# Research Concerning the Optimized Positional Synthesis of the Lynx 5 Robot System 

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#### Abstract

The paper presents a method that permits the calculus of the inverse geometric model of the Lynx 5 robot system when an optimization criterion is adopted. The optimization criterion imposes that the displacements of the motor axes of the robot system for achieving an imposed position of the gripper characteristic point is minimum compared to the zero configuration of the robot (the configuration when all of the robot coordinates are equal to zero). Finally, some simulation results are presented.


Key words: robot, position, orientation, optimization

## Introduction

In many cases, when the robots are used in different operations of manipulation, their mechanism has more degrees of freedom than necessary to fulfill a task. These robots are redundant in relation to the task to be performed. So, they can perform a task, whilst meeting certain performance criteria, such as: minimum energy consumption, the fulfillment of some conditions for avoiding obstacles or singular configurations of the robot mechanism etc.

In this paper a method that allows the calculus of the inverse geometric model of the Lynx 5 robot system (fig. 1) by considering an optimization criterion is presented. The optimization criterion imposes that the displacements of the motor axes of the robot system for achieving an imposed position of the gripper characteristic point is minimum compared to the zero configuration of the robot (the configuration when all of the robot coordinates are equal to zero).

## Theoretical Considerations and Verification Results

In figure 1 , the cinematic scheme of the mechanism of the Lynx 5 robot system is presented. The systems of coordinates $\left(O_{i} x_{i} y_{i} z_{i}\right), i=\overline{0,4}$, have been attached to each component module $i$, $i=\overline{0,4}$. The values of the geometric parameters $a, b, c$ and $d$ are: $a=79 \mathrm{~mm} ; b=95.25 \mathrm{~mm}$; $c=95.25 \mathrm{~mm} ; d=124.3 \mathrm{~mm}$.

The rotation matrices ${ }^{i} R_{i+1}, i=\overline{0,3}$, corresponding to the relative orientation of the component modules $i+1$ and $i$ have the following expressions:

$$
{ }^{0} R_{1}=R\left(z, q_{1}\right)=\left[\begin{array}{ccc}
c 1 & -s 1 & 0  \tag{1}\\
s 1 & c 1 & 0 \\
0 & 0 & 1
\end{array}\right] ; \quad{ }^{i-1} R_{i}=R\left(x, q_{i}\right)=\left[\begin{array}{ccc}
c i & -s i & 0 \\
s i & c i & 0 \\
0 & 0 & 1
\end{array}\right] ; i=\overline{2,4}
$$

where:

$$
\left\{\begin{array}{l}
s i=\sin q_{i}  \tag{2}\\
c i=\cos q_{i}
\end{array} \quad i=\overline{1,4}\right.
$$



Fig. 1. Lynx 5 robot mechanism

The position of the characteristic point $P$ of the robot gripper (fig. 1) relative to the system of coordinates $\left(O_{0} x_{0} y_{0} z_{0}\right)$ can be determined with the following relation:

$$
\begin{equation*}
{ }^{(0)} O_{0} P={ }^{(0)} O_{0} O_{1}+{ }^{0} R_{1} \cdot{ }^{(1)} O_{1} O_{2}+{ }^{0} R_{2} \cdot{ }^{(2)} O_{2} O_{3}+{ }^{0} R_{3} \cdot{ }^{(3)} O_{3} O_{4}+{ }^{0} R_{4} \cdot{ }^{(4)} O_{4} P \tag{3}
\end{equation*}
$$

where:

$$
\begin{align*}
& { }^{(0)} O_{0} O_{1}=0 ;{ }^{(1)} O_{1} O_{2}=\left[\begin{array}{lll}
0 & 0 & a
\end{array}\right]^{\mathrm{T}} ;{ }^{(2)} O_{2} O_{3}=\left[\begin{array}{lll}
0 & 0 & b
\end{array}\right]^{\mathrm{T}} ; \\
& { }^{(3)} O_{3} O_{4}=\left[\begin{array}{lll}
0 & -c & 0
\end{array}\right]^{\mathrm{T}} ;{ }^{(4)} O_{4} P=\left[\begin{array}{lll}
0 & -d & 0
\end{array}\right]^{\mathrm{T}} \tag{4}
\end{align*}
$$

$$
\begin{gather*}
{ }^{0} R_{2}={ }^{0} R_{1} \cdot{ }^{1} R_{2}=\left[\begin{array}{ccc}
c 1 & -s 1 \cdot c 2 & s 1 \cdot s 2 \\
s 1 & c 1 \cdot c 2 & -c 1 \cdot s 2 \\
0 & s 2 & c 2
\end{array}\right]  \tag{5}\\
{ }^{0} R_{3}={ }^{0} R_{2} \cdot{ }^{2} R_{3}=\left[\begin{array}{ccc}
c 1 & -s 1 \cdot c 23 & s 1 \cdot s 23 \\
s 1 & c 1 \cdot c 23 & -c 1 \cdot s 23 \\
0 & s 23 & c 23
\end{array}\right]  \tag{6}\\
{ }^{0} R_{4}={ }^{0} R_{3} \cdot{ }^{3} R_{4}=\left[\begin{array}{ccc}
c 1 & -s 1 \cdot c 234 & s 1 \cdot s 234 \\
s 1 & c 1 \cdot c 234 & -c 1 \cdot s 234 \\
0 & s 234 & c 234
\end{array}\right] \tag{7}
\end{gather*}
$$

where:

$$
\left\{\begin{array}{l}
s 23=\sin \left(q_{2}+q_{3}\right)  \tag{8}\\
c 23=\cos \left(q_{2}+q_{3}\right)
\end{array} ;\left\{\begin{array}{l}
s 234=\sin \left(q_{2}+q_{3}+q_{4}\right) \\
c 234=\cos \left(q_{2}+q_{3}+q_{4}\right)
\end{array}\right.\right.
$$

