# On Determining the Solution of a Problem Concerning the Positioning and Orientation of the Lynx 6 Robot System 

Dorin Bădoiu

Universitatea Petrol-Gaze din Ploieşti, Bd. Bucureşti 39, Ploieşti
e-mail: badoiu@mail.upg-ploiesti.ro


#### Abstract

The paper presents a method that permits the solving of a problem concerning the positioning and orientation of the Lynx 6 robot system mechanism. The generalized coordinates of the robot mechanism are calculated when the position of the characteristic point of the robot is imposed and an axis of the tool frame has a given orientation.


Key words: robot, joints, position, orientation

## Introduction

Many times, when the industrial robots accomplish different tasks, they must to have a precise position and orientation of the frame attached to the gripper. The analysis of this kind of tasks, that the robots have to do, requires the design of the positional model that permits the calculus of the relative position and orientation between the component modules and between the grasping module and the base of the robot.
These models have to be very precise and, generally, they are verified using the control system of the robot.

In this paper it is presented a method that allows the calculus of the generalized coordinates of the robot mechanism when the position of the characteristic point of the robot is imposed and an axis of the tool frame has a given orientation.

## Theoretical Considerations and Analysis Results

In fig. 1 , the cinematic scheme of the mechanism of the Lynx 6 robot is presented. The systems of coordinates $\left(O_{i} x_{i} y_{i} z_{i}\right), i=\overline{0,5}$, has been attached to each component module $i, i=\overline{0,5}$, (the module zero is the fixed part of the mechanism). The tool frame ( $O_{T} x_{T} y_{T} z_{T}$ ) has the same orientation as $\left(O_{5} x_{5} y_{5} z_{5}\right)$ system of coordinates.

The rotation matrices corresponding to the relative orientation of the modules $i+1$ and $i, i=\overline{0,4}$, can be calculated with the following relations [1]:

$$
\begin{gather*}
{ }^{0} R_{1}=R\left(z, q_{1}\right)=\left[\begin{array}{ccc}
c 1 & -s 1 & 0 \\
s 1 & c 1 & 0 \\
0 & 0 & 1
\end{array}\right]  \tag{1}\\
{ }^{i-1} R_{\mathrm{i}}=R\left(x, q_{i}\right)=\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & c i & -s i \\
0 & s i & c i
\end{array}\right] ; i=\overline{2,4}  \tag{2}\\
{ }^{4} R_{5}=R\left(y, q_{5}\right)=\left[\begin{array}{ccc}
c 5 & 0 & s 5 \\
0 & 1 & 0 \\
-s 5 & 0 & c 5
\end{array}\right] \tag{3}
\end{gather*}
$$

where:

$$
\left\{\begin{array}{l}
s i=\sin q_{i}  \tag{4}\\
c i=\cos q_{i}
\end{array} \quad i=\overline{1,5}\right.
$$



Fig. 1. Lynx 6 robot mechanism
We consider that the characteristic point $P$ has an imposed position and the $y$ axis of the tool frame has a given orientation. The point $P$ in the imposed position verifies the following relation (fig. 1):

$$
\begin{equation*}
{ }^{(0)} O_{0} P={ }^{(0)} O_{0} O_{4}-(d+e) \cdot{ }^{(0)} u \tag{5}
\end{equation*}
$$

where: ${ }^{(0)} u$ is the unit vector corresponding to the imposed direction of the $y_{T}$ axis.

The vector ${ }^{(0)} \mathrm{O}_{0} \mathrm{O}_{4}$ can be determined with the relation [4]:

$$
\begin{equation*}
{ }^{(0)} O_{0} O_{4}={ }^{(0)} O_{0} O_{1}+{ }^{0} R_{1}{ }^{.(1)} O_{1} O_{2}+{ }^{0} R_{2} \cdot{ }^{(2)} O_{2} O_{3}+{ }^{0} R_{3}{ }^{(3)} O_{3} O_{4} \tag{6}
\end{equation*}
$$

where:

$$
\begin{align*}
& { }^{(0)} O_{0} O_{1}=0 ;{ }^{(1)} O_{1} O_{2}=\left[\begin{array}{lll}
0 & 0 & a
\end{array}\right]^{\mathrm{T}} ;{ }^{(2)} O_{2} O_{3}=\left[\begin{array}{lll}
0 & 0 & b
\end{array}\right]^{\mathrm{T}} ;{ }^{(3)} O_{3} O_{4}=\left[\begin{array}{lll}
0 & -c & 0
\end{array}\right]^{\mathrm{T}} \\
& { }^{0} R_{2}=R_{1} R_{1}{ }^{1} R_{2} ; \quad{ }^{0} R_{3}={ }^{0} R_{2} \cdot{ }^{2} R_{3} \tag{7}
\end{align*}
$$

After doing the calculi, we obtain:

$$
{ }^{(0)} O_{0} O_{4}=\left[\begin{array}{c}
\sin q_{1} \cdot\left(b \cdot \sin q_{2}+c \cdot \cos \left(q_{2}+q_{3}\right)\right)  \tag{8}\\
-\cos q_{1} \cdot\left(b \cdot \sin q_{2}+c \cdot \cos \left(q_{2}+q_{3}\right)\right) \\
a+b \cdot \cos q_{2}-c \cdot \sin \left(q_{2}+q_{3}\right)
\end{array}\right]
$$

