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Loading State in the Leaning Zone of the Cylindrical - Vertical Boilers on Rigid Foundations. Theoretical Study

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Abstract

The paper sets out two ways of analyzing of the loading states developed in the joint zone of the cylindrical bodies of the vertical boilers with the bottoms leaning on rigid foundations of concrete. The theory with bending moments is taken into account, using the equations of continuity of the deformations - radial displacements and rotations.

Key words: *cylindrical boiler, rigid foundation*

Assuming the theory with moments of the revolution shells, in the separation plan of the cylindrical shell rings of the vertical boilers with flat bottoms leaning on rigid foundations, it is noted that the boiler bottom is required on the periphery of the radial bending moments (of joint) M_0.

Loaded with transverse load $q = p_h + p_m$, the boiler bottom has a curved annular marginal portion (Fig. 1), while a central area of r_0 radius remains flat.

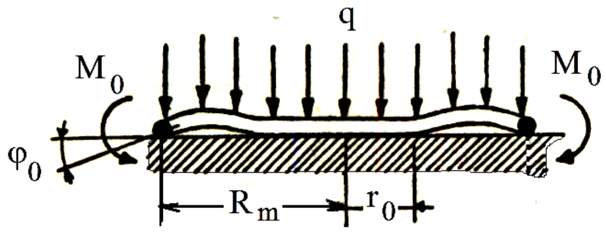

Fig. 1 Annular marginal portion of the boiler bottom

The analysis of the loading state of the boiler bottom is made in two types of study.

Variant I [1 - 3]

For the uniformly loaded annular portion the expression of the vertical displacement can be written in the form of:

$$w = C_1 + C_2 \cdot \ln r + C_3 \cdot r^2 + C_4 \cdot r^2 \cdot \ln r + \frac{q \cdot r^4}{64 \cdot \Re_f}\,, \tag{1}$$

for determining the constants of integration disposing of the limit conditions:

$$\left(w\right)_{r=R_m} = 0\,; \qquad \left(M_r\right)_{r=R_m} = -\,M_0^{\bullet}\,; \tag{2}$$

$$\left(w\right)_{r=r_0} = 0\,; \qquad \left(\frac{d\,w}{d\,r}\right)_{r=r_0} = 0\,; \quad \left(M_r\right)_{r=r_0} = 0\,, \tag{3}$$

where the expression of the total radial bending moment developed at the bottom of the boiler (as opposed to the indication from [1, 3] the bending moment developed of the P_0 cutting load, moved along the medium fiber of the bottom is taken into account, too) is inserted:

$$M_0^{\bullet} = 0{,}5 \cdot \delta_f \cdot P_0 + M_0\,, \tag{4}$$

(the positive direction of the M_0 radial bending moment corresponds to that of figure 1, and of the P_0 radial load, when it is directed outwards from the bottom of the boiler).

It seeks, on the other hand, that the φ_0 angle of rotation is small, so the tangential component to the deformed surface of the bottom of the boiler to the circumference of R_m radius is practically constant with the P_0 radial load ($P_0 \cdot \cos\varphi_0 \approx P_0$).

Five algebraically equations containing the constants of integration from the (1) structures of the vertical displacement, written in the form [1 - 3] are concluded:

$$C_1 + C_2 \cdot \ln R_m + C_3 \cdot R_m^2 + C_4 \cdot R_m^2 \cdot \ln R_m = -\frac{q \cdot R_m^4}{64 \cdot \Re_f}\,; \tag{5}$$

$$C_1 + C_2 \cdot \ln r_0 + C_3 \cdot r_0^2 + C_4 \cdot r_0^2 \cdot \ln r_0 = -\frac{q \cdot r_0^4}{64 \cdot \Re_f}\,; \tag{6}$$

$$C_2 \cdot \frac{\nu_f - 1}{R_m^2} + 2 \cdot C_3 \cdot \left(\nu_f + 1\right) + C_4 \cdot \begin{pmatrix} 3 + 2 \cdot \ln R_m + \\ + 2 \cdot \nu_f \cdot \ln R_m + \nu_f \end{pmatrix} =$$
$$= -\frac{q \cdot R_m^2}{16 \cdot \Re_f} \cdot \left(3 + \nu_f\right) + \frac{M_0^{\bullet}}{\Re_f}\,; \tag{7}$$

