].

The Friction Effect between the Plate and the Simple Support

Generalities

On the support/supports, in the deformation phase of the plate loaded with different exterior forces, a friction force is developed, dependent on the friction coefficient, μ, typical for any materials in contact, in relative motion, and for the intensity of the normal distributed force.

For example, considering a concentrated force S (fig. 1), under its action the plate deforms properly, so that between the inferior face of the plate and the fixed simple support a friction force is developed – uniformly distributed, $\overline{F}_f$.

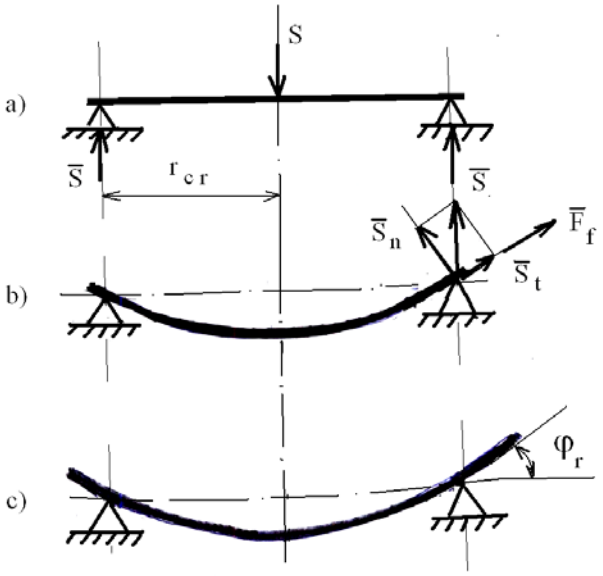

Fig. 1. Sketching the development of the friction force between the plate and the fixed simple support

Based on the exposed facts it is determined:

$$\overline{S} = \frac{S}{2 \cdot \pi \cdot r_{cr}}; \quad \overline{S}_n = \overline{S} \cdot \cos \varphi_r ;$$

$$\overline{S}_t = \overline{S} \cdot \sin \varphi_r . \qquad (1)$$

Considering that the rotation angle of the plate on the fixed simple support φ_r – figure 1 – has a small value then:

$$\cos \varphi_r \approx 1;$$

$$\sin \varphi_r \approx \varphi_r , \qquad (2)$$

and:

$$\overline{S}_n \approx \overline{S}; \quad \overline{S}_t \approx \overline{S} \cdot \varphi_r \qquad (3)$$

The uniformly distributed friction force has the following expression:

$$\overline{F}_f = \mu \cdot \overline{S}_n = \mu \cdot \overline{S} \cdot \cos \varphi_r \approx \mu \cdot \overline{S} . \qquad (4)$$

To make a more precise analysis of the plate's loadings it is necessary to obtain the translation of friction force, $\overline{F}_f$, through bringing it at the level of the medium surface of the plate and introducing the adequate bending moment:

$$M_f = 0{,}5 \cdot \delta_p \cdot \overline{F}_f . \qquad (5)$$

Taking into account (3) and (4), the longitudinal stress acts along the elastic surface of the plate:

$$Q = \left(\mu + \varphi_r \right) \cdot \overline{S} . \qquad (6)$$

<u>**Note**</u>: In the case of a sliding simple support, the friction bending moment has the following expression:

$$M_f^{\bullet} = \overline{F}_f \cdot h_r , \qquad (7)$$

where h_r represents the height of the support.

