

# Comparative Analysis of the Assembly by Tightening the Cone and the Assembly with Tapered Rings

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## Abstract

*This work carried out an comparative analysis of the assembly with tightening on the cone and the assembly with tapered rings. Fundamental criterion for comparison is the torque capable of assembling analyzed. It is found that tapered ring assembly are better than assembly with tightening on the cone, because for the transmission of exterior torsion moment is necessary a smaller moment of friction forces.*

**Key words:** *assembly, yield squeeze, tapered rings, torque bearing capacity.*

## Introduction

Assembly by elastic tightening is made by forced contact between the conjugated surfaces of assembled parts.

From the point of view of the mounting and contact surface we distinguish: [1, 2, 3, 6]:

- assembly by elastic tightening without intermediate bodies tightening with conical contact surface:
- assembly by elastic tightening with intermediate body tightening.

This paper makes a comparative analysis of torque capable in the case of the assembly with tightening on the cone and assembly with tapered rings.

## Assembly with Tightening on the Cone

At this type of assembly, normal force necessary to ensure slip-free torque transmission is accomplished by exerting axial force developed using threaded parts (fig.1).

In Figure 1, the normal force  $F_n$  is considered concentrated in the middle of contact surface.

From the projection equation on the assembly axis we obtain [5]:

$$F_a = F_n \cdot (\sin\alpha + \mu \cos\alpha) , \quad (1)$$

where  $\mu$  is the coefficient of friction

$$\mu = \operatorname{tg}\varphi , \quad (2)$$

$\varphi$  – angle of friction.

In these conditions,  $F_a$  has the form:

$$F_a = F_n \cdot \frac{\sin(\alpha + \varphi)}{\cos \varphi} \quad , \quad (1')$$

from which we obtain:

$$F_n = F_a \cdot \frac{\cos \varphi}{\sin(\alpha + \varphi)} \quad , \quad (1'')$$

Load transmitting through friction, the moment of friction forces is given by (where  $\mu$  represents the coefficient of friction,  $\mu = 0.10 \dots 0.20$ ):

$$M_f = \mu F_n \cdot \frac{d_m}{2} = \mu F_a \cdot \frac{\cos \varphi}{\sin(\alpha + \varphi)} \cdot \frac{d_m}{2} \quad , \quad (3)$$

Functional condition of a transmitting through friction a exterior torsion moment is:

$$M_f = \mu F_a \cdot \frac{\cos \varphi}{\sin(\alpha + \varphi)} \cdot \frac{d_m}{2} \geq M_{tc} = c_s \cdot M_t \quad , \quad (4)$$

where  $c_s$  is the safety coefficient.

## Assembly with Tapered Rings

At these assemblies, each pair of rings consists of an interior ring and an exterior ring, surfaces which are in contact are tapered (fig. 2) and surfaces in contact with parts assembled (shaft, hub) are cylindrical (fig. 3).

In the case of assembling with tapered rings, rings are tight using a fixing device so that through slip axial and their elastic deformation resulting tightening pressure on contact surfaces.

To establish transmission assembly capacity is calculated radial tightening forces  $Q_1$ , depending on value of axial force compression  $F_a$ .

According to Figure 2, the relationship between  $Q_1$  and  $F_{a1}$  is established by writing successively the equilibrium condition after direction of axial forces for each interior and exterior ring based on specified forces system in Figure 4. We obtain [4]:

$$F_{a1} = \mu \cdot Q_1 + F_1 = \mu \cdot Q_1 + Q_1 \cdot \operatorname{tg}(\alpha + \varphi) = Q_1 \cdot [\operatorname{tg} \varphi + \operatorname{tg}(\alpha + \varphi)] \quad , \quad (5)$$

from which it results:

$$Q_1 = \frac{F_{a1}}{\operatorname{tg} \varphi + \operatorname{tg}(\alpha + \varphi)} = \frac{1}{\frac{\sin(\alpha + 2\varphi)}{\cos \varphi \cdot \cos(\alpha + \varphi)}} \cdot F_{a1} \quad , \quad (5')$$

$$Q_1 = \frac{\cos \varphi \cdot \cos(\alpha + \varphi)}{\sin(\alpha + 2\varphi)} \cdot F_{a1} \quad , \quad (5'')$$

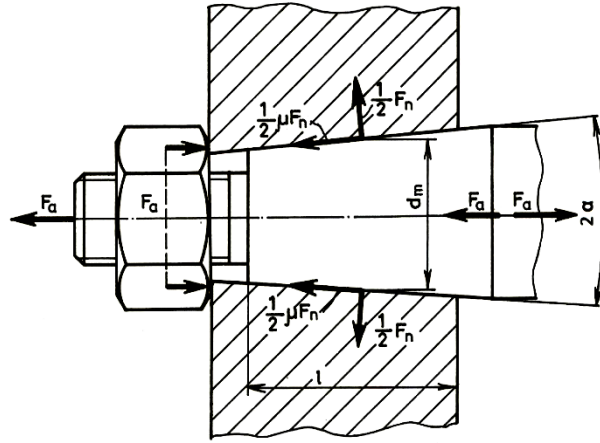


Fig. 1. Assembly with tightening on the cone

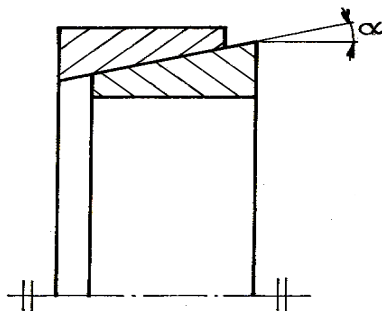


Fig. 2. Tapered rings.

