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On the Calculus of the Motor Moment of a Sucker Rod Pumping Unit

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Abstract

In this paper some results concerning the calculus of the variation on a cinematic cycle of the motor moment at the cranks shaft for a pumping unit with conventional type geometry are presented. The calculus of the motor moment is based on the expressing of the dynamic equilibrium in instantaneous powers of all the forces and moments that work on the component links of the mechanism of the sucker rod pumping unit. Finally, the simulation results obtained in the case of a C-640D-365-144 pumping unit are compared with the measurements results carried out at a well in exploitation.

Key words: *pumping unit mechanism, motor moment, instantaneous powers*

Introduction

Many researches conducted over time on the operating behavior of the sucker rod pumping installations [1, 2, 3, 4] have been focused mainly on the study of the dynamics of the sucker rods column and on the issues related to the dynamics of the mechanism of the pumping units with significant impact on the operation of the whole installation: the calculus of the variation on a cinematic cycle of the motor moment, the mechanical balancing of the pumping unit with the determination of the value of the counterweights and the calculus of their position, the establishing of the dynamic reaction forces in the bearings connections and of the mechanical stresses to which are subject the component elements of the pumping units.

In this paper a method that permits the determination of the variation on a cinematic cycle of the motor moment at the cranks shaft of a pumping unit with conventional type geometry is presented. The method is based on the expressing of the dynamic equilibrium in instantaneous powers of all the forces and moments that work on the component links of the mechanism of the sucker rod pumping unit. The simulation results obtained in the case of a C-640D-365-144 pumping unit are compared with the measurements results carried out at a well in exploitation.

Theoretical Considerations and Simulation Results

In figure 1 the cinematic scheme of the mechanism of a sucker rod pumping unit with conventional type geometry is presented. C_1, C_2 and C_3 are the mass centers of the cranks, connecting rods and of the rocker, respectively. m_{CGR} is the mass of the balancing counterweights, m_{L1} is the mass of the connecting bearings between the cranks and the connecting rods, m_{L2} is the mass of the spherical connecting bearing, m_{tr} is the mass of the equalizer traverse, m_{CB} is the mass of the rocker head.

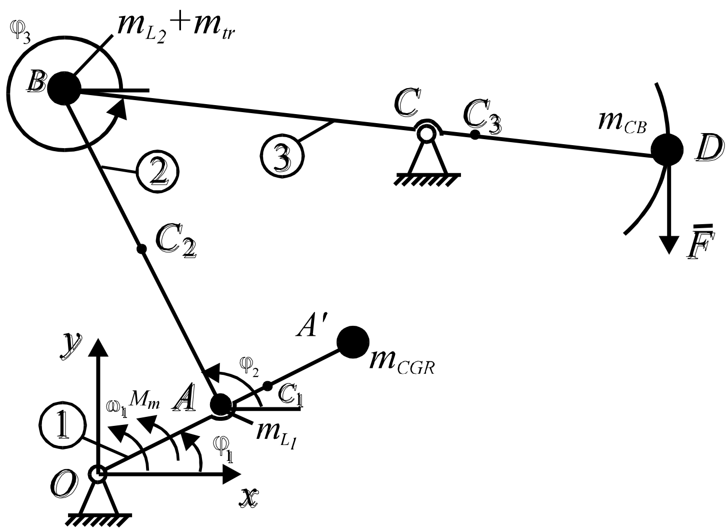

Fig. 1. Cinematic scheme of a pumping unit mechanism with conventional type geometry

By expressing the dynamic equilibrium in instantaneous powers of all external and inertial forces and moments that work on the mechanism of a pumping unit with conventional type geometry, the motor moment at the cranks shaft M_m can be calculated with the following relation [3]:

$$\begin{aligned} &\overline{M}_m \cdot \overline{\omega}_1 + \sum_{j=1}^{3} \overline{G}_j \cdot \overline{v}_{C_j} + \overline{G}_{CGR} \cdot \overline{v}_{A'} + \overline{G}_{L1} \cdot \overline{v}_A + (\overline{G}_{L2} + \overline{G}_{tr}) \cdot \overline{v}_B + \overline{G}_{CB} \cdot \overline{v}_D + \\ &+ \sum_{j=1}^{3} (\overline{F}_{ij} \cdot \overline{v}_{C_j} + \overline{M}_{ij} \cdot \overline{\omega}_j) + \overline{F}_{iCGR} \cdot \overline{v}_{A'} + \overline{F}_{iL1} \cdot \overline{v}_A + (\overline{F}_{iL2} + \overline{F}_{itr}) \cdot \overline{v}_B + \overline{F}_{iCB} \cdot \overline{v}_D + \overline{F} \cdot \overline{v}_D = 0 \end{aligned} \tag{1}$$

where:

ω_1 is the angular speed of the cranks;

$\overline{G}_j = m_j \cdot \overline{g}; j = \overline{1,3}$, where: m_1, m_2 and m_3 are the masses of the cranks, connecting rods and of the rocker, respectively, and $\overline{g}$ is the gravitational acceleration vector that has the same direction as (Oy) axis, but to the contrary of this axis;

$\overline{v}_{C_j}; j = \overline{1,3}$, are the speeds of the mass centers of the cranks, connecting rods and of the rocker, respectively;

$\overline{G}_{CGR} = m_{CGR} \cdot \overline{g}$; $\overline{v}_{A'}$ is the speed of the point where the mass of the counterweights is concentrated (*OA'* is the distance from the cranks shaft to the point where the counterweights are mounted);

$\overline{G}_{L1} = m_{L1} \cdot \overline{g}$; $\overline{G}_{L2} = m_{L2} \cdot \overline{g}$; $\overline{G}_{tr} = m_{tr} \cdot \overline{g}$; $\overline{G}_{CB} = m_{CB} \cdot \overline{g}$;

$\overline{F}_{ij} = -m_j \cdot \overline{a}_{C_j}; j = \overline{1,3}$, are the inertial forces corresponding to the cranks, connecting rods and to the rocker, respectively, where: $\overline{a}_{C_j}; j = \overline{1,3}$, are the accelerations of their mass centers;

$\overline{M}_{ij} = -J_{Cj} \cdot \overline{\varepsilon}_j;\ j = \overline{1,3}$, are the inertial moments corresponding to the cranks, connecting rods and to the rocker, respectively, where: $J_{C_j};\ j = \overline{1,3}$, represent their mass moments of inertia and $\overline{\omega}_j, \overline{\varepsilon}_j;\ j = \overline{1,3}$, are their angular speed and accelerations;

$\overline{F}_{iCGR} = -m_{CGR} \cdot \overline{a}_{A'}$, where $\overline{a}_{A'}$ is the acceleration of the point where the mass of the counterweights is concentrated;

$\overline{F}_{iL1} = -m_{L1} \cdot \overline{a}_A$, where $\overline{a}_A$ is the acceleration of the point *A*;

$\overline{F}_{iL2} = -m_{L2} \cdot \overline{a}_B$, where $\overline{a}_B$ is the acceleration of the point *B*;

$\overline{F}_{itr} = -m_{tr} \cdot \overline{a}_B$; $\overline{F}_{iCB} = -m_{CB} \cdot \overline{a}_D$, where $\overline{a}_D$ is the acceleration of the point *D*;

F is the force at the polished rod.

Starting from the relation (1) it can be highlighted the influence of all the components that are involved in the calculus of the motor moment M_m (the weight forces of the components of the mechanism of the pumping unit and the weight force of the balancing counterweights, the force at the polished rod and the inertial forces and moments) with the following relation:

$$M_m = M_m^g + M_m^{f_i} + M_m^{m_i} + M_m^f \tag{2}$$

where: M_m^g is the component corresponding to the weight forces of the components of the mechanism of the pumping unit and to the weight force of the balancing counterweights, having the following expression:

$$M_m^g = -\frac{1}{\omega_1} \cdot \left(\sum_{j=1}^{3} \overline{G}_j \cdot \overline{v}_{C_j} + \overline{G}_{CGR} \cdot \overline{v}_{A'} + \overline{G}_{L1} \cdot \overline{v}_A + (\overline{G}_{L2} + \overline{G}_{tr}) \cdot \overline{v}_B + \overline{G}_{CB} \cdot \overline{v}_D \right) \tag{3}$$

