Design of Stratified Composite Materials Reinforced with Continuous Fibers

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Abstract

In this paper are presented some of the particularities of stratified composites reinforced with continuous fibers, which are the most commonly used composite category worldwide. Thus, took into account the specific elements of the manufacturing process and some problems in terms of their resistance calculation. Concepts and calculation methods are customized in an application, showing in detail how to determine the specific deformations depending on the loads applied.

Key words: composite materials, multi-ply laminates, lamina, laminates.

Realization of Stratified Composite Materials Reinforced with Fibers

Because operating conditions become increasingly severe, mechanical structures must have properties becoming more efficient. Thus, appear more frequently situations where traditional materials can not cope entirely with these conditions. Since the geometric configuration of the structure is imposed, the only solution is the use of new materials with special qualities. Thus appear the composite materials, which are combinations of two or more components. Properties of components complement each other, resulting in a material with properties superior to those specific components [1, 2]. These components will cooperate so that component failures will be replaced by the qualities of others, giving ensemble properties that no component can have.

Layered composite materials reinforced with fiber are the most common category worldwide. Due to their configuration and reduced number of elastic constants which are characterized, analyzing structures made of layered composite materials reinforced with fiber can be done with great precision. They are among the few composite materials for which the strength calculations can be performed regardless of the complexity of the structure [3]. Late 80s marked a significant increase in the use of fiber reinforced composite materials, introducing the concept of "material design", based on the technical requirements of the product. In each of these new materials is found a central tenet of development of composite materials, namely, the principle of improving one or more properties by coupling materials in different ways.

A fiber-reinforced stratified composite is obtained by hot pressing (or rolling) and bonding of several layers (laminae, plies) with different orientations of fibers (fig. 1). Usually, this technology is used to obtain stratified composite materials with high strength and stiffness in different directions. These products are mainly used in aerospace industry.

A lamina is composed of a single layer plane (or sometimes curved) fiber (or fabric) arranged in matrix material. Its thickness depends on the thickness of the fibers from which is achieved. For example, a lamina from graphite fibers embedded in an epoxy resin matrix may have a thickness of 0.127 mm.

Realization of laminae is done usually from impregnated material (fiber or fabric). This material is encountered as "prepreg" (fig. 2). Bonding of laminae is making through lamination, which allows obtaining very long stratified composite materials. In this case, for the designation of stratified composite material and that the component layers are used pair of concepts "laminate" - "lamina", which comes from the rolling technological process.

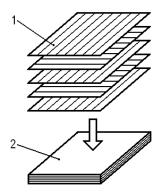


Fig. 1. Structure of a stratified composite materials: 1- layers (laminae) of continuous fiber reinforced resin; laminae are stacked and fibers oriented in different directions to improve the mechanical properties of stratified material; 2 - stratified composite plate obtained by hot pressing of laminae.

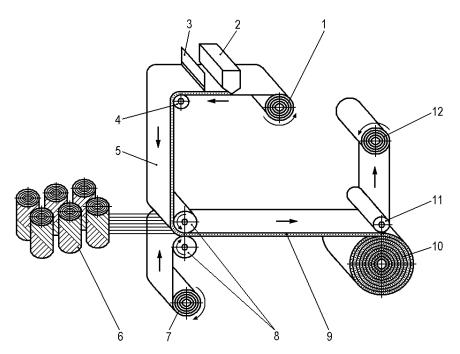


Fig. 2. Scheme of installation for the production of prepreg tapes: 1- coil of support paper; 2- heated resin tank; 3 - knife for uniformity of resin layer; 4 - cylinder guide; 5 - resin deposited on the backing paper;
6 - coils of fiber; 7 - coil of transport paper; 8 - heated cylinder for press; 9 - prepreg tape sandwiched between transport paper and backing paper; 10 - coil of prepreg wrapped with transport paper; 11 - cylinder guide; 12 - coil of waste backing paper.

The prepreg is made of polymer resin reinforced with continuous fibers, characterized in that the resin is only partially solidified. This material delivered by the manufacturer in the form of rolled tape coils, from which then can be obtained the desired product by modeling and complete solidification, without the need to add resin. To achieve of prepregs are used as thermoplastic and thermosetting resins. The common reinforcement materials are carbon fiber, glass, and aramid. Manufacture of pieces using prepreg begins with its location on the surface prepared, after removing transport paper. Normally sits a number of layers to ensure the desired thickness.

