BULETINUL Universităţii Petrol – Gaze din Ploieşti	Vol. LXVII No. 3/2015	25 – 30	Seria Tehnică

The Influence of Time Application of a Force by Shock on the Dynamic Response of a Mechanical Structure

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Abstract

Mechanical shocks are complex dynamic phenomena which can appear during the functioning of a structure or equipment. They can be encountered in normal operation (case of forges or automatic pile drivers) or they can be accidentally produced. In the much known cases of shock, a body with a certain mass strikes another body or a structure. Likewise, shocks can be encountered when a force is suddenly applied on a structure or when bodies with known masses (motors) are suddenly turned on or off. The aim of this paper is to determine the dynamic response of a structure due to a mechanical shock produced by a suddenly applied force. The influence of time variation of a force on the dynamic response (shock response spectrum for displacements) of a given structure like a pillar-mounted slewing jib will be analyzed.

Key words: *mechanical shocks, response spectrum, suddenly applied force, a pillar-mounted slewing jib.*

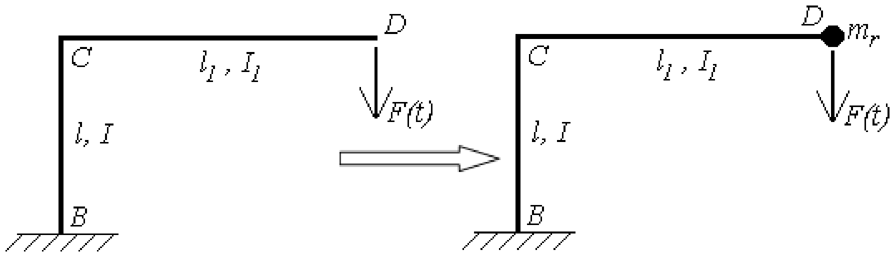
General Considerations: the Pillar-Mounted Slewing Jib Case

Regardless of their origin, mechanical shocks are accompanied by a big quantity of kinetic energy in a very short time [1], [2], [3]. The dynamic response of a structure during shocks action is also called “initial response” and the one which is obtained after shock applying (free vibration of the structure) is called “residual response”. In order to exemplify, it will be considered in the following the case of a mechanical shock produced by a sudden variation of a suspended hook load of a pillar-mounted slewing jib [7], of type that Figure 1 presents.

a. Unloaded pillar-mounted slewing jib   b. Loaded pillar-mounted slewing jib

Fig. 1. Pillar-mounted slewing jib [7]

This situation corresponds to the case when the suspended weight (as a given load) falls accidentally. These types of pillar-mounted slewing jibs are ideal equipments for weights handling in case of indoor use. From a dynamic point of view, they are structures like "continuous systems" [5]. Because the shock is applied in a known point, the study of dynamic phenomena requires their modeling as a structure with concentrated masses (discrete system) [2], [3]. So, the real structure which scheme is shown in Figure 2, a, with mass per length $\overline{m}$ on the zone *BC* and $\overline{m}_1$ on the zone *CD*, is modeled as the Figure 2, b presents, with a concentrated mass m_r in point D, named reduced mass.



a. Scheme as continuous system b. Scheme with reduced mass

Fig. 2. Pillar-mounted slewing jib scheme

Determining the Reduced Mass

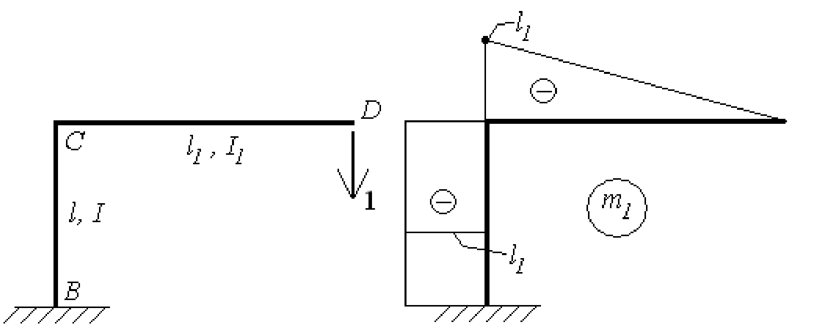
For the dynamic model, the reduced mass value from point D is determined from the condition that the kinetic energy of the distributed mass must be equal to the one of the reduced mass m_r, using the formula [2]:

$$m_r = \int_{(l)} \overline{m} \left(\frac{v_x}{v_D} \right)^2 dx \qquad (1)$$

where: v_x is the bending deflection of an arbitrary point of the structure produced by a force equal to 1 statically applied in the point where the shock action occurs; v_D is the bending deflection of the point *D* (where the force *F* is applied by shock).

The integral from (1) is calculated for both beams of the structure from Figure 2.

A force equal to 1 is applied in point *D*, on the shock direction (see Figure 3, a), and then the bending moments diagram m_1 is drawn (see Figure 3, b).



a. Applying a force equal to 1 b. Bending moments diagram, m_1

Fig. 3. Structure scheme and bending moments diagram

Using the “boundary parameters method” [2]:

$$v_x = v_0 + \varphi_0 x - \frac{M_0}{EI} \cdot \frac{x^2}{2} - \frac{T_0}{EI} \cdot \frac{x^3}{6} + \bar{v}, \tag{2}$$

the bending deflection equations v_x on *BC* beam and v_{x1} on *CD* beam can be determined.

Considering the origin of the axes system in point *B* and taking into account the following conditions:

$$v_0 = v_B = 0;\ \varphi_0 = \varphi_B = 0;\ M_0 = M_B = -l_1;\ T_0 = T_B = 0; \bar{v} = 0, \tag{3}$$

v_x is:

$$v_x = \frac{l_1}{EI} \cdot \frac{x^2}{2} \tag{4}$$

and also:

$$\varphi_x = v_x' = \frac{l_1}{EI} \cdot x. \tag{5}$$

For $x = l$, it is obtained: $\varphi_C = \frac{l_1 \cdot l}{EI}$.

For the *CD* beam, considering the origin of axes system being in point *C* and taking into account the conditions:

$$v_0 = v_C = 0;\ \varphi_0 = \varphi_C = \frac{l_1 \cdot l}{EI};\ M_0 = M_C = -l_1;\ T_0 = T_C = 1;\ \bar{v} = 0, \tag{6}$$

the bending deflection, depending on x_1, is:

$$v_{x1} = \frac{l_1 \cdot l}{EI} x_1 + \frac{l_1}{EI_1} \frac{x_1^2}{2} - \frac{1}{EI_1} \frac{x_1^3}{6} \tag{7}$$

Using Maxwell-Mohr formula [4], the displacement of point *D* is:

$$v_D = d_{11} = \frac{1}{EI} \cdot l \cdot l_1^2 + \frac{l_1^3}{3EI}, \tag{8}$$

which can be also obtained from (7) for $x_1 = l_1$.

The reduced mass is then obtained with:

$$m_r = m = \bar{m} \int_0^l \left(\frac{v_x}{v_D} \right)^2 dx + \bar{m}_1 \int_0^{l_1} \left(\frac{v_{x1}}{v_D} \right)^2 dx_1, \tag{9}$$

where v_x, v_{x1} and v_D are given by (4), (7) and (8).

Obtaining the Dynamic Initial and Residual Responses

When the load suspended in the pillar-mounted slewing jib’s hook accidentally falls, the force becomes a function of time and its law is presented in Figure 4.

