BULETINUL Universităţii Petrol – Gaze din Ploieşti	Vol. LXIV No. 3/2012	11 - 16	Seria Tehnică

An Approximate Method Used in the Calculation of the Critical Load for Beams with Variable Cross Sectional Area

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Abstract

In the paper is presented an approximate method that can be used in order to calculate the critical load for beams with different cross sectional area, loaded with different axial forces. The results obtained are compared with those obtained by using the second order theory of the buckling of beams.

Key words: *deflection, slope, elastic curve, buckling*

The Classical Method

A cantilever with two geometrical parts characterized by two different cross sectional areas is considered (fig. 1). The first part of the beam has the length l_1, the diameter of the cross circular area 1.5d and the moment of inertia in relation with z axis I_1. The second part has the length l_2, the diameter of the cross circular area d and the moment of inertia in relation with z axis $I_2 = I_o$.

Using the classical relations from the second order theory of the buckling of the beams [1] the following transcendent equation is obtained:

$$tg(k_1 l_1) \cdot tg(k_2 l_2) = \frac{P_1}{k_1 k_2 E I_2} = \sqrt{\frac{P_1}{P_2}} \cdot \sqrt{\frac{I_1}{I_2}}\ , \tag{1}$$

where P_i represents the axial force on the i part of the beam, I_i the moment of inertia (in relation with z axis) of the cross sectional area of the same part of the beam and k_i the coeficient that characterizes the buckling of the beam [1] :

$$k_i = \sqrt{\frac{P_i}{E I_i}} \tag{2}$$

The above method has been applied in order to calculate the critical load as the solution of the (1) equation, for different lengths of the two parts of the beam. The results obtained are presented in table 1.

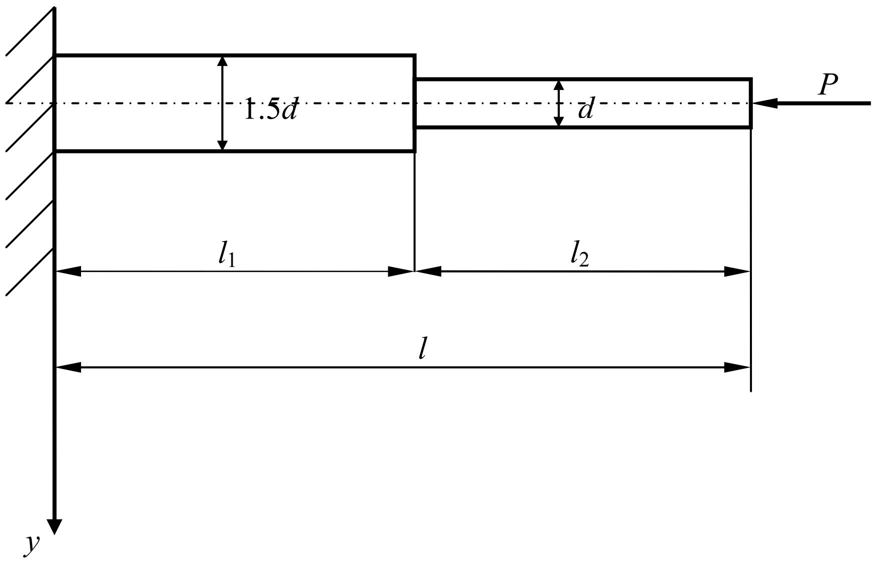
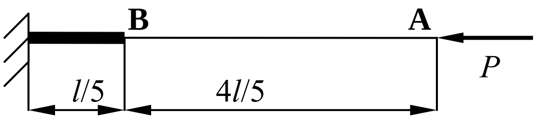
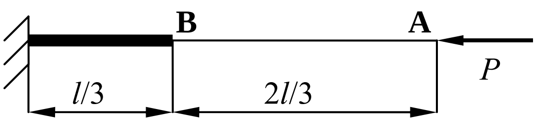
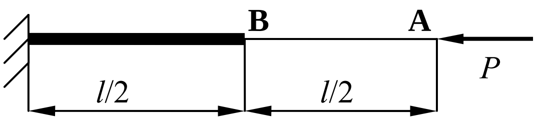
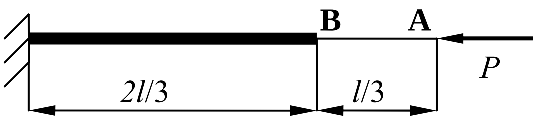