$$\begin{aligned} & C_2 \cdot \frac{\nu_f - 1}{r_0^2} + 2 \cdot C_3 \cdot \left(\nu_f + 1\right) + \\ & + C_4 \cdot \begin{pmatrix} 3 + 2 \cdot \ln r_0 + \\ + 2 \cdot \nu_f \cdot \ln r_0 + \nu_f \end{pmatrix} = -\frac{q \cdot r_0^2}{16 \cdot \Re_f} \cdot \left(3 + \nu_f\right)\,; \end{aligned} \tag{8}$$

$$C_2 \cdot \frac{1}{r_0} + 2 \cdot C_3 \cdot r_0 + C_4 \cdot r_0 \cdot \left(2 \cdot \ln r_0 + 1\right) = -\frac{q \cdot r_0^3}{16 \cdot \Re_f}\,. \tag{9}$$

After some necessary changes (the decrease of the (5) and (6) equations and the replacement of the C_2 constant based of C_4, from the equality resulted by the subtracting of the (7) and (8) equations, inclusive in (5), the original unchanged), a new system is inferred, after the form [2]:

$$C_1 + C_3 \cdot R_m^2 - C_4 \cdot \left(A_2 - R_m^2 \right) \cdot \ln R_m = - \frac{q \cdot R_m^4}{64 \cdot \Re_f} + A_3 \cdot \ln R_m + A_4 \cdot M_0^{\bullet} \cdot \ln R_m ; \tag{10}$$

$$C_3 \cdot \left(R_m^2 - r_0^2 \right) - C_4 \cdot \left(A_2 \cdot \ln \frac{R_m}{r_0} - R_m^2 \cdot \ln R_m + r_0^2 \cdot \ln r_0 \right) = - \frac{q}{64 \cdot \Re_f} \cdot \left(R_m^4 - r_0^4 \right) + A_3 \cdot \ln \frac{R_m}{r_0} + A_4 \cdot M_0^{\bullet} \cdot \ln \frac{R_m}{r_0} ; \tag{11}$$

$$C_3 \cdot 2 \cdot \left(1 + \nu_f \right) + C_4 \cdot \left(A_1 + \frac{1 - \nu_f}{R_m^2} \cdot A_2 \right) = - \frac{q \cdot R_m^2}{16 \cdot \Re_f} \cdot \left(3 + \nu_f \right) + \left(\frac{1}{\Re_f} - \frac{1 - \nu_f}{R_m^2} \cdot A_4 \right) \cdot M_0^{\bullet} - \frac{1 - \nu_f}{R_m^2} \cdot A_3 ; \tag{12}$$

$$C_2 + C_4 \cdot A_2 = - A_3 - A_4 \cdot M_0^{\bullet} ; \tag{13}$$

$$C_3 \cdot 2 \cdot r_0 - C_4 \cdot \left[\frac{A_2}{r_0} + r_0 \cdot \left(1 + 2 \cdot \ln r_0 \right) \right] = - \frac{q \cdot r_0^3}{16 \cdot \Re_f} + \frac{A_3}{r_0} + \frac{A_4}{r_0} \cdot M_0^{\bullet} \tag{14}$$

The analytical solving of the system consists of the (10) ... (14) equalities lead to the following results, accepting known the total radial bending moment:

$$C_1 = - \left\{ \frac{R_m^2}{2 \cdot r_0} \cdot \left[\frac{A_2}{r_0} + r_0 \cdot \left(2 \cdot \ln r_0 + 1 \right) \right] - \left(A_2 - R_m^2 \right) \cdot \ln R_m \right\} \cdot C_4 - \frac{q \cdot R_m^4}{64 \cdot \Re_f} + \left(A_3 + A_4 \cdot M_0^{\bullet} \right) \cdot \ln R_m ; \tag{15}$$