The initial dynamic response, available during the variation of the force, for $t \le T$, is given by [2], [5], [6]:

$$\eta = C_1 \cos pt + C_2 \sin pt + \frac{1}{mp} \int_0^t F(\tau) \cdot \sin p(t - \tau) d\tau, \tag{10}$$

After observing Figure 4, it should be noted that:

$$F(\tau) = F^0\left(1 - \frac{\tau}{t}\right), \text{ with } \tau \le T \tag{11}$$

and (10) becomes:

$$\eta(t) = C_1 \cos pt + C_2 \sin pt + \frac{F^0}{mp}\left[1 - \cos pt - \frac{t}{T}\left(1 - \frac{1}{pt}\sin pt\right)\right], \tag{12}$$

where p is the angular frequency of the given structure:

$$p = \sqrt{\frac{1}{md_{11}}}\,. \tag{13}$$

The constants C_1 and C_2 are determined from initial conditions (for $t = 0)$:

$$\eta(0) = v_D = F^0 \cdot d_{11} = \frac{F^0}{mp^2};\ \ \dot{\eta}(0) = 0 \tag{14}$$

and their values are given by:

$$C_1 = \frac{F^0}{mp^2};\ C_2 = 0\,. \tag{15}$$

With these known values, the dynamic initial response is:

$$\eta(t) = \frac{F^0}{mp^2}\left[1 - \frac{t}{T}\left(1 - \frac{1}{pt}\sin pt\right)\right] \tag{16}$$

The residual response, which occurs after applying by shock the force *F*(*t*) (for $t > T$), is determined with:

$$\eta_1(t) = \eta(t)\cos p(t-T) + \frac{1}{T}\dot{\eta}(T)\sin p(t-T) \tag{17}$$

where $\eta(t)$ and $\dot{\eta}(t)$ are obtained from (16). So, the displacement as function of time $\eta_1(t)$ is:

$$\eta_1(t) = \frac{F^0}{mp^2}\cdot\frac{1}{p\cdot T}\left[\sin pT\cdot\cos p(t-T) - (1-\cos pT)\cdot\sin p(t-T)\right] \tag{18}$$

Taking into account (16) and (18), the dynamic response of the structure (initial and residual) can be written as it follows:

$$\eta(t) = \begin{cases} \dfrac{F^0}{mp^2}\left[1 - \dfrac{t}{T}\left(1 - \dfrac{1}{pt}\sin pt\right)\right], & for\ t \le T \\ \dfrac{F^0}{mp^2}\cdot\dfrac{1}{p\cdot T}\left[\sin pT\cdot\cos p(t-T) - (1-\cos pT)\cdot\sin p(t-T)\right], & for\ t > T \end{cases} \tag{19}$$

The Influence of Time of Application *T* on Displacements: a Case Study

Let be a pillar-mounted slewing jib as it is shown in Figure 2, a, having in the zone *BC* a round pillar (with annular cross-section) with $A = 5969mm^2$, the second moment of area (with respect to z-axis) $I = 27\cdot 10^6 mm^4$, height $l = 2.5m$ and in the zone *CD* an I-beam profile jib, with area $A_1 = 4000mm^2$, the second moment of area $I_1 = 28.733\cdot 10^6 mm^4 = 1.0642\cdot I$ and the outreach $l_1 = 3m = 1.2\cdot l$.

The distributed masses on the two zones *BC* and *CD* are: $\overline{m} = A \cdot \frac{\gamma}{g} = 4.775 \cdot 10^{-5} t/mm$, $\overline{m}_1 = 3.2 \cdot 10^{-5} t/mm$ and the structure's mass is $M = \overline{m} \cdot l + \overline{m}_1 \cdot l_1 = 209.37 kg$. With (4) it is obtained:

$v_x = \frac{1.2 \cdot l}{EI} \cdot \frac{x^2}{2} = 0.6 \cdot \frac{l}{EI} \cdot x^2$, and with (7), $v_{x1} = \frac{1.2 \cdot l^2}{EI} x_1 + 0.5638 \frac{l}{EI} \cdot x_1^2 - 0.1566 \frac{x_1^3}{EI}$.

With (8) it is determined $v_D = d_{11} = 1.981 \frac{l^3}{EI}$ and with (9) the reduced mass is $m_r = 31.27 kg$.

The angular frequency of the structure, using (13), is $p = 76.54\ rad/s$. For different values of *T*, the dynamic response is shown in Figure 5.