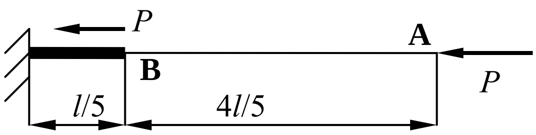
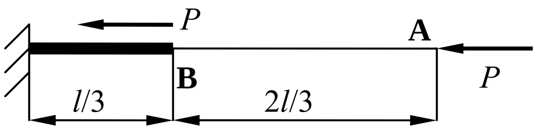
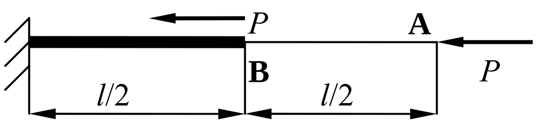
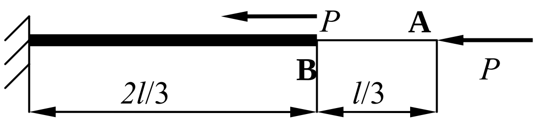
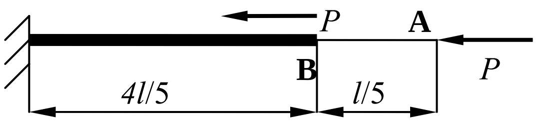
Fig.1. A cantilever axially compressed with P force

Table 1. Critical loads for different types of geometries

Crt. No.	Geometrical scheme and loads	Critical load
1		$P_{cr} = 1.42\dfrac{\pi^2 EI_o}{(2l)^2}$
2		$P_{cr} = 1.858\dfrac{\pi^2 EI_o}{(2l)^2}$
3		$P_{cr} = 2.704\dfrac{\pi^2 EI_o}{(2l)^2}$
4		$P_{cr} = 3.95\dfrac{\pi^2 EI_o}{(2l)^2}$

The same classical method has been applied in order to calculate the critical loads for the same beams with those presented in table 1 when a axial force *P* acts in the point that delimitates the two parts of the beam (see table 2). For this situation the axial loads are different for each part : for the first part the axial force is $P_1 = 2P$ and for the second part $P_2 = P$.

Table 2. Critical loads for different types of geometries

Crt. No.	Geometrical scheme and loads	Critical load
1		$P_{cr} = 1.4186 \frac{\pi^2 EI_o}{(2l)^2}$
2		$P_{cr} = 1.84 \frac{\pi^2 EI_o}{(2l)^2}$
3		$P_{cr} = 2.59 \frac{\pi^2 EI_o}{(2l)^2}$
4		$P_{cr} = 3.18 \frac{\pi^2 EI_o}{(2l)^2}$
5		$P_{cr} = 3.068 \frac{\pi^2 EI_o}{(2l)^2}$

Analysing the results from the tables 1 and 2 it can be noticed that the critical loads from table 2 are different from those presented in table 1 especially for the cases when the second force P acts far away from the embedded point (for $l_1 \geq l/2$).

All the cases presented in tables 1 and 2 required a numerical solving of the (1) equation in order to find the first positive solution and the critical load *P*.

An Approximate Method

Because solving the transcendent equation (1) takes some time, an approximate method is suggested. The method can be named the reducing forces method and consists of the similarity between the critical load that acts in a current point of a beam and the critical load that acts at the end of the beam (fig. 2).

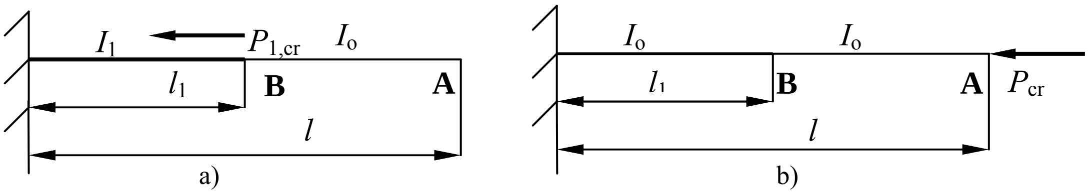


Fig. 2. The equivalence between critical loads

If the critical load is calculated with Euler's formula for the case presented in Figure 2a is obtained:

$$P_{1,cr} = \frac{\pi^2 EI_1}{(2l_1)^2} \tag{3}$$

Applying the same Euler's formula for the case presented in Figure 2b, the critical load can be expressed as:

$$P_{cr} = \alpha \frac{\pi^2 EI_o}{(2l)^2} \quad , \tag{4}$$

where α is the coeficient presented in table 1 and depends on the geometry of the beam.

Analysing the expressions (3) and (4) it can be noticed that:

$$P_{cr} = \alpha \frac{\pi^2 EI_1}{(2l_1)^2} \cdot \frac{I_o}{I_1} \cdot \left(\frac{l_1}{l}\right)^2 = P_{1,cr} \cdot \alpha \frac{I_o}{I}\left(\frac{l_1}{l}\right)^2 \tag{5}$$

The expression (5) can be considered as a possibility of reducing the effect of the force P_1 of the point B in the point A, regarding only the buckling effect. (fig. 2). If a beam is subjected to many axial loads every one can be reduced at the end of a beam, where the critical load can be more easily calculated.

