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# On Determining the Solution of a Problem Concerning the Positioning and Orientation of the Lynx 6 Robot System

### Dorin Bădoiu

Universitatea Petrol-Gaze din Ploiești, Bd. București 39, Ploiești e-mail: badoiu@mail.upg-ploiesti.ro

### Abstract

The paper presents a method that permits the solving of a problem concerning the positioning and orientation of the Lynx 6 robot system mechanism. The generalized coordinates of the robot mechanism are calculated when the position of the characteristic point of the robot is imposed and an axis of the tool frame has a given orientation.

Key words: robot, joints, position, orientation

# Introduction

Many times, when the industrial robots accomplish different tasks, they must to have a precise position and orientation of the frame attached to the gripper. The analysis of this kind of tasks, that the robots have to do, requires the design of the positional model that permits the calculus of the relative position and orientation between the component modules and between the grasping module and the base of the robot.

These models have to be very precise and, generally, they are verified using the control system of the robot.

In this paper it is presented a method that allows the calculus of the generalized coordinates of the robot mechanism when the position of the characteristic point of the robot is imposed and an axis of the tool frame has a given orientation.

# **Theoretical Considerations and Analysis Results**

In fig. 1, the cinematic scheme of the mechanism of the Lynx 6 robot is presented. The systems of coordinates  $(O_i x_i y_i z_i), i = \overline{0,5}$ , has been attached to each component module *i*,  $i = \overline{0,5}$ , (the module zero is the fixed part of the mechanism). The tool frame  $(O_T x_T y_T z_T)$  has the same orientation as  $(O_5 x_5 y_5 z_5)$  system of coordinates.

The rotation matrices corresponding to the relative orientation of the modules i+1 and i,  $i = \overline{0,4}$ , can be calculated with the following relations [1]:

$${}^{0}R_{1} = R(z,q_{1}) = \begin{bmatrix} c1 & -s1 & 0\\ s1 & c1 & 0\\ 0 & 0 & 1 \end{bmatrix}$$
(1)

$${}^{i-1}R_{i} = R(x,q_{i}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & ci & -si \\ 0 & si & ci \end{bmatrix}; i = \overline{2,4}$$
(2)

$${}^{4}R_{5} = R(y,q_{5}) = \begin{bmatrix} c5 & 0 & s5 \\ 0 & 1 & 0 \\ -s5 & 0 & c5 \end{bmatrix}$$
(3)

where:

$$\begin{cases} si = \sin q_i \\ ci = \cos q_i \end{cases} \quad i = \overline{1,5} \tag{4}$$

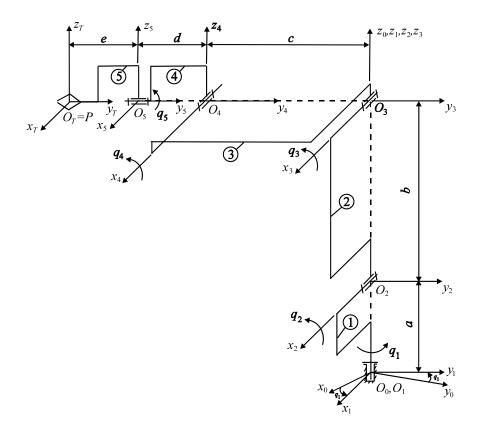


Fig. 1. Lynx 6 robot mechanism

We consider that the characteristic point P has an imposed position and the y axis of the tool frame has a given orientation. The point P in the imposed position verifies the following relation (fig. 1):

$${}^{(0)}O_0P = {}^{(0)}O_0O_4 - (d+e) \cdot {}^{(0)}u \tag{5}$$

where:  ${}^{(0)}u$  is the unit vector corresponding to the imposed direction of the  $y_T$  axis.

The vector  ${}^{(0)}O_0O_4$  can be determined with the relation [4]:

$${}^{(0)}O_0O_4 = {}^{(0)}O_0O_1 + {}^0R_1 \cdot {}^{(1)}O_1O_2 + {}^0R_2 \cdot {}^{(2)}O_2O_3 + {}^0R_3 \cdot {}^{(3)}O_3O_4$$
(6)

where:

$${}^{(0)}O_0O_1 = 0; {}^{(1)}O_1O_2 = \begin{bmatrix} 0 & 0 & a \end{bmatrix}^{\mathrm{T}}; {}^{(2)}O_2O_3 = \begin{bmatrix} 0 & 0 & b \end{bmatrix}^{\mathrm{T}}; {}^{(3)}O_3O_4 = \begin{bmatrix} 0 & -c & 0 \end{bmatrix}^{\mathrm{T}}$$

$${}^{0}R_2 = {}^{0}R_1 \cdot {}^{1}R_2; {}^{0}R_3 = {}^{0}R_2 \cdot {}^{2}R_3$$
(7)