Examples

Circular plate with no console area, exterior to the edge, with pressure (on a face) and the weight uniformly distributed

Under the action of the uniformly distributed pressure, p, the friction force is uniformly distributed on the radius, r_{cr}, of the simple support and has the expression:

$$\overline{F}_f = \frac{\mu}{2}\cdot r_{cr}\cdot\left(c_g\cdot p_G + c_p\cdot p\right) = \frac{\mu}{2}\cdot r_{cr}\cdot p_T\,, \tag{8}$$

and the friction bending moment:

$$M_f = \frac{\mu}{4}\cdot r_{cr}\cdot\delta_p\cdot\left(c_g\cdot p_G + c_p\cdot p\right) = \frac{\mu}{4}\cdot r_{cr}\cdot\delta_p\cdot p_T\,. \tag{9}$$

Note: When selecting c_g and c_p coefficients, the following will be met:

- $c_g = 1{,}0$ − when the weight is taken into consideration; $c_g = 0{,}0$ – when it is not;
- $c_p = 1{,}0$ − when the uniformly distributed pressure p is taken into consideration and has the same direction as the weight; $c_p = -1{,}0$ − when the uniformly distributed pressure has the other direction then the weight G_p - uniformly distributed weight; $c_p = 0{,}0$ − when the uniformly distributed pressure p is not taken into consideration.

To evaluate the displacement w and the rotation φ the following expressions are used [2, 4]:

$$w = \frac{p_T\cdot r_{cr}^3}{8\cdot\Re_p}\cdot\left[\frac{r_{cr}}{8}\cdot\left(\frac{5+\nu_p}{1+\nu_p} - 2\cdot\frac{r^2}{r_{cr}^2}\cdot\frac{3+\nu_p}{1+\nu_p} + \frac{r^4}{r_{cr}^4}\right) + \frac{c_f\cdot\mu\cdot\delta_p\cdot}{1+\nu_p}\cdot\left(1-\frac{r^2}{r_{cr}^2}\right)\right]\cdot\frac{1}{1+c_f\cdot\alpha_f}, \tag{10}$$

$$\varphi = \frac{p_T\cdot r}{4\cdot\Re_p}\cdot\left[\frac{r^2}{4}\cdot\left(\frac{r_{cr}^2}{r^2}\cdot\frac{3+\nu_p}{1+\nu_p} - 1\right) + \frac{c_f\cdot\mu\cdot r_{cr}\cdot\delta_p}{1+\nu_p}\right]\cdot\frac{1}{1+c_f\cdot\alpha_f}; \tag{11}$$

The maximum displacement is at $r = 0$:

$$w_{max} = \frac{p_T\cdot r_{cr}^3}{8\cdot\left(1+\nu_p\right)\cdot\Re_p}\cdot\left[\frac{r_{cr}}{8}\cdot\left(5+\nu_p\right) + c_f\cdot\mu\cdot\delta_p\right]\cdot\frac{1}{1+c_f\cdot\alpha_f}\,, \tag{12}$$

where the rotation is zero, while the rotation has a maximum value on the support:

$$\varphi_{max} = \varphi_r = \frac{p_T\cdot r_{cr}^2}{4\cdot\Re_p}\cdot\left[\frac{r_{cr}}{4}\cdot\left(\frac{3+\nu_p}{1+\nu_p} - 1\right) + \frac{c_f\cdot\mu\cdot\delta_p}{1+\nu_p}\right]\cdot\frac{1}{1+c_f\cdot\alpha_f};\ \alpha_f = \frac{\overline{F}_f\cdot r_{cr}^2}{4{,}2\cdot\Re_p}. \tag{13}$$

Note: In the above expressions c_f was introduced as a selection coefficient which can have the value $1{,}0$ when the friction is taken into consideration or value 0,0 (zero) when it is neglected.

Note: In the case of analyzing the plate joined with other constructive elements, then the radial displacement on the superior or inferior surface of the plate has to be taken also into consideration:

$$u = \pm\,0{,}5\cdot\delta_p\cdot\varphi_{max}\,. \tag{14}$$

Regarding the evaluation of the radial and circumferential stress the known expressions [2] can be used, suited for this case:

$$\sigma_r = \pm\frac{E_p\cdot\delta_p}{2\cdot\left(1-\nu_p^2\right)}\cdot\left(\frac{d\varphi}{dr} + \nu_p\cdot\frac{\varphi}{r}\right) + \frac{1}{2\cdot\delta_p}\cdot\left(c_f\cdot\mu + \varphi_r\right)\cdot p_T\cdot r_{cr}\,; \tag{15}$$