Using the above approximate method the critical load has been evaluated for the cases presented in table 2. For example, for the case 1 from table 2, the above method has the following steps:

- the calculation of the reducing force from B to A, using the relation (5):

$$P_{AB} = P \cdot \alpha_1 \cdot \frac{I_o}{I_1} \cdot \left(\frac{l_1}{l}\right)^2 = P \cdot (1.42) \cdot \frac{I_o}{5.0625 I_o}\left(\frac{1}{5}\right)^2 = 0.0112 \cdot P \tag{6}$$

- the calculation of the total axial force that acts in point A :

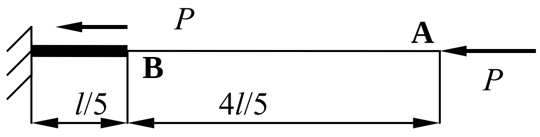
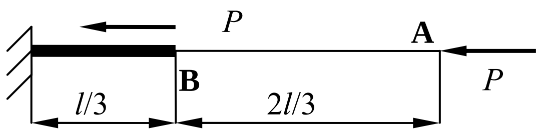
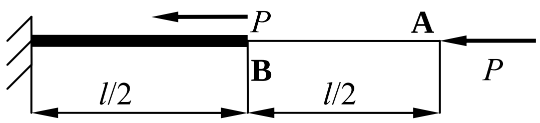
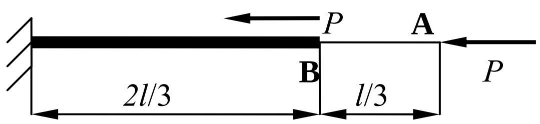
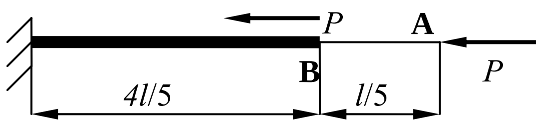
$$P_A = P_{AB} + P_{AA} = 0.0112 \cdot P + P = 1.0112 \cdot P \tag{6}$$

- the calculation of the approximate critical load using the results presented in table 1:

$$P_A = P_{cr} \Rightarrow 1.0112 \cdot P = 1.42 \frac{\pi^2 EI_o}{(2l)^2} \Rightarrow P_{cr} = 1.404 \frac{\pi^2 EI_o}{(2l)^2} \tag{8}$$

If the result (8) is compared with those presented in table 2 (case 1), it can be noticed that an error of 1% occurs between the exact and approximate method. Some similar calculations have been made for all the cases presented in table 2. The results obtained and the errors that appear are presented in table 3.

Table 3. Critical loads obtained using the approximate method

Crt. No.	Geometrical scheme and loads	Critical load	Errors
1		$P_{cr} = 1.404\dfrac{\pi^2 EI_o}{(2l)^2}$	1%
2		$P_{cr} = 1.785\dfrac{\pi^2 EI_o}{(2l)^2}$	3%
3		$P_{cr} = 2.385\dfrac{\pi^2 EI_o}{(2l)^2}$	7.9%
4		$P_{cr} = 2.933\dfrac{\pi^2 EI_o}{(2l)^2}$	7.77%
5		$P_{cr} = 2.985\dfrac{\pi^2 EI_o}{(2l)^2}$	2.77%

Analysing the values obtained in table 3 it can be noticed that the errors do not exceed 8%. This result validate the approximate method that is more easily to be applied for beams with similar geometry with those presented in the above table. For beams that contains more than two geometrical steps the approximate method can be easy extended.

Conclusions

In the paper is presented an approximate method that allows the calculation of the critical load for beams with different cross sectional area and axial forces. The main principle is similar to all the principles that reduce the effect of a force that acts in a point into another point of a structure, the only difference consisting in applying of this principle only to the effect of the buckling of a beam axially compressed.

The results obtained are compared with those obtained by using the second order theory of buckling of beams. The errors obtained (less than 8%) validate the approximate method and recommend it in order to be used for beams with similar geometry with those presented in this paper.

References

1. Posea N. – *Rezistenţa materialelor* , Editura Didactică şi Pedagogică, Bucureşti, 1979.
2. Vasilescu Ş., Ilinca C. – A method of the origin parameters used in the calculation of the critical load for beams with variable cross sectional area. *Buletinul U.P.G. Ploieşti, Seria Tehnică*, Vol. LXIV, Nr. 1/2012, pp. 41-44.
3. Vasilescu Ş., Talle V. – *Rezistenţa materialelor - solicitări fundamentale*, Editura U.P.G., Ploieşti, 2007.

O metodă aproximativă utilizată în determinarea sarcinii critice pentru bare cu secţiunea variabilă

Rezumat

În lucrare se prezintã o metodã aproximativa ce poate fi utilizata in determinarea sarcinii critice pentru bare cu sectiunea variabila. Rezultatele obţinute sunt comparate cu cele furnizate de teoria de ordinul II a flambajului barelor drepte.