After calculations, it results:

$$
{ }^{(0)} O_{0} P=\left[\begin{array}{c}
s 1 \cdot(s 2 \cdot b+c 23 \cdot c+c 234 \cdot d)  \tag{9}\\
-c 1 \cdot(s 2 \cdot b+c 23 \cdot c+c 234 \cdot d) \\
a+c 2 \cdot b-s 23 \cdot c-s 234 \cdot d
\end{array}\right]
$$

By imposing the position of the point $P$, the inverse geometric model of the Lynx 5 robot system will be solved by considering an optimization criterion. The optimization criterion imposes that the displacements of the motor axes of the robot system for achieving an imposed position of the point $P$ is minimum compared to the zero configuration of the robot (the configuration when all of the robot coordinates are equal to zero). So, this optimization criterion imposes that the following function must take a minimum value:

$$
\begin{equation*}
F=\frac{1}{2} \cdot \sum_{i=1}^{4} q_{i}^{2} \tag{10}
\end{equation*}
$$

where $q_{i}, i=\overline{1,4}$, verify the relation (9), when the position of the point $P$ is imposed.
By introducing the following function [3]:

$$
L=F+\lambda^{T} \cdot\left({ }^{(0)} O P-\left[\begin{array}{l}
x_{P}  \tag{11}\\
y_{P} \\
z_{P}
\end{array}\right]\right)
$$

where: $x_{P}, y_{P}$ and $z_{P}$ are the imposed values of the coordinates of the point $P$ relative to the fixed system of coordinates $\left(O_{0} x_{0} y_{0} z_{0}\right)$ and $\lambda=\left[\begin{array}{lll}\lambda_{1} & \lambda_{2} & \lambda_{3}\end{array}\right]^{\mathrm{T}}$, where $\lambda_{i}, i=\overline{1,3}$, are three unknown parameters (Lagrange's parameters), we must find the values of the coordinates $q_{i}, i=\overline{1,4}$, for which the function $L$ is minimum and the coordinates of the point $P$ given by the relation (9) take the imposed values: $x_{P}, y_{P}$ and $z_{P}$.

After calculations, it results the following system of equations:

$$
\left\{\begin{array}{l}
f_{1}-x_{P}=0 ; f_{2}-y_{P}=0 ; f_{3}-z_{P}=0  \tag{12}\\
q_{i}+\frac{\partial f_{1}}{\partial q_{i}} \cdot \lambda_{1}+\frac{\partial f_{2}}{\partial q_{i}} \cdot \lambda_{2}+\frac{\partial f_{3}}{\partial q_{i}} \cdot \lambda_{3}=0 ; \mathrm{i}=\overline{1,4}
\end{array}\right.
$$

where:

$$
\left\{\begin{array}{l}
f_{1}=s 1 \cdot(s 2 \cdot b+c 23 \cdot c+c 234 \cdot d)  \tag{13}\\
f_{2}=-c 1 \cdot(s 2 \cdot b+c 23 \cdot c+c 234 \cdot d) \\
f_{3}=a+c 2 \cdot b-s 23 \cdot c-s 234 \cdot d
\end{array}\right.
$$

The method has been transposed into a computer program. As a numerical example, by considering: $x_{P}=-34 \mathrm{~mm} ; y_{P}=-223.5 \mathrm{~mm} ; z_{P}=100 \mathrm{~mm}$, the following values for the coordinates $q_{i}, i=\overline{1,4}$, have been obtained: $q_{1}=-8.65^{\circ} ; q_{2}=11.43^{\circ} ; q_{3}=6^{\circ} ; q_{4}=3.2^{\circ}$.

## Conclusions

The paper presents a method that allows the calculus of the inverse geometric model of the Lynx 5 robot system by considering an optimization criterion. The optimization criterion imposes that the displacements of the motor axes of the robot system for achieving an imposed position of the gripper characteristic point is minimum compared to the zero configuration of the robot. The method is very useful to be applied for operations of manipulation, when the robot mechanism has more degrees of freedom than necessary to fulfill a task.

## References

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## Cercetări privind sinteza pozițională optimizată a sistemului robot Lynx 5

## Rezumat

Articolul prezintă o metodă care permite calculul modelului geometric invers al sistemului robot Lynx 5 atunci când se adoptă un criteriu de optimizare. Criteriul de optimizare impune ca deplasările axelor motoare ale sistemului robot pentru atingerea unei poziții impuse a punctului caracteristic al griperului să fie minime in raport cu configurația zero a robotului (configurația în care toate coordonatele robotului sunt egale cu zero). In final, sunt prezentate o serie de rezultate ale simulărilor.