$$C_2 = - A_2 \cdot C_4 - A_3 - A_4 \cdot M_0^{\bullet} ; \tag{16}$$

$$C_3 = \frac{1}{2 \cdot r_0} \cdot \left[\frac{A_2}{r_0} + r_0 \cdot \left(2 \cdot \ln r_0 + 1 \right) \right] \cdot C_4 - \frac{q \cdot r_0^2}{32 \cdot \Re_f} + \frac{1}{2 \cdot r_0^2} \cdot \left(A_3 + A_4 \cdot M_0 \right) ; \tag{17}$$

$$C_4 = \frac{A_6 + A_7 \cdot M_0^{\bullet}}{A_5} = \frac{A_9 + A_{10} \cdot M_0^{\bullet}}{A_8} , \tag{18}$$

with notations:

$$A_1 = 3 + \nu_f + 2 \cdot \left(1 + \nu_f\right) \cdot \ln R_m ; \tag{19}$$

$$A_2 = \frac{2 \cdot \left(1 + \nu_f\right)}{1 - \nu_f} \cdot \frac{R_m^2 \cdot r_0^2}{R_m^2 - r_0^2} \cdot \ln \frac{R_m}{r_0} ; \tag{20}$$

$$A_3 = \frac{3 + \nu_f}{16 \cdot \left(1 - \nu_f\right)} \cdot \frac{q}{\Re_f} \cdot R_m^2 \cdot r_0^2 ; \tag{21}$$

$$A_4 = \frac{R_m^2 \cdot r_0^2}{\left(1 - \nu_f\right) \cdot \left(R_m^2 - r_0^2\right) \cdot \Re_f} ; \tag{22}$$

$$A_5 = \frac{R_m^2 - r_0^2}{2 \cdot r_0} \cdot \left[\frac{A_2}{r_0} + r_0 \cdot \left(2 \cdot \ln r_0 + 1\right) \right] - \\ - A_2 \cdot \ln \frac{R_m}{r_0} + R_m^2 \cdot \ln R_m - r_0^2 \cdot \ln r_0 ; \tag{23}$$

$$A_6 = - \frac{q}{64 \cdot \Re_f} \cdot \left(R_m^2 - r_0^2\right) - A_3 \cdot \left(\frac{R_m^2 - r_0^2}{2 \cdot r_0^2} - \ln \frac{R_m}{r_0} \right) ; \tag{24}$$

$$A_7 = - \left(\frac{R_m^2 - r_0^2}{2 \cdot r_0^2} - \ln \frac{R_m}{r_0} \right) \cdot \frac{R_m^2 \cdot r_0^2}{\left(1 - \nu_f\right) \cdot \left(R_m^2 - r_0^2\right) \cdot \Re_f} ; \tag{25}$$

$$A_8 = \left(1 + \nu_f\right) \cdot \left(1 + 2 \cdot \ln r_0\right) + A_1 + \left(\frac{1 + \nu_f}{r_0^2} + \frac{1 - \nu_f}{R_m^2} \right) \cdot A_2 ; \tag{26}$$

$$A_9 = - \frac{q}{16 \cdot \Re_f} \cdot \left[\left(3 + \nu_f\right) \cdot R_m^2 - \left(1 + \nu_f\right) \cdot r_0^2 \right] - \left(\frac{1 - \nu_f}{R_m^2} + \frac{1 + \nu_f}{r_0^2} \right) \cdot A_3 ; \tag{27}$$

$$A_4 = \frac{1}{\Re_f} - \left(\frac{1 - \nu_f}{R_m^2} + \frac{1 + \nu_f}{r_0^2} \right) \cdot A_4 . \tag{28}$$

The equalization of the reports from (18) leads to:

$$M_0^\bullet = \frac{A_5 \cdot A_9 - A_6 \cdot A_8}{A_7 \cdot A_8 - A_5 \cdot A_{10}} , \tag{29}$$

expression that permit the determination ok the r_0 radius position, knowing the value of the $M_0^\bullet$ radial bending moment.

Variant II

Note: In this study variant the P_0 and M_0, boundary loads, are unknown.