By doing the following notations in the relation (5):

$$
{ }^{(0)} O_{0} P+(d+e) \cdot{ }^{(0)} u=\left[\begin{array}{lll}
d_{1} & d_{2} & d_{3} \tag{9}
\end{array}\right]^{\mathrm{T}}
$$

and by considering the expression of the vector ${ }^{(0)} O_{0} O_{4}$ in the relation (8), after solving the system of equations (5), we obtain the following expressions for the generalized coordinates $q_{1}, q_{2}$ and $q_{3}$ :

$$
\left\{\begin{array}{l}
q_{1}=\operatorname{arctg}\left(-\frac{d_{1}}{d_{2}}\right)+k \pi ; \quad k \in Z  \tag{10}\\
q_{3}=(-1)^{k} \cdot \arcsin \left(\frac{b^{2}+c^{2}-\left(\frac{d_{1}}{\sin q_{1}}\right)^{2}-\left(d_{3}-a\right)^{2}}{2 \cdot b \cdot c}\right)+k \pi ; \quad k \in Z \\
q_{2}=\operatorname{ATAN2} 2\left(\Delta_{1}, \Delta_{2}\right)
\end{array}\right.
$$

where:

$$
\left\{\begin{array}{l}
\Delta_{1}=\frac{d_{1}}{\sin q_{1}} \cdot\left(b-c \cdot \sin q_{3}\right)-c \cdot \cos q_{3} \cdot\left(d_{3}-a\right)  \tag{11}\\
\Delta_{2}=\left(b-c \cdot \sin q_{3}\right) \cdot\left(d_{3}-a\right)+\frac{d_{1}}{\sin q_{1}} \cdot c \cdot \cos q_{3}
\end{array}\right.
$$

and ATAN2 $(y, x)$ calculates $\operatorname{arctg}(y / x)$ by taking into account the signs of $x$ and $y$ [4]. The condition that the $y$ axis of the tool frame has a given orientation can be transposed into the following relation:

$$
\begin{equation*}
{ }^{(0)} u={ }^{0} R_{5}{ }^{(5)} y_{T} \tag{12}
\end{equation*}
$$

where:

$$
\left\{\begin{array}{l}
{ }^{0} R_{5}={ }^{0} R_{3} \cdot{ }^{3} R_{4} \cdot{ }^{4} R_{5}  \tag{13}\\
{ }^{(5)} y_{T}=\left[\begin{array}{lll}
0 & 1 & 0
\end{array}\right]^{\mathrm{T}}
\end{array}\right.
$$

From the relation (12), we can obtain the following matrix equation:

$$
\begin{equation*}
{ }^{3} R_{0} \cdot{ }^{(0)} u={ }^{3} R_{4} \cdot{ }^{4} R_{5} \cdot{ }^{(5)} y_{T} \tag{14}
\end{equation*}
$$

where:

$$
\left\{\begin{array}{l}
{ }^{3} R_{0}={ }^{0} R_{3}^{\mathrm{T}}  \tag{15}\\
{ }^{3} R_{4} \cdot{ }^{4} R_{5} \cdot{ }^{(5)} y_{T}=\left[\begin{array}{lll}
0 & \cos q_{4} & \sin q_{4}
\end{array}\right]^{\mathrm{T}}
\end{array}\right.
$$

By doing the following notations:

$$
{ }^{3} R_{0}{ }^{(0)} u=\left[\begin{array}{lll}
f_{1} & f_{2} & f_{3} \tag{16}
\end{array}\right]^{\mathrm{T}}
$$

from the relation (14), we obtain:

$$
\begin{equation*}
q_{4}=\operatorname{ATAN} 2\left(f_{3}, f_{2}\right) \tag{17}
\end{equation*}
$$

and the generalized coordinate $q_{5}$ can take any value in its corresponding variation interval.

## Conclusions

The analysis developed in this paper permits the calculus of the relative position and orientation between the component modules and between the grasping module and the base of the Lynx 6 robot. In the paper the analytical expressions of the generalized coordinates of the Lynx 6 robot mechanism are calculated when the position of the characteristic point of the robot is imposed and an axis of the tool frame has a given orientation.

## References

1. B ă d o i u, D . - Analiza structurală şi cinematică a mecanismelor, Editura Tehnică, 2001;
2. Craig, J. J. - Introduction to robotics: mechanics and control, Addison-Wesley, 1986;
3. Dombre, E., Khalil, W. - Modélisation et commande des robots, Ed. Hermès, Paris, 1988.

## Asupra determinării soluției unei probleme privind poziționarea şi orientarea sistemului robot Lynx 6

## Rezumat

Articolul prezintă o metodă care permite rezolvarea unei probleme privind poziționarea şi orientarea mecanismului sistemului robot Lynx 6. Coordonatele generalizate ale mecanismului robotului sunt determinate atunci când poziția punctului caracteristic al robotului este impusă şi o axă a sistemului de coordonate ataşat sculei are o orientare dată.