$$\sigma_\theta = \pm\frac{E_p\cdot\delta_p}{2\cdot\left(1-\nu_p^2\right)}\cdot\left(\frac{\varphi}{r} + \nu_p\cdot\frac{d\varphi}{dr}\right) + \frac{1}{2\cdot\delta_p}\cdot\left(c_f\cdot\mu + \varphi_r\right)\cdot p_T\cdot r_{cr}\,, \tag{16}$$

using for this expression (11) and taking into account the observation made for c_f the selection coefficient.

Note: It can be seen that introducing the angle on the support φ_r the radial strain created through the deformation of the plate has an effect on the values for radial and circumferential stress. At low deformations the influence on the mentioned angle is small, but at big rotation angles this value can be considerable.

Annular plate simply supported on the interior edge, with pressure (on a face) and the weight uniformly distributed

This time, the total load is give by:

$$p_T = c_g \cdot p_G + c_p \cdot p,$$

with the observations made above regarding the selection/influence coefficients.

The friction force, uniformly distributed on the support, has the following expression:

$$\overline{F}_f = \frac{\mu}{2} \cdot \frac{r_{cr}^2 - r_0^2}{r_0} \cdot p_T, \tag{17}$$

also the friction moment distributed on the same edge, as a result of making the translation of friction force established by the expression (5).

As follows, the displacements and rotation of the plate have the expressions:

$$w = k_{9\,p_T}^{w} \cdot p_T; \quad \varphi = k_{9\,p_T}^{\varphi} \cdot p_T, \tag{18}$$

where:

$$\begin{aligned} k_{9\,p_T}^{w} &= \frac{r_{cr}^4}{4\cdot\Re_p}\cdot\left[\frac{x^2\cdot(1-\ln x)-1}{2\cdot\alpha^2} - \frac{x^4-1}{16\cdot\alpha^4}\right] - \\ &- \frac{r_{cr}^4}{4\cdot\Re_p}\cdot\left[\frac{x^2-1}{2}\cdot\left(\begin{array}{c}\dfrac{3+\nu_p}{1+\nu_p}\cdot\dfrac{1+\alpha^2}{4\cdot\alpha^4} + \\ + \dfrac{\ln\alpha}{\alpha^2-1} - \dfrac{1-\nu_p}{2\cdot(1+\nu_p)\cdot\alpha^2}\end{array}\right)\right] - \\ &- \frac{r_{cr}^4}{4\cdot\Re_p}\cdot\left[\frac{\ln x}{1-\nu_p}\cdot\left(\frac{3+\nu_p}{4\cdot\alpha^2} - (1+\nu_p)\cdot\frac{\ln\alpha}{\alpha^2-1}\right)\right] - \\ &- \frac{c_f\cdot\mu\cdot\left(r_{cr}^2 - r_0^2\right)\cdot r_{cr}^2\cdot\delta_p}{8\cdot\Re_p\cdot r_0\cdot(1+\nu_p)\cdot(1-\beta^2)}\cdot\left(1-\rho^2 - 2\cdot\frac{1+\nu_p}{1-\nu_p}\cdot\beta^2\cdot\ln\rho\right); \end{aligned} \tag{19}$$