This time, a boiler whose weight of the metal part and possibly of the thermal insulation and / or of the deposited snow on its cover is G_r, a p_0 under pressure of a gas above the stored liquid,

having the ρ_f density and the H_f height, leaning on a rigid foundation of concrete is considered.
In equations (10) ... (14) is added:

$$w_{rb} = 0 \; ; \quad \left(\frac{dw}{dr} \right)_{r = R_m} = \vartheta_m \; , \tag{30}$$

where the w_{rb} total radial displacement at the boiler base is expressed as:

$$w_{rb} = -\frac{1}{4 \cdot k_m^4 \cdot \Re_m} \cdot \left[\rho_f \cdot g \cdot H_f + \frac{\nu_m \cdot G_r}{2 \cdot \pi \cdot R_m^2} + \frac{1}{2} \cdot \left(2 - \nu_m \right) \cdot p_0 \right] +$$

$$+ \frac{2 \cdot k_m \cdot R_m^2}{\delta_m \cdot E_m} \cdot P_0 - \frac{2 \cdot k_m^2 \cdot R_m^2}{\delta_m \cdot E_m} \cdot M_0 \; , \tag{31}$$

respectively the ϑ_m rotation of the median surface at the boiler base:

$$\vartheta_m = \frac{\rho_f \cdot g}{4 \cdot k_m^4 \cdot \Re_m} - \frac{2 \cdot k_m^2 \cdot R_m^2}{\delta_m \cdot E_m} \cdot P_0 + \frac{4 \cdot k_m^3 \cdot R_m^2}{\delta_m \cdot E_m} \cdot M_0 \; . \tag{32}$$

Were also used, the action meanings of the P_0 cutting load and of the M_0 bending moment, both on the boiler bottom and the lower shell rings of its.
Differentiating the (1) expression and customizing the result for the circumference of R_m radius is reached:

$$\left(\frac{dw}{dr} \right)_{r = R_m} = \frac{1}{R_m} \cdot C_2 + 2 \cdot R_m \cdot C_3 + R_m \cdot \left(1 + 2 \cdot \ln R_m \right) \cdot C_4 + \frac{q \cdot R_m^3}{16 \cdot \Re_f} \; . \tag{33}$$

In the effectuated calculations the $q = p_h + p_0$ load will take into account.
The G_r weight was considered uniform distributed on the boundary of R_m radius, helping to keep the boiler bottom in contact with the foundation and making possible the hypothesis of its undeformation in radial direction (see equation $(30)_1$).
And this time, too, the (4) expression of the total radial bending moment will be used, which action on the contact boundary with lower shell rings of the boiler. From the $(30)_1$ condition the expression of the cutting load is established:

$$P_0 = \frac{B_2}{B_1} \cdot M_0 + \frac{B_3}{B_1} \; , \tag{34}$$

and from the (4) equality:

$$M_0 = B_4 \cdot M_0^{\bullet} + B_5 \; . \tag{35}$$

After replacing the (34) and (35) expressions in the $(30)_2$ equation the expression of the C_4 constant is determined which allows for the deduction of the relations for C_2 from (13), respectively for C_3 from (14).
Finally, the (10) equation helps to establish the expression for C_1. As such, this time:

$$C_1 = B_{15} - B_{16}\cdot M_0^{\bullet}\,; \qquad C_2 = B_{11} - B_{12}\cdot M_0^{\bullet}\,; \tag{36}$$

$$C_3 = B_{13} + B_{14}\cdot M_0^{\bullet}\,; \qquad C_4 = \frac{B_9}{B_8} + \frac{B_{10}}{B_8}\cdot M_0^{\bullet}\,, \tag{37}$$

where the notations are used:

$$B_1 = \frac{2\cdot k_m\cdot R_m^2}{\delta_m\cdot E_m}\,; \qquad B_2 = \frac{2\cdot k_m^2\cdot R_m^2}{\delta_m\cdot E_m}\,; \tag{38}$$

$$B_3 = \frac{1}{4\cdot k_m^4\cdot \Re_m}\cdot\left[\rho_f\cdot g\cdot H_f + \frac{\nu_m\cdot G_r}{2\cdot\pi\cdot R_m^2} + \frac{1}{2}\cdot\left(2-\nu_m\right)\cdot p_0\right]; \tag{39}$$