After doing the calculi, we obtain:

$$^{(0)}O_{0}O_{4} = \begin{bmatrix} \sin q_{1} \cdot (b \cdot \sin q_{2} + c \cdot \cos(q_{2} + q_{3})) \\ -\cos q_{1} \cdot (b \cdot \sin q_{2} + c \cdot \cos(q_{2} + q_{3})) \\ a + b \cdot \cos q_{2} - c \cdot \sin(q_{2} + q_{3}) \end{bmatrix}$$
(8)

By doing the following notations in the relation (5):

$${}^{(0)}O_0P + (d+e) \cdot {}^{(0)}u = \begin{bmatrix} d_1 & d_2 & d_3 \end{bmatrix}^{\mathrm{T}}$$
(9)

and by considering the expression of the vector  ${}^{(0)}O_0O_4$  in the relation (8), after solving the system of equations (5), we obtain the following expressions for the generalized coordinates  $q_1, q_2$  and  $q_3$ :

$$\begin{cases} q_1 = \operatorname{arctg}\left(-\frac{d_1}{d_2}\right) + k\pi; \quad k \in \mathbb{Z} \\ q_3 = (-1)^k \cdot \operatorname{arcsin}\left(\frac{b^2 + c^2 - \left(\frac{d_1}{\sin q_1}\right)^2 - (d_3 - a)^2}{2 \cdot b \cdot c}\right) + k\pi; \quad k \in \mathbb{Z} \end{cases}$$
(10)
$$q_2 = \operatorname{ATAN2}(\Delta_1, \Delta_2)$$

where:

$$\begin{cases} \Delta_1 = \frac{d_1}{\sin q_1} \cdot \left( b - c \cdot \sin q_3 \right) - c \cdot \cos q_3 \cdot (d_3 - a) \\ \Delta_2 = \left( b - c \cdot \sin q_3 \right) \cdot (d_3 - a) + \frac{d_1}{\sin q_1} \cdot c \cdot \cos q_3 \cdot \end{cases}$$
(11)

and ATAN2(y,x) calculates arctg(y/x) by taking into account the signs of x and y [4]. The condition that the y axis of the tool frame has a given orientation can be transposed into the following relation:

$${}^{(0)}u = {}^{0}R_{5} \cdot {}^{(5)}y_{T} \tag{12}$$

where:

$$\begin{cases} {}^{0}R_{5} = {}^{0}R_{3} \cdot {}^{3}R_{4} \cdot {}^{4}R_{5} \\ {}^{(5)}y_{T} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^{\mathrm{T}} \end{cases}$$
(13)

From the relation (12), we can obtain the following matrix equation:

$${}^{3}R_{0} \cdot {}^{(0)}u = {}^{3}R_{4} \cdot {}^{4}R_{5} \cdot {}^{(5)}y_{T}$$
(14)

where:

$$\begin{cases} {}^{3}R_{0} = {}^{0}R_{3}^{\mathrm{T}} \\ {}^{3}R_{4} \cdot {}^{4}R_{5} \cdot {}^{(5)}y_{T} = \begin{bmatrix} 0 & \cos q_{4} & \sin q_{4} \end{bmatrix}^{\mathrm{T}} \end{cases}$$
(15)

By doing the following notations:

$${}^{3}R_{0} \cdot {}^{(0)}u = \begin{bmatrix} f_{1} & f_{2} & f_{3} \end{bmatrix}^{\mathrm{T}}$$
 (16)

from the relation (14), we obtain:

$$q_4 = \operatorname{ATAN2}(f_3, f_2) \tag{17}$$

and the generalized coordinate  $q_5$  can take any value in its corresponding variation interval.

#### Conclusions

The analysis developed in this paper permits the calculus of the relative position and orientation between the component modules and between the grasping module and the base of the Lynx 6 robot. In the paper the analytical expressions of the generalized coordinates of the Lynx 6 robot mechanism are calculated when the position of the characteristic point of the robot is imposed and an axis of the tool frame has a given orientation.

#### References

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# Asupra determinării soluției unei probleme privind poziționarea și orientarea sistemului robot Lynx 6

#### Rezumat

Articolul prezintă o metodă care permite rezolvarea unei probleme privind poziționarea și orientarea mecanismului sistemului robot Lynx 6. Coordonatele generalizate ale mecanismului robotului sunt determinate atunci când poziția punctului caracteristic al robotului este impusă și o axă a sistemului de coordonate atașat sculei are o orientare dată.