$$\begin{aligned} k_{9\,p_T}^{\varphi} &= \frac{r_{cr}^4}{4\cdot\Re_p\cdot r_0}\cdot\left\{\frac{x\cdot(1-2\cdot\ln x)}{2\cdot\alpha^2} + \frac{x^3}{4\cdot\alpha^4} - \right. \\ &- x\cdot\left(\frac{3+\nu_p}{4\cdot(1+\nu_p)}\cdot\frac{1+\alpha^2}{\alpha^4} + \frac{\ln\alpha}{\alpha^2-1} - \frac{1-\nu_p}{2\cdot(1+\nu_p)\cdot\alpha^2}\right) - \\ &\left. - \frac{1}{1-\nu_p}\cdot\left[\frac{3+\nu_p}{4\cdot\alpha^2} - (1+\nu_p)\cdot\frac{\ln\alpha}{\alpha^2-1}\right]\cdot\frac{1}{x}\right\} - \\ &- \frac{c_f\cdot\mu\cdot\left(r_{cr}^2 - r_0^2\right)\cdot\delta_p\cdot r_{cr}}{2\cdot\Re_p\cdot(1+\nu_p)\cdot r_0}\cdot\frac{1}{1-\beta^2}\cdot\left(\rho + \frac{1+\nu_p}{1-\nu_p}\cdot\frac{\beta^2}{\rho}\right), \end{aligned} \tag{20}$$

x and α have the expressions:

$$x = r/r_0;\ x = 1,\ \text{for } r = r_0; \qquad x = \alpha,\ \text{for } r = r_{cr}; \quad \alpha = r_{cr}/r_0;$$
$$\rho = r/r_{cr}; \quad \beta = r_0/r_{cr}.$$

Radial and circumferential stress can be evaluated with:

$$\sigma_r = \pm\frac{E_p\cdot\delta_p}{2\cdot\left(1-\nu_p^2\right)}\cdot\left(\frac{d\varphi}{dr}+\nu_p\cdot\frac{\varphi}{r}\right)+\frac{1}{2\cdot\delta_p}\cdot\left(c_f\cdot\mu+\varphi_r\right)\cdot p_T\cdot r_0\cdot\left(1-\frac{r_{cr}^2}{r^2}\right); \tag{21}$$

$$\sigma_\theta = \pm\frac{E_p\cdot\delta_p}{2\cdot\left(1-\nu_p^2\right)}\cdot\left(\frac{\varphi}{r}+\nu_p\cdot\frac{d\varphi}{dr}\right)+\frac{1}{2\cdot\delta_p}\cdot\left(c_f\cdot\mu+\varphi_r\right)\cdot p_T\cdot r_0\cdot\left(1+\frac{r_{cr}^2}{r^2}\right). \tag{22}$$

Annular plate simply supported on the exterior edge, with pressure (on a face) and the weight uniformly distributed

In the given conditions, the displacement and the rotation of the plate have the expressions:

$$w = k_{10p_T}^w\cdot p_T; \quad \varphi = k_{10p_T}^\varphi\cdot p_T, \tag{23}$$

where the influence factors are:

$$k_{10p_T}^w = \frac{r_{cr}^4}{64\cdot\Re_p}\cdot\left\{\begin{array}{l}\left(\dfrac{x}{\alpha}\right)^4-1+\dfrac{8\cdot x^2\cdot(1-\ln x)}{\alpha^4}-\dfrac{2\cdot\left(3+\nu_p\right)}{1+\nu_p}\cdot\\ \cdot\dfrac{\left(x^2-\alpha^2\right)\cdot\left(\alpha^2+1\right)}{\alpha^4}+4\cdot\dfrac{3+\nu_p}{\left(1-\nu_p\right)\cdot\alpha^2}\cdot\ln\dfrac{\alpha}{x}\end{array}\right\}+$$
$$+\frac{r_{cr}^4}{64\cdot\Re_p}\cdot\left\{\begin{array}{l}4\cdot\dfrac{1-\nu_p}{1+\nu_p}\cdot\dfrac{x^2-\alpha^2}{\alpha^4}+8\cdot\dfrac{\ln\alpha-1}{\alpha^2}+8\cdot\dfrac{\ln\alpha}{\left(\alpha^2-1\right)\cdot\alpha^2}\cdot\\ \cdot\left(x^2-\alpha^2+2\cdot\dfrac{1+\nu_p}{1-\nu_p}\cdot\ln\dfrac{x}{\alpha}\right)\end{array}\right\}-$$
$$-\frac{c_f\cdot\left(r_{cr}^2-r_0^2\right)\cdot r_{cr}^2\cdot\delta_p}{8\cdot\Re_p\cdot\left(1+\nu_p\right)\cdot\left(1-\beta^2\right)\cdot r_0}\cdot\left(1-\rho^2-2\cdot\frac{1+\nu_p}{1-\nu_p}\cdot\beta^2\cdot\ln\rho\right); \tag{24}$$