$$B_4 = 0{,}5\cdot\delta_f\cdot\left(B_2/B_1\right)+1\,; \quad B_5 = 0{,}5\cdot\delta_f\cdot\left(B_3/B_1\right); \tag{40}$$

$$B_6 = -\frac{q}{4}\cdot\left(\frac{g}{k_m^4\cdot\Re_m} + \frac{R_m^3}{4\cdot\Re_f}\right) + B_2\cdot B_5\cdot\left(2\cdot k_m - \frac{B_2}{B_1}\right) - \frac{B_2\cdot B_3}{B_1}\,; \tag{41}$$

$$B_7 = B_2\cdot B_4\cdot\left(2\cdot k_m - B_2/B_1\right); \tag{42}$$

$$B_8 = -\frac{A_2}{R_m} + \frac{R_m}{r_0}\cdot\left[\frac{A_2}{r_0} + r_0\cdot\left(1+2\cdot\ln r_0\right)\right] + R_m\cdot\left(1+2\cdot\ln R_m\right); \tag{43}$$

$$B_9 = A_3\cdot\left(\frac{1}{R_m} - \frac{R_m}{r_0^2}\right) + \frac{q\cdot R_m\cdot r_0^2}{16\cdot\Re_f} + B_6\,; \tag{44}$$

$$B_{10} = A_4\cdot\left(\frac{1}{R_m} - \frac{R_m}{r_0^2}\right) - B_7\,; \tag{45}$$

$$B_{11} = -A_2\cdot\frac{B_9}{B_8} - A_3\,; \qquad B_{12} = A_2\cdot\frac{B_{10}}{B_8} + A_4\,; \tag{46}$$

$$B_{13} = \frac{1}{2\cdot r_0}\cdot\frac{B_9}{B_8}\cdot\left[\frac{A_2}{r_0} + r_0\cdot\left(1+2\cdot\ln r_0\right)\right] - \frac{q\cdot r_0^2}{32\cdot\Re_f} + \frac{A_3}{2\cdot r_0^2}\,; \tag{47}$$

$$B_{14} = \frac{1}{2\cdot r_0}\cdot\frac{B_{10}}{B_8}\cdot\left[\frac{A_2}{r_0} + r_0\cdot\left(1+2\cdot\ln r_0\right)\right] + \frac{A_4}{2\cdot r_0^2}\,; \tag{48}$$

$$B_{15} = -B_{13}\cdot R_m^2 + \frac{B_9}{B_8}\cdot\left(A_2 - R_m^2\right)\cdot\ln R_m - \frac{q\cdot R_m^4}{64\cdot\Re_f} + A_3\cdot\ln R_m\,; \tag{49}$$

$$B_{16} = -B_{14}\cdot R_m^2 + \left[\frac{B_{10}}{B_8}\cdot\left(A_2 - R_m^2\right) + A_4\right]\cdot\ln R_m\,. \tag{50}$$

Substituting the (37) expressions of the C_3 and C_4 constants in the (12) and (13) equalities, the relationship of the $M_0^{\bullet}$ total radial bending moment is deduced as:

$$M_0^{\bullet} = \frac{B_{18}}{B_{17}} = \frac{B_{20}}{B_{19}}\,, \tag{51}$$

which the equality is established, too:

$$B_{18} \cdot B_{19} = B_{17} \cdot B_{20}\,, \tag{52}$$

which allows the r_0 radius evaluation of the circumference that separates the two areas of the boiler bottom, the outer curved and the inner planar shape, in contact with the rigid foundation. In the (51) and (52) equalities the notations have used:

$$B_{17} = B_{14} \cdot \left(R_m^2 - r_0^2 \right) - \frac{B_{10}}{B_8} \cdot \left(\begin{array}{c} A_2 \cdot \ln \dfrac{R_m}{r_0} - \\ - R_m^2 \cdot \ln R_m + r_0^2 \cdot \ln r_0 \end{array} \right) - A_4 \cdot \ln \frac{R_m}{r_0}\,; \tag{53}$$