It should be noted that the reproducibility of the mechanical characteristics of layered composite materials reinforced with fiber is difficult because many defects that may occur in the manufacturing process: incomplete coverage of the fibers with resin; excess matrix material to the fiber; insufficient polymerization of the matrix material; incomplete bonding of laminae; introduction of air bubbles between the laminae; excessive porosity, inclusions of foreign materials; incorrect orientation of the fibers; the appearance of folds and bumps because incorrect pressing etc.

System of Coordinate Axes Used to Design Stratified Composite Materials Reinforced with Fibers

Each lamina from the structure of a stratified composite material has associated a local coordinate system, located in the plan of lamina: axis 1 is parallel to the fiber direction and axis 2 is perpendicular to the fiber direction (fig. 3). Layered composite material, as a whole, is characterized by global axis system Oxyz. This system of axes is characterized in that the axes x and y are contained in the median plane of stratified composite materials, and the z axis is perpendicular to this plane [4].

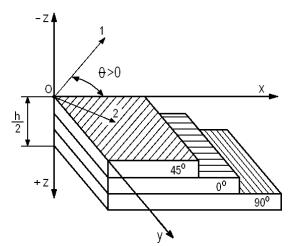


Fig. 3. Stratified composite material made of laminae with different orientations of reinforcing fibers.

Angle between axis 1 (from local coordinate system) and the *x*-axis (from global coordinate system) is denoted by θ . This angle defines the orientation of lamina in the structure of layered composite material. Angle θ is between 0° and 90°:

- if angle $\theta = 0^{\circ}$, the local coordinate system axes correspond to global axes system and lamina is called *unidirectional* lamina;
- if the angle θ is greater than 0°, lamina is called *angular*.

In order to obtain specific mechanical properties, a layered composite material contains both unidirectional laminae and angular laminae.

Structure of a Stratified Composite Material Reinforced with Fiber

Mechanical performance of the stratified composite material (strength, stiffness) depends on the following construction characteristics: number of layers (laminae); value of angle θ , which indicates the orientation of fibers in a lamina (axis 1) to the x axis of stratified composite material (fig. 3); sequence of laminae with different fiber orientations in the stratified material. Laminae location in a stratified structure is described from the upper surface of the stratified material is located at elevation z = -h/2. Lower surface of the stratified material is located at elevation z = +h/2 (fig. 4). In Figures 3 and 4 is presented the same stratified composite material. The difference between the figures is that in Figure 3 are shown only laminae located below the median plan xOy, i.e. laminae 4, 5 and 6 in Figure 4.

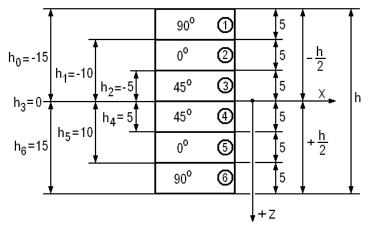


Fig. 4. Notation of laminae thicknesses and positions in the structure of a stratified composite material with formula [90/0/45]_s.

To describe the structure of a stratified composite material are used formulas that show the number of layers (laminae) and fiber orientation of each lamina (angle θ). A group of laminae with the same orientation of the fibers is marked with an index that shows the number of laminae in the group. For example, the stratified composite material described by the formula $[0/90_3/0/45]$ contains six laminae: three of which have fibers oriented at $\theta = 90^\circ$ to the axis of x, one has fibers oriented at $\theta = 45^\circ$ to the axis x and two have fibers oriented in the x axis direction ($\theta = 0^\circ$).

It tells of a stratified material that possesses mirror symmetry, if the lamina identical type and orientation of fibers are found symmetrically disposed on both sides of the median plane xOy (fig. 4). For example, the stratified composite material described by the formula $[90/02/-45/45]_s$ is achieved of 10 laminae symmetrically disposed (see index "s") from the median plane. The fibers are oriented towards the x axis under the following angles: 90° (two laminae), 0° (four laminae), -45° (two laminae) and 45° (two laminae).

Strength Calculation of Unidirectional Lamina

Materials commonly used in the construction of industrial equipment are metals and their alloys. It should be noted that the calculations of strength of structures made from these materials is based on the premise that they are homogeneous and isotropic. Composite materials, unlike metals, are inhomogeneous (with properties ranging from one point to another) and anisotropic (i.e. have different properties in different directions). Inhomogeneity and anisotropy are characteristics of composite materials.

