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On the Dynamic Analysis of a Family of Quadrilateral Mechanisms

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Abstract

In the paper some results concerning the dynamic analysis of a family of quadrilateral mechanisms are presented. The mechanisms in the family differ by the length of the fixed link. Elements concerning the positional and cinematic analysis are also highlighted. Finally, some interesting simulation results regarding the variation of the equilibrium moment that can then be used for an optimum design of the analyzed mechanisms are presented.

Key words: *quadrilateral mechanism, dynamics, equilibrium moment*

Introduction

An optimum design of the mechanisms requires a rigorous analysis of their running behavior and the setting of the variations curves of different positional, cinematic or dynamical parameters. In this paper some quadrilateral mechanisms that differ by the length of the fixed link are dynamically analyzed. Elements concerning the positional and cinematic analysis are also highlighted. Some interesting 3D results regarding the variation of the equilibrium moment that can then be used for an optimum design of the analyzed mechanisms are presented.

Theoretical Considerations and Verification Results

In figure 1 the cinematic scheme of a quadrilateral mechanism is presented. The following elements are considered to be known:

- the dimensions of the component links: *OA*=0.2 m; *AB*=0.6 m; *BC*=0.45 m. The length of the fixed link *OC* can varies between 0.35 m and 0.6 m. The mass centers: C_1, C_2, C_3 are on the middle of the corresponding links.
- the mass of the component links: $m_1 = 1.5\text{kg}$; $m_2 = 6\text{kg}$; $m_3 = 4.5\text{kg}$.
- the moments of inertia of the links: $I_{C_1} = 0.025\ \text{kgm}^2$; $I_{C_2} = 0.3\ \text{kgm}^2$; $I_{C_3} = 0.17\ \text{kgm}^2$.
- the technological moment: $M_{ru} = 120\ \text{Nm}$.
- the nominal angular speed of the leader link of the mechanism: $\omega_1 = 10\,\text{rad/s}$.

The method of the projection of the independent vector circuits [1] has been used for the positional analysis. The independent contour: $O-A-B-C-O$ has been projected on the x and y axes (fig. 1) and the following system of equations was obtained:

$$\begin{cases} l_1 \cdot \cos\varphi_1 + l_2 \cdot \cos\varphi_2 + l_3 \cdot \cos\varphi_3 - l_0 = 0 \\ l_1 \cdot \sin\varphi_1 + l_2 \cdot \sin\varphi_2 + l_3 \cdot \sin\varphi_3 = 0 \end{cases} \tag{1}$$

where: $l_1 = OA$; $l_2 = AB$; $l_3 = BC$; $l_0 = OC$.

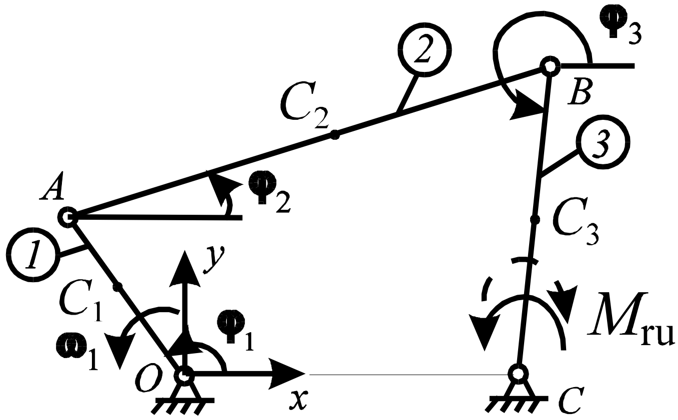

Fig. 1. Quadrilateral mechanism

By solving the system of equations (1), the unknown parameters: φ_2, φ_3 can be calculated from the following relations:

$$\begin{cases} A_2 \cdot \sin\varphi_2 + B_2 \cdot \cos\varphi_2 = C_2 \\ A_3 \cdot \sin\varphi_3 + B_3 \cdot \cos\varphi_3 = C_3 \end{cases} \tag{2}$$

where:

$$\begin{cases} A_2 = 2 \cdot l_1 \cdot l_2 \cdot \sin\varphi_1 \\ B_2 = 2 \cdot l_1 \cdot l_2 \cdot \cos\varphi_1 - 2 \cdot l_0 \cdot l_2 \\ C_2 = l_3^2 - l_1^2 - l_2^2 - l_0^2 + 2 \cdot l_0 \cdot l_1 \cdot \cos\varphi_1 \end{cases} \tag{3}$$

$$\begin{cases} A_3 = 2 \cdot l_1 \cdot l_3 \cdot \sin\varphi_1 \\ B_3 = 2 \cdot l_1 \cdot l_3 \cdot \cos\varphi_1 - 2 \cdot l_0 \cdot l_3 \\ C_3 = l_2^2 - l_1^2 - l_3^2 - l_0^2 + 2 \cdot l_0 \cdot l_1 \cdot \cos\varphi_1 \end{cases} \tag{4}$$