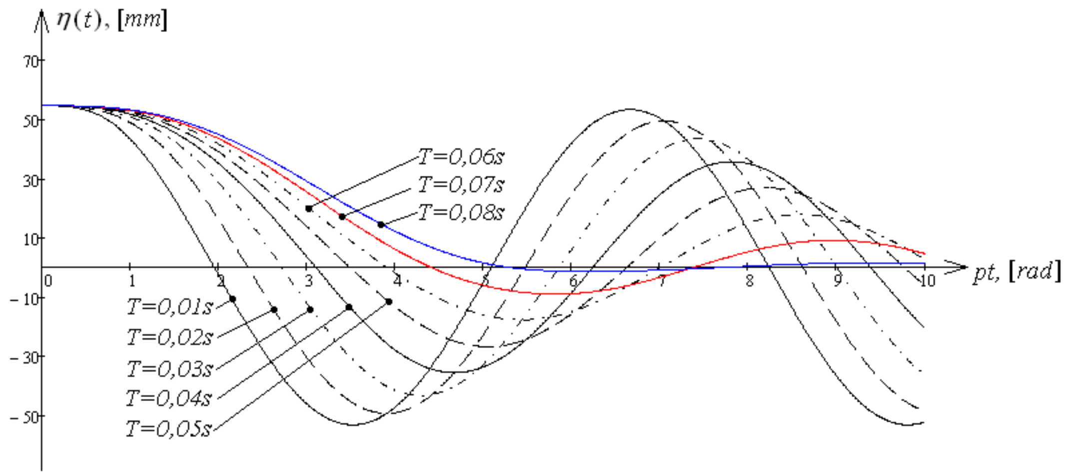


Fig. 5. Displacements versus time, for different values of *T* (spectrum for displacements)

Analyzing this chart, it can be observed that:

- after applying *F(t)* by shock, the structure effectuates free vibrations; the influence of damping was ignored;
- after shock, the displacements decrease with the increase of time of applying the force;
- if the time of application of the force is bigger than 0.1 *s*, the structure practically will not vibrate after shock.

Conclusions

1. Shocks are complex dynamic phenomena, characterized by the transmission of large amounts of kinetic energy in a very short time.

2. In the present work it was determined the dynamic response of a structure due to a mechanical shock produced by a suddenly applied force.

3. The concretely analyzed structure in the case study is a pillar-mounted slewing jib, which is, from a dynamical point of view, a continuous system. For this reason, in order to do the necessary calculus, the slewing jib must be modeled as a structure with discrete mass ("discrete system"), named concentrated mass, which is placed in the point where the force is applied by shock.

4. The dynamic response of the structure was determined as a spectrum of displacements and it depends of *T*, which is the time of application of force *F*(*t*). After shock, the displacements decrease with the increase of time of applying the force and if this time is bigger than 0.1 s, the structure practically will not vibrate after shock.

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Influenţa timpului de aplicare a unei sarcini prin şoc asupra răspunsului dinamic al unei structuri

Rezumat

Şocurile mecanice sunt fenomene dinamice complexe care apar în timpul funcţionării unor instalaţii sau structuri. Ele pot corespunde unor situaţii normale de funcţionare (cazul forjelor, al sonetelor de bătut piloni) sau se pot produce accidental. În cazurile clasice de şoc mecanic, un corp de o anumită masă loveşte un alt corp sau o structură. Tot şocuri sunt şi cazurile în care o forţă se aplică brusc asupra unei structuri sau în cazul opririlor sau pornirilor bruşte ale unor corpuri de tipul motoarelor. În lucrarea de faţă se va determina răspunsul dinamic al unei structuri la un şoc mecanic produs de aplicarea bruscă a unei forţe. Se va analiza influenţa timpului de variaţie a forţei care se aplică prin şoc asupra răspunsului dinamic (spectrul de răspuns în deplasări) al unei structuri de tipul macara pivotantă.