Higher or lower degree of anisotropy or inhomogeneity is given by the structure of this type of material, which may include elements with characteristics very different. These characteristics lead to accentuated complication of calculations of strength and the use of approximate methods for rapid determination of mechanical properties of composite materials. For example, metallic materials (isotropic) are characterized by a single value for each module of elasticity E (Young's modulus), transverse modulus G and transverse contraction ratio (Poisson coefficient) ν . These values are valid regardless of the load direction.

Opposed to isotropic materials, to define composite materials reinforced with continuous fiber (*anisotropic*), in the most general case, it takes 36 elastic constants. This situation is imposed by the fact that the longitudinal modulus E, transverse modulus G, and Poisson's ratio ν are no longer constant. So, instead of E (for isotropic materials), we have E_1, E_2 and E_3 , instead of G, we also have G_{12}, G_{31} and G_{23} , and instead ν we have ν_{12} , ν_{31} , and ν_{23} . So the constitutive equation of such a material can be written in matrix form as [4]:

$$\begin{bmatrix} \sigma_{1} \\ \sigma_{2} \\ \sigma_{3} \\ \tau_{23} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{1} \\ \varepsilon_{2} \\ \varepsilon_{3} \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix}$$
(1)

or

$$\{\sigma\} = [C]\{\varepsilon\},\tag{2}$$

where [C] is the stiffness matrix. A material whose behavior is described by equation (2, 1) is called triclinic material.

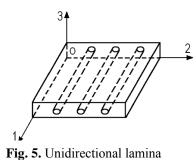
In the constitutive equation are involved, in fact, only 21 independent constants because $C_{ij} = C_{ji}$. The large number of elastic constants which characterize such material makes it difficult both their determination and strength calculation of structures made from materials of this type.

For the study a unidirectional lamina is adopted the local reference system, guided by its natural direction of symmetry: axis 1 in the direction of the fibers; axis 2 perpendicular to axis 1 and in the plane of plate; axis 3, perpendicular to lamina (fig. 5).

In the general case of solicitation, the tensor of stresses from lamina has the following components: $\sigma_1, \sigma_2, \sigma_3, \tau_{12}, \tau_{23}, \tau_{31}$. Since the thickness is much smaller than the other two dimensions of them, the size of the tensions σ_3, τ_{23} and τ_{31} may allow them to be neglected, and the plate is considered in a plane state of stresses. In this case, the nonzero components of the stresses tensor will be σ_1, σ_2 and τ_{12} (fig. 6). It is considered that the domain of solicitation is linear - elastic, and the material is *orthotropic*, requested according to a plane stress state (biaxial state of stress) [5, 6].

Orthotropic material presents three planes of symmetry, orthogonal to each other. Elastic behavior of the material is described by only 9 independent elastic constants and the relation between stresses and deformations is as follows:

$$\begin{cases} \sigma_{1} \\ \sigma_{2} \\ \sigma_{3} \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{cases} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{1} \\ \varepsilon_{2} \\ \varepsilon_{3} \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix}$$
(3)



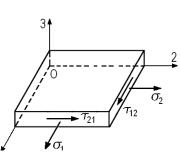


Fig. 6. Tensions from lamina, for the general case of solicitation

The inverse of constitutive equation (3) is written as:

$$\begin{array}{c} \varepsilon_{1} \\ \varepsilon_{2} \\ \varepsilon_{3} \\ \gamma_{23} \\ \gamma_{12} \end{array} \right\} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{21} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{31} & S_{32} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_{1} \\ \sigma_{2} \\ \sigma_{3} \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{bmatrix}$$
(4)

or

$$\{\varepsilon\} = [S]\{\sigma\},\tag{5}$$

where [S] is the matrix of compliance:

$$[S] = \begin{bmatrix} \frac{1}{E_1} & \frac{-\nu_{21}}{E_2} & \frac{-\nu_{31}}{E_3} & 0 & 0 & 0\\ \frac{-\nu_{12}}{E_1} & \frac{1}{E_2} & \frac{-\nu_{21}}{E_3} & 0 & 0 & 0\\ \frac{-\nu_{13}}{E_1} & \frac{-\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{13}} \end{bmatrix}$$
(6)

If at any point there is a plan in which the material mechanical properties are the same in all directions, this is called orthotropic material with transverse isotropy. From this category belong

stratified composite materials reinforced with fibers. In this case the stiffness matrix contains only five independent constants obtained by customizing the stiffness matrix of orthotropic material [equation (3)], as follows:

$$E_2 = E_3; \quad G_{12} = G_{13}; \quad v_{12} = v_{13}.$$
 (7)