$$\begin{aligned} B_{18} = & - \frac{q}{64 \cdot \Re_f} \cdot \left(R_m^4 - r_0^4 \right) + A_3 \cdot \ln \frac{R_m}{r_0} - B_{13} \cdot \left(R_m^2 - r_0^2 \right) + \\ & + \frac{B_9}{B_8} \cdot \left(A_2 \cdot \ln \frac{R_m}{r_0} - R_m^2 \cdot \ln R_m + r_0^2 \cdot \ln r_0 \right); \end{aligned} \tag{54}$$

$$B_{19} = 2 \cdot B_{14} \cdot \left(1 + \nu_f \right) + \frac{B_{10}}{B_8} \cdot \left(A_1 + \frac{1 - \nu_f}{R_m^2} \cdot A_2 \right) - \frac{1}{\Re_f} + \frac{1 - \nu_f}{R_m^2} \cdot A_4\,; \tag{55}$$

$$\begin{aligned} B_{20} = & - \frac{q \cdot R_m^2}{16 \cdot \Re_f} \cdot \left(3 + \nu_f \right) - \frac{1 - \nu_f}{R_m^2} \cdot A_3 - \\ & - 2 \cdot B_{13} \cdot \left(1 + \nu_f \right) - \frac{B_9}{B_8} \cdot \left(A_1 + \frac{1 - \nu_f}{R_m^2} \cdot A_2 \right). \end{aligned} \tag{56}$$

To estimate the loading state developed in the cylindrical shell rings and in the bottom of the boiler has to be adopted the following algorithm:

- From the (52) equality the r_0 radius position is determined;
- The (51) expression allows the calculation of the $M_0^{\bullet}$ total radial bending moment, and further, the M_0 contour radial bending moment with the (35) equality and P_0 cutting load with the (34) relation;
- The induced stress from the lower shell rings is assessed with the known relations [2], which based the equivalent stresses are determined;
- Using the (36), (37) constants of integration, the boiler bottom vertical displacement expression, the median fiber rotation of it, respectively the annular and radial stresses with known relations, respectively of the equivalent stresses are established.

Notations

q – normal load at the median surface of the bottom boiler; p_h – developed hydrostatic pressure of the stored fluid; p_m – equivalent pressure given by the metallic mass of the vertical cylindrical boiler; M_0 – unit radial bending moment of contour; R_m – radius of the median circumference of the cylindrical body of the boiler; r_0 – radius of the plan circumference of the bottom of the boiler in service; w – vertical displacement of the bottom of the boiler under the action of the external loads; r – current radius of the boiler bottom; $\Re_m$, $\Re_f$ – cylindrical rigidity of the boiler body, respectively of the boiler bottom; M_r – unit radial bending moment; P_0 – liaison cutting load, developed in the plan for separating of the cylindrical shell rings of the bottom of the boiler; φ_0 – rotation angle of the bottom of the boiler; w_{rb} – total radial displacement at the base of the boiler; ϑ_m – rotation of the cylindrical median surface at the boiler bottom; δ_m – thickness of the cylinder wall; ρ_f – density of the stored fluid; ν_m – transverse contraction coefficient of the material of the boiler body; H_f – height of the column of the stored fluid; E_m – elasticity longitudinal modulus of the material of the boiler; k_m – attenuation factor for the base cylindrical shell rings of the boiler [2]; G_r – the constructive weight of the boiler; $C_1, \cdots, C_4$ – integration constants.

Conclusions

Resorting to the theory with bending moments, based on the continuity equations of the deformation - radial displacements and rotations - by two types of variants analysis, the liaison loads are determined. Based on these, the radial and annular stresses, respectively the annular and equivalent stresses are determined.

In this way, the bearing capacity of the structure is evaluated.

Subsequent examples will illustrate both the deformation of the boiler bottom and the lower mantle, and developed induced stresses.

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Starea de solicitare în zona de rezemare a rezervoarelor cilindrice pe fundații rigide. Studiu teoretic

Rezumat

Lucrarea expune două moduri de analiză a stărilor de solicitare dezvoltate în zona de îmbinare a corpurilor cilindrice ale rezervoarelor verticale cu fundurile rezemate pe fundații rigide din beton. Se are în vedere teoria cu momente încovoietoare, utilizând ecuațiile de continuitate a deformațiilor – deplasări radiale şi rotiri.