The angular and linear speeds and accelerations distributions have been determined by deriving with time the variation functions of the corresponding position parameters. Then, the variation of the equilibrium moment M_e [2] has been obtained using the relation:

$$\overline{M}_e \cdot \overline{\omega}_1 + \sum_{j=1}^{3} \overline{G}_j \cdot \overline{v}_j + \sum_{j=1}^{3} \left(\overline{F}_{ij} \cdot \overline{v}_j + \overline{M}_{ij} \cdot \overline{\omega}_j \right) + \overline{M}_{ru} \cdot \overline{\omega}_3 = 0 \tag{5}$$

where: $\overline{M}_e$ is the equilibrium moment, $\overline{G}_j, j = \overline{1,3}$, are the weight forces corresponding to the component links, $\overline{F}_{ij}$ and $\overline{M}_{ij}$ are the resultant inertia force and the resultant inertia moment corresponding to the j link, $\bar{v}_j$ is the speed of the mass centre of the j link, $\overline{\omega}_j$ is the angular speed of the j link.

Having the variation of the equilibrium moment on a cinematic cycle, the motor moment M_m which is necessary for the running in good conditions of the mechanism can be calculated with the following relation [2]:

$$M_m = \frac{\int_{\Phi c} M_e(\varphi_1)\,\mathrm{d}\varphi_1}{\Phi_c} \tag{6}$$

where: Φ_c is the angular value corresponding to the cinematic cycle ($\Phi_c = 2\pi$).

The analysis has been transposed into a computer program using the Maple software.

In figure 2 the variation of the equilibrium moment M_e, when $l_0 = OC$ varies between 0.4 m and 0.56 m is represented. By applying the relation (6), the values for the motor moment when $l_0 = OC$ varies between 0.4m and 0.56m have been obtained. Some values are given in the following table.

Table 1. The values of the motor moment

l_0[m]	0.4	0.44	0.48	0.52	0.56
M_m[Nm]	56.31	49.64	45.11	41.85	39.40

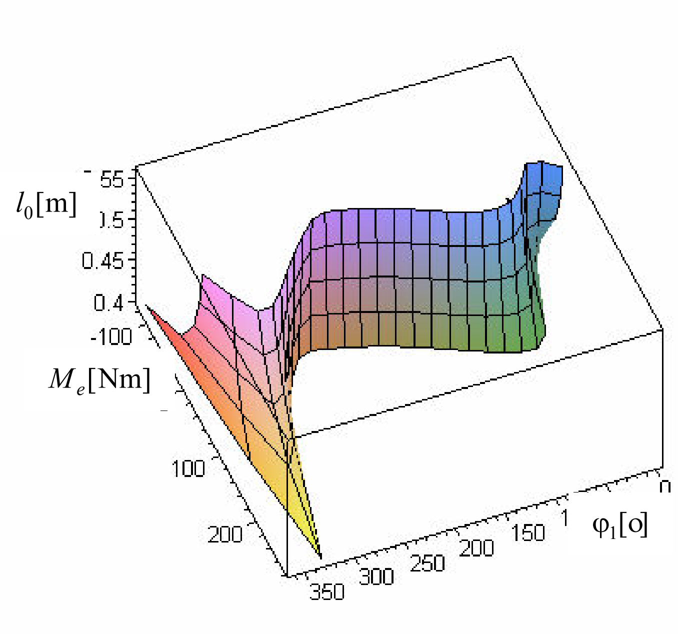

Fig. 2. The variation of the equilibrium moment

Conclusions

In this paper some results concerning the dynamics of a family of quadrilateral mechanisms are presented. A simulation program has been developed. That computer program allowed the obtaining of the variation for many cinematic and dynamic parameters. In the paper the variation of the equilibrium moment is highlighted. There are also given the values of the motor moments for some quadrilateral mechanisms of the family. These results can then be used for an optimum design of the analyzed mechanisms.

References

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Asupra analizei dinamice a unei familii de mecanisme patrulater

Rezumat

In articol sunt prezentate o serie de rezultate privind analiza dinamică a unei familii de mecanisme patrulater. Mecanismele din familie diferă prin lungimea elementului fix. Elemente privind analiza poziţională şi cinematică sunt de asemenea evidenţiate. In final, sunt prezentate o serie de rezultate interesante ale simulărilor care pot fi folosite pentru o proiectare optimă a mecanismelor analizate.