Thus, taking into account the relations (7), in the case of a plane stress state (when are neglected σ_3 , τ_{23} and τ_{31}), expression (4) becomes:

$$\begin{cases} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{cases} = \begin{vmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{vmatrix} \begin{cases} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{cases},$$
(8)

The elements of matrix of compliance are calculated with relations:

$$S_{11} = \frac{1}{E_1}; \quad S_{12} = \frac{-\nu_{12}}{E_1}; \quad S_{22} = \frac{1}{E_2}; \quad S_{66} = \frac{1}{G_{12}},$$
 (9)

and by substitution in relation (8) is obtained:

$$\begin{cases} \varepsilon_{1} \\ \varepsilon_{2} \\ \gamma_{12} \end{cases} = \begin{bmatrix} \frac{1}{E_{1}} & \frac{-\nu_{21}}{E_{2}} & 0 \\ \frac{-\nu_{12}}{E_{1}} & \frac{1}{E_{2}} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{cases} \sigma_{1} \\ \sigma_{2} \\ \tau_{12} \end{cases},$$
(10)

that is:

$$\varepsilon_1 = \frac{\sigma_1}{E_1} - v_{21} \frac{\sigma_2}{E_2}; \ \varepsilon_2 = \frac{\sigma_2}{E_2} - v_{12} \frac{\sigma_1}{E_1}; \ \gamma_{12} = \frac{\tau_{12}}{G_{12}}.$$
 (11)

where E_1 and E_2 are longitudinal modules of elasticity in the direction of axis 1, respectively, axis 2; v_{12} - "principal" transverse contraction coefficient (stress after axis 1, deformation after axis 2); v_{21} - "secondary" transverse contraction coefficient (stress after axis 2, deformation after axis 1); $\gamma_{12} = 0$. Indicated that $v_{12} \neq v_{21}$ and there is relation:

$$\frac{v_{12}}{E_1} = \frac{v_{21}}{E_2} \,. \tag{12}$$

The elastic constants $E_1, E_2, G_{12}, v_{12}, v_{21}$ are called technical constants. Orthotropic material is characterized by five independent technical constants, which are usually determined experimentally. Table 1 presents technical constants of several types of laminae with unidirectional fibers, in relation to directions of natural coordinate system axes.

Through the inversion of relation (10) the relation between stresses and deformations are obtained:

$$\begin{cases} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{cases} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{cases} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{cases},$$
(13)

or

$$[\sigma] = [Q] \{\varepsilon\}, \tag{14}$$

where [Q] is the reduced stiffness matrix.

Type of composite material	Materials	Percentage by volume of fibers	E ₁ [GPa]	E ₂ [GPa]	<i>G</i> ₁₂ [GPa]	<i>v</i> ₁₂
T300/5208	Graphite/Epoxi	0.70	181.00	10.30	7.17	0.28
B(4)/5505	Bor/Epoxi	0.50	204.00	18.50	5.59	0.23
AS/3501	Graphite/Epoxi	0.66	138.00	8.96	7.10	0.30
Scotch/ply 1002	Glass/Epoxi	0.45	38.60	8.27	4.14	0.26
Kevlar 49/Epoxi	Aramid/Epoxi	0.60	76.00	5.50	2.30	0.34

Table 1. Technical constants of some types of laminae reinforced with fiber.

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Elements of reduced stiffness matrix are related to elements of compliances matrix [S] by the relations:

$$Q_{11} = \frac{S_{22}}{S_{11}S_{22} - S_{12}^2} = \frac{E_1}{1 - v_{12}v_{21}};$$

$$Q_{12} = \frac{-S_{12}}{S_{11}S_{22} - S_{12}^2} = \frac{v_{12} \cdot E_2}{1 - v_{12}v_{21}};$$

$$Q_{22} = \frac{S_{11}}{S_{11}S_{22} - S_{12}^2} = \frac{E_2}{1 - v_{12}v_{21}};$$

$$Q_{66} = \frac{1}{S_{66}} = G_{12}.$$
(15)

Constitutive matrix equation (14) establishes the connection between stresses which load a lamina $(\sigma_1, \sigma_2, \tau_{12})$ and resulting deformations $(\varepsilon_1, \varepsilon_2, \gamma_{12})$.

Application

A unidirectional lamina made from epoxy resin reinforced with graphite fibers is required for the following stress (fig. 5, fig. 7): $\sigma_1 = 4 MPa$; $\sigma_2 = -2 MPa$; $\tau_{12} = 3 MPa$.