$$k_{10p_T}^\varphi = \frac{r_{cr}^4}{16\cdot\Re_p\cdot r_0}\cdot\left[\frac{x^3+2\cdot x\cdot(1-2\cdot\ln x)}{\alpha^4}-\frac{3+\nu_p}{1+\nu_p}\cdot\frac{\alpha^2+1}{\alpha^4}\cdot x-\right.$$
$$\left.-\frac{3+\nu_p}{\left(1-\nu_p\right)\cdot\alpha^2\cdot x}+2\cdot\frac{\left(1-\nu_p\right)\cdot x}{\left(1+\nu_p\right)\cdot\alpha^4}+\frac{4\cdot\ln\alpha}{\left(\alpha^2-1\right)\cdot\alpha^2}\cdot\left(x-\frac{1+\nu_p}{1-\nu_p}\cdot\frac{1}{x}\right)\right]-$$
$$-\frac{c_f\cdot\left(r_{cr}^2-r_0^2\right)\cdot r_{cr}\cdot\delta_p}{4\cdot\Re_p\cdot\left(1+\nu_p\right)\cdot r_0}\cdot\frac{1}{1-\beta^2}\cdot\left(\rho+\frac{1+\nu_p}{1-\nu_p}\cdot\frac{\beta^2}{\rho}\right), \tag{25}$$

and $\rho = r/r_{cr}$; $\beta = r_0/r_{cr}$.

This time the radial and circumferential stress can be evaluated with:

$$\sigma_r = \pm\frac{E_p\cdot\delta_p}{2\cdot\left(1-\nu_p^2\right)}\cdot\left(\frac{d\varphi}{dr}+\nu_p\cdot\frac{\varphi}{r}\right)+\frac{1}{2\cdot\delta_p}\cdot\left(c_f\cdot\mu+\varphi_r\right)\cdot p_T\cdot\frac{r_{cr}^2}{r_0}\cdot\left(1-\frac{r_0^2}{r^2}\right); \quad (26)$$

$$\sigma_\theta = \pm\frac{E_p\cdot\delta_p}{2\cdot\left(1-\nu_p^2\right)}\cdot\left(\frac{\varphi}{r}+\nu_p\cdot\frac{d\varphi}{dr}\right)+\frac{1}{2\cdot\delta_p}\cdot\left(c_f\cdot\mu+\varphi_r\right)\cdot p_T\cdot\frac{r_{cr}^2}{r_0}\cdot\left(1+\frac{r_0^2}{r^2}\right). \quad (27)$$

Other notations: ν_p – Poisson's coefficient; E_p – Young's modulus; δ_p – plate thickness; r_0 – interior radius of the plate; $\Re_p$ – bending stiffness.

Conclusions

This paper highlights the influence of the friction force which can manifest between the surface of the circular or annular plate and the simple support on which it is positioned, while keeping the medium surface from deformation.

At the same time, the weight of the plate is inserted in the analysis, considered as a concentrated force or uniformly distributed. In this context selection coefficients for the exterior loadings are introduced due to associated influences.

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Aspecte teoretice privind efectul frecării plăcilor circulare şi inelare pe reazeme simple

Rezumat

Lucrarea indică influența frecării existente între o placă circulară sau inelară simplu rezematǎ, asupra stării de deformare şi de tensiuni. Se introduc coeficienți de selectare pentru departajarea sarcinilor exterioare şi a influențelor lor parțiale.