$M_m^{f_i}$ and $M_m^{m_i}$ are the components corresponding to the inertial forces and moments:

$$M_m^{f_i} = -\frac{1}{\omega_1} \cdot \left(\sum_{j=1}^{3} \overline{F}_{ij} \cdot \overline{v}_{C_j} + \overline{F}_{iCGR} \cdot \overline{v}_{A'} + \overline{F}_{iL1} \cdot \overline{v}_A + (\overline{F}_{iL2} + \overline{F}_{itr}) \cdot \overline{v}_B + \overline{F}_{iCB} \cdot \overline{v}_D \right) \tag{4}$$

$$M_m^{m_i} = -\frac{1}{\omega_1} \cdot \sum_{j=1}^{3} \overline{M}_{ij} \cdot \overline{\omega}_j \tag{5}$$

M_m^f is the component corresponding to the force at the polished rod:

$$M_m^f = -\frac{1}{\omega_1} \cdot \overline{F} \cdot \overline{v}_D \tag{6}$$

The cinematic parameters that appear in the relations above have been determined with the exact cinematic theory [1,2] by using the method of the projection of the independent and closed vectorial contours [6,7].

In [8] the values of the angles φ_2 and φ_3 (fig. 1) have been calculated from the following systems of equations obtained by projecting the contour $O - A - B - C - O$ on the *x* and *y* axes:

$$\begin{cases} l_1 \cdot \cos\varphi_1 + l_2 \cdot \cos\varphi_2 + l_3 \cdot \cos\varphi_3 - x_C = 0 \\ l_1 \cdot \sin\varphi_1 + l_2 \cdot \sin\varphi_2 + l_3 \cdot \sin\varphi_3 - y_C = 0 \end{cases} \tag{7}$$

where: $l_1 = OA$; $l_2 = AB$; $l_3 = BC$.

Then, the angular speeds and accelerations: $\omega_j, \varepsilon_j,\ j = 2{,}3$, of the links *2* and *3* can be calculated by deriving with time the variation functions corresponding to the angles φ_2 and φ_3 with the following relations [6]:

$$\omega_j = \dot{\varphi}_j = \frac{\mathrm{d}\varphi_j}{\mathrm{d}\varphi_1} \cdot \frac{\mathrm{d}\varphi_1}{\mathrm{d}t} = \omega_1 \cdot \frac{\mathrm{d}\varphi_j}{\mathrm{d}\varphi_1};\ \ j = 2{,}3 \tag{8}$$

$$\varepsilon_j = \ddot{\varphi}_j = \varepsilon_1 \cdot \frac{\mathrm{d}\varphi_j}{\mathrm{d}\varphi_1} + \omega_1^2 \cdot \frac{\mathrm{d}^2\varphi_j}{\mathrm{d}\varphi_1^2};\ \ j = 2{,}3 \tag{9}$$

The speed v_D and the acceleration a_D (fig. 1) of the end of the polished rod can be calculated with the following relations: $v_D = \omega_3 \cdot l_{3p}$; $a_D = \varepsilon_3 \cdot l_{3p}$, where $l_{3p} = CD$. The speeds and the accelerations of the remaining points that appear in the relation (1) have been calculated by applying *Euler* formula and *Rivals* formula, respectively [6].

The variation on a cinematic cycle of the motor moment at the cranks shaft has been determined beginning with the value of the crank angle φ_{1d}, corresponding to the starting of the upward movement of the polished rod. In [8] the angle φ_{1d} has been calculated from the following equations system, obtained by projecting the contour $O-A-B-C-O$ on the *x* and *y* axes when the rod *2* is in the prolongation of the crank *1*:

$$\begin{cases} (l_1 + l_2)\cdot\cos\varphi_{1d} + l_3\cdot\cos\varphi_{3d} - x_C = 0 \\ (l_1 + l_2)\cdot\sin\varphi_{1d} + l_3\cdot\sin\varphi_{3d} - y_C = 0 \end{cases} \tag{10}$$

where: φ_{3d} is the value of the angle φ_3 for this extreme position of the rocker of the mechanism.

The relations above have been transposed into a computer program using Maple programming language that has integrated powerful functions for symbolical calculus [5]. For obtaining the analytical expressions of the speeds and of the accelerations $\omega_j, \varepsilon_j,\ j = 2{,}3$, the derivatives with respect to the crank angle φ_1 of the angles φ_2 and φ_3 have been calculated using the derivation function *diff* in Maple programming language [5].

The simulation results obtained in the case of a C-640D-365-144 pumping unit have been compared with the measurements results carried out at the well *Colibaşi* 256, belonging to *OMV PETROM.* The values of the force at the polished rod on a cinematic cycle depending on the values of the crank angle φ_1 (beginning with φ_{1d}, that in this case has the value 87.078º) are presented in table *1*.