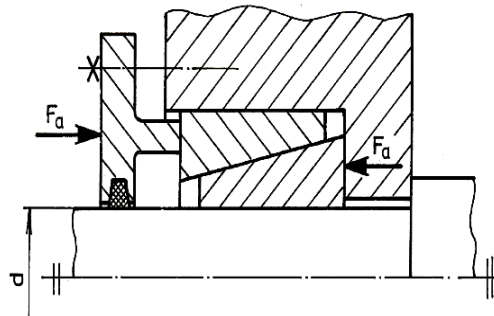


Fig. 3. Assembly with a pair of conical rings with axial support off the hub

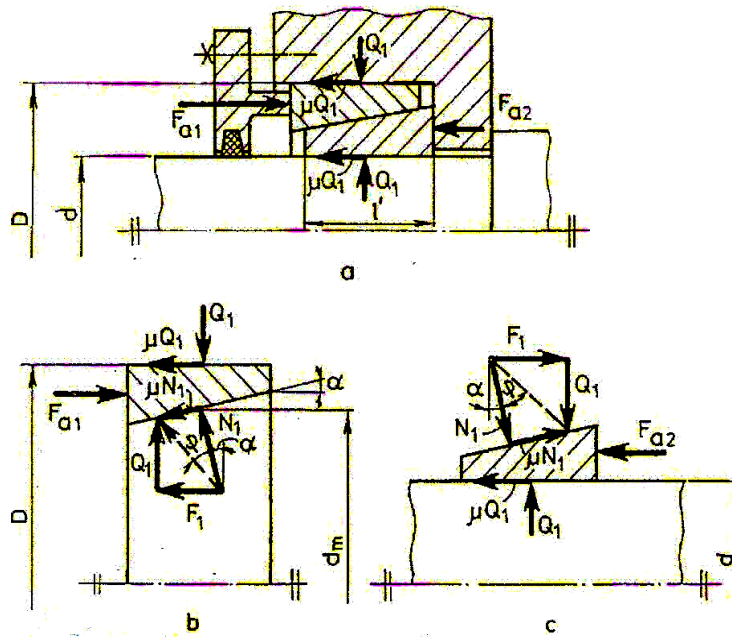


Fig. 4. Some geometric elements of assembly with on pair of tapered ring  
a; b- forces which act on the external ring; c- force forces which act on internal ring

The torque can be transmitted through the tapered rings is minimal on the periphery of shaft where slip can occur.

$$M_{f1} = \mu \cdot Q_1 \cdot \frac{d}{2} = \mu \cdot F_{a1} \cdot \frac{\cos \varphi \cdot \cos(\alpha + \varphi)}{\sin(\alpha + 2\varphi)} \cdot \frac{d}{2}, \quad (6)$$

and must be bigger than the exterior torsion moment:

$$M_{f1} = \mu \cdot Q_1 \cdot \frac{\cos \varphi \cdot \cos(\alpha + \varphi)}{\sin(\alpha + 2\varphi)} \cdot \frac{d}{2} \geq M_{tc} = \beta \cdot M_t, \quad (7)$$

where  $\beta$  is the safety coefficient (overload) at slip ( $\beta=1.5...3$ )

From comparing relationships (4) and (7) is found that to transmit exterior torque,  $M_t$  by friction, the difference is given by the expression:

$$a_1 = \frac{\cos(\alpha + \varphi)}{\sin(\alpha + 2\varphi)}, \quad (8)$$

$$a_2 = \frac{1}{\sin(\alpha + \varphi)}, \quad (9)$$

that you can compare for some values of angles  $\alpha$  and  $\varphi$ . So for  $\alpha=16^\circ$  and  $\varphi=6^\circ$ , we obtained  $a_1=2.21$  and  $a_2=2.95$ , while for  $\alpha=5^\circ$  and  $\varphi=6^\circ$  it is obtained that  $a_1=3.734$  and  $a_2=5.817$ .

It is found that for external torque transmission, assembly tapered rings are more favorable than the assembly with tightening on the cone, meaning that requires a lower moment of friction force.

## Conclusions

This paper presents a comparative analysis between the assembly with tightening on cone and assembly with tapered rings of light transmission of torque exterior through friction. It was found that, from this point of view, assemblies with tapered rings are more favorable.

Of course it should be taken into account that the assembly with tapered ring according to the assembly with tightening on the cone, shows more constructive complexity, a greater number of elements, which leads to a much higher price, and must be made a very careful analysis when it comes to use one or other assembly with tapered rings or assembly with tightening on the cone.

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## Analiza comparativă a asamblărilor prin strângere pe con şi a asamblărilor cu inele tronconice

### Rezumat

*Se prezintă o analiză comparativă între asamblările prin strângere pe con şi asamblările cu inele tronconice. Criteriul de comparaţie îl constituie momentul de torsiune al forţelor de frecare constatându-se că din acest punct de vedere la asamblările cu inele tronconice este necesar, pentru transmiterea momentului de torsiune exterior, un moment al forţelor de frecare mai mic decât în cazul asamblărilor prin strângere pe con.*

*Desigur trebuie avut in vedere că totuşi asamblările cu inele tronconice prezintă un număr mai mare de elemente componente, o mai mare complexitate constructivă faţă de asamblările prin strângere pe con şi deci trebuie făcută o analiză minuţioasă atunci când se pune problema utilizării asamblărilor cu inele tronconice, sau a asamblării prin strângere pe con.*