It is required to determine:

- 1. Elements of compliances matrix.
- 2. Secondary Poisson's coefficient.
- 3. Reduced stiffness matrix.
- 4. Lamina specific deformations under the action of stress.

The following elastic constants of the material are known [7]:

$$E_1 = 181 \ GPa, \ E_2 = 10.3 \ GPa, \ v_{12} = 0.28, \ G_{12} = 7.17 \ GPa.$$

Solution:

1. The elements of compliances matrix, involved in the expression (8), are calculated using the relations (9):

$$S_{11} = \frac{1}{181 \times 10^{9}} = 0.5525 \times 10^{-11} Pa^{-1},$$

$$S_{12} = -\frac{0.28}{181 \times 10^{9}} = -0.1547 \times 10^{-11} Pa^{-1},$$

$$S_{22} = \frac{1}{10.3 \times 10^{9}} = 0.9709 \times 10^{-10} Pa^{-1},$$

$$S_{66} = \frac{1}{7.17 \times 10^{9}} = 0.1395 \times 10^{-9} Pa^{-1}.$$

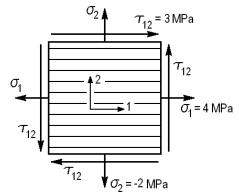


Fig. 7. Stresses which require the unidirectional lamina.

2. Secondary Poisson's coefficient is determined by the relation (12):

$$v_{21} = \frac{0.28}{181 \times 10^9} (10.3 \times 10^9) = 0.01593.$$

3. The elements of reduced stiffness matrix involved in the expression (14) are calculated using the relations (15):

$$\begin{aligned} \mathcal{Q}_{11} &= \frac{181 \times 10^9}{1 - 0.28 \times 0.01593} = 181.8 \times 10^9 \ Pa \ , \\ \mathcal{Q}_{12} &= \frac{0.28 \times 10.3 \times 10^9}{1 - 0.28 \times 0.01593} = 2.897 \times 10^9 \ Pa \ , \\ \mathcal{Q}_{22} &= \frac{10.3 \times 10^9}{1 - 0.28 \times 0.01593} = 10.35 \times 10^9 \ Pa \ , \\ \mathcal{Q}_{66} &= 7.17 \times 10^9 \ Pa \ . \end{aligned}$$

Reduced stiffness matrix [Q] can be obtained also by inverting the compliances matrix [S]:

$$[Q] = [S]^{-1} = \begin{bmatrix} 0.5525 \times 10^{-11} & -0.1547 \times 10^{-11} & 0 \\ -0.1547 \times 10^{-11} & 0.9709 \times 10^{-10} & 0 \\ 0 & 0 & 0.1395 \times 10^{-9} \end{bmatrix}^{-1} = \\ = \begin{bmatrix} 181.8 \times 10^9 & 2.897 \times 10^9 & 0 \\ 2.897 \times 10^9 & 10.35 \times 10^9 & 0 \\ 0 & 0 & 7.17 \times 10^9 \end{bmatrix} Pa.$$

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4. To determinate the lamina specific deformations relation (10) is used:

$$\begin{cases} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{cases} = \begin{bmatrix} 0.5525 \times 10^{-11} & -0.1547 \times 10^{-11} & 0 \\ -0.1547 \times 10^{-11} & 0.9709 \times 10^{-10} & 0 \\ 0 & 0 & 0.1395 \times 10^{-9} \end{bmatrix} \begin{bmatrix} 4 \times 10^6 \\ -2 \times 10^6 \\ 3 \times 10^6 \end{bmatrix} = \begin{bmatrix} 25.19 \\ -200.37 \\ 418.50 \end{bmatrix} (10^{-6}).$$

So the specific deformations values are:

$$\varepsilon_1 = 25.19 \, \frac{\mu m}{m}; \ \varepsilon_2 = -200.37 \, \frac{\mu m}{m}; \ \gamma_{12} = 418.50 \, \frac{\mu m}{m}$$

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Proiectarea materialelor compozite stratificate armate cu fibre continui

Rezumat

În cadrul lucrării de față se prezintă câteva dintre particularitățile materialelor compozite stratificate armate cu fibre continui, care constituie categoria de compozite cea mai folosită la nivel mondial. Astfel, s-au avut în vedere atât elementele caracteristice procesului de fabricație, cât și unele probleme deosebite în ceea ce privește calculul lor de rezistență. Noțiunile și metodele de calcul specifice domeniului enunțat sunt particularizate în cadrul unei aplicații, care prezintă în mod detaliat cum se determină deformațiile specifice ale acestui tip de material în funcție de sarcinile aplicate.