For the C-640D-365-144 pumping unit with which is equipped the well *Colibaşi* 256 the following elements are known:

- the dimensions of the component links: $OA = 1.19\text{m}$; $OA' = 2.2\text{m}$; $AB = 3.72\text{m}$; $BC = 3.05\text{m}$; $CD = 4.55\text{m}$; $x_C = 3.05\text{m}$; $y_C = 3.72\text{m}$;
- the mass of the balancing counterweights: $m_{CGR} = 4808\text{kg}$;
- the linear mass of the cranks: 722kg/m;
- the linear mass of the connecting rods: 34kg/m;
- the linear mass of the rocker: 300kg/m;

- the total mass of the connecting bearings between the cranks and the connecting rods: $m_{L1} = 88\text{kg}$;
- the mass of the spherical connecting bearing: $m_{L2} = 169\text{kg}$;
- the mass of the equalizer traverse: $m_{tr} = 580\text{kg}$;
- the mass of the rocker head: $m_{CB} = 840\text{kg}$;
- the working angular speed in rotation per minute: $n_1 = 4.71\ \text{rot/min}$.

Table 1. The values of the force at the polished rod for the well *Colibaşi* 256

φ_1 [º]	*F* [N]	φ_1 [º]	*F* [N]	φ_1 [º]	*F* [N]
87.078	63351	207.078	70287	327.078	56168
102.078	66388	222.078	69429	342.078	54330
117.078	73603	237.078	66599	357.078	56168
132.078	77630	252.078	67497	372.078	58554
147.078	70287	267.078	66198	387.078	58449
162.078	73603	282.078	65114	402.078	57559
177.078	74325	297.078	59474	417.078	60945
192.078	69429	312.078	53117	432.078	62802

In fig. 2 the variation on a cinematic cycle of the motor moment at the cranks shaft for the pumping unit with which is equipped the well *Colibaşi* 256, beginning with the value of the crank angle φ_{1d}, corresponding to the starting of the upward movement of the polished rod, is presented.

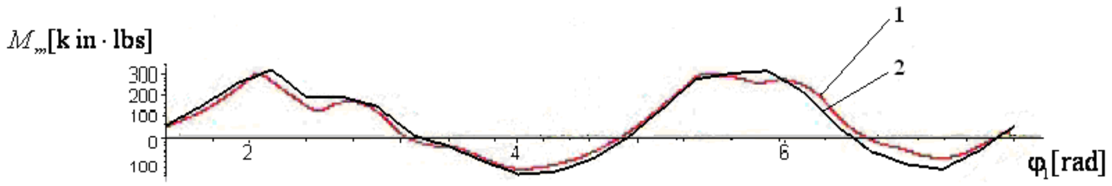

Fig. 2. The variation on a cinematic cycle of the motor moment at the cranks shaft for the pumping unit with which is equipped the well *Colibaşi* 256 (curve *1* corresponds to the measurements results and curve *2* to the simulation results)

Curve *1* corresponds to the measurements results and curve *2* to the simulation results. Because the values of the motor moment obtained after performing the measurements are expressed in [kin·lbs], the simulation results have been transformed using the same measurements units.

Conclusions

In this paper a method that permits the determination of the variation on a cinematic cycle of the motor moment at the cranks shaft of a pumping unit with conventional type geometry has been presented. The method based on the expressing of the dynamic equilibrium in instantaneous powers of all the forces and moments that work on the component links of the mechanism of the sucker rod pumping unit has been transposed into a computer program. The simulation results obtained in the case of a C-640D-365-144 pumping unit are in a good accordance with the measurements results carried out at a well in exploitation.

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Asupra calculului momentului motor al unei unităţi de pompare cu prăjini

Rezumat

In articol sunt prezentate o serie de rezultate privind calculul variaţiei pe un ciclu cinematic al momentului motor la arborele manivelelor pentru o unitate de pompare de construcţie clasică. Calculul momentului motor se bazează pe exprimarea echilibrului dinamic în puteri instantanee al tuturor forţelor şi momentelor care acţionează asupra elementelor componente ale unităţii de pompare cu prăjini. In final, rezultatele simulărilor obţinute în cazul unei unităţi de pompare C-640D-365-144 sunt comparate cu rezultatele măsurătorilor realizate la o sondă în exploatare.