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A Method of the Origin Parameters Used in the Calculation of the Critical Load for Beams with Variable Cross Sectional Area

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Abstract

In the paper is presented a method of the origin parameters that can be used in order to calculate the critical load for a axially compressed beam with variable cross sectional area. The results obtained are analysed in a calculus example.

Key words: *deflection, slope, elastic curve, buckling*

General Equations

A cantilever with a variable cross sectional area is considered (fig. 1). The beam has a linear depth h_x, the length l, the constant thickness b ($b > h$) and is externally loaded with a axial concentrated force at the free end (P).

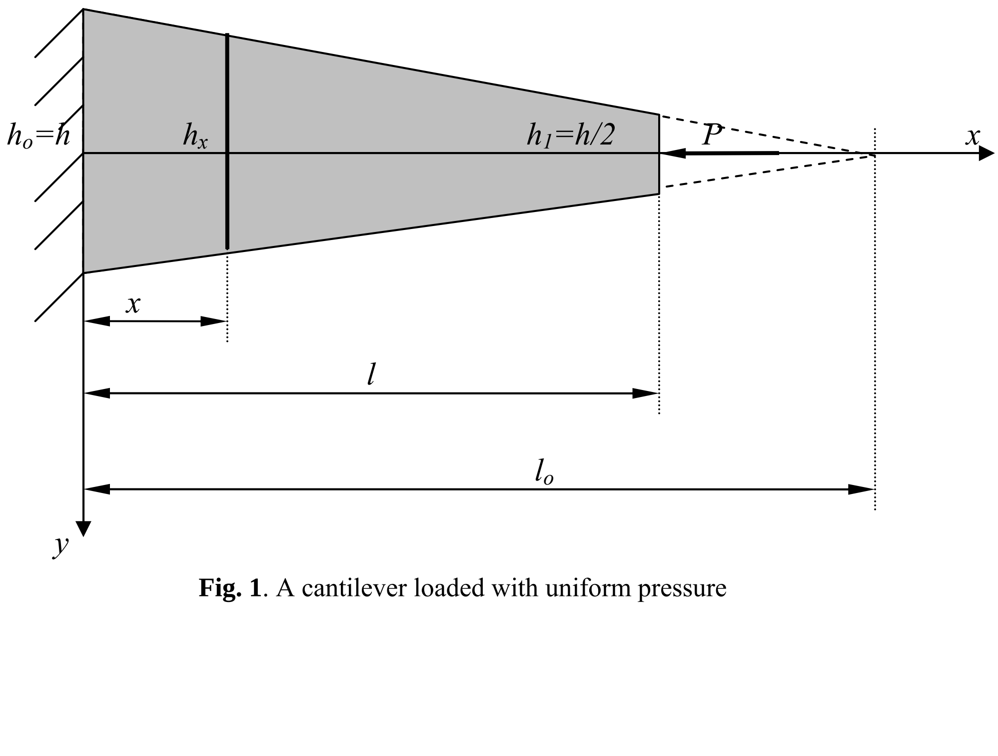

Fig. 1. A cantilever loaded with uniform pressure

The beam is embedded at the left end and its depth presents a linear variation along the length. If the depth in the origin is h the current depth of the beam can be expressed from the geometry of the beam as : $h_x = h\left(1-\frac{x}{2l}\right)$, where l represents the entire length of the beam. The main geometrical characteristics of the cross sectional area (the inertia moment and the area) can be calculated with the relations :

$$I_z(x) = b\frac{h_x^3}{12} = b\left(1-\frac{x}{2l}\right)^3 \cdot \frac{h^3}{12} = I_{zo}\left(1-\frac{x}{2l}\right)^3 \quad (1)$$

$$A(x) = b \cdot h_x = b \cdot h\left(1-\frac{x}{2l}\right) = A_o\left(1-\frac{x}{2l}\right) \quad (2)$$

In order to evaluate the critical load for the beam presented in figure 1 an energetical method is used. In this respect the equality between the external and internal energy can be written under the form:

$$\frac{1}{2}P_{cr}\int_0^l v'^2(x)\cdot dx = \frac{1}{2}\int_0^l \frac{M^2(x)\cdot dx}{EI_z(x)} \quad (3)$$

If the bending moment for the cantilever presented in figure 1 is expressed as:

$$M(x) = -P_{cr}(v_{\max} - v) \quad , \quad (4)$$

where v_{max} represents the maximum deflection from the free end, from the (3) equality it can be expressed the critical load under the form :

$$P_{cr} = \frac{\int_o^l v'^2(x)\cdot dx}{\int_o^l \frac{(v_{\max}-v)^2}{EI}dx} \quad (5)$$

In order to evaluate the critical load with the (5) equality it is necessary to admit a general function that describes the buckling deformed shape of the beam. Usually some trigonometric functions are chosen because these functions satisfied more easy the limit conditions.

In this paper a general function that contains the origin parameters is used, because such a function can be used for any others limit conditions or beam supports.

The differential equation that describes the bending of the beam has the form:

$$\frac{d^2v}{dx^2} = -\frac{M(x)}{EI_z(x)} \quad , \quad (6)$$

where v is the deflection and $M(x)$ the bending moment in the current section from x abscissa. The bending moment in the current section of a general beam can be expressed by reducing all the moments from the left side, using the origin parameters:

$$M_z(x) = M_o + T_o \cdot x \quad , \quad (7)$$

where M_o and T_o represent the bending moment and shear force from the origin.

Integrating twice, step by step, the (6) differential equation and taking into account that the limit conditions that have to be satisfied are: $x = 0 \Rightarrow v = v_o, v' = \varphi_o$ (where v_0 and φ_0 represent the deflection and the slope of the beam in the origin), the general expressions for the deflection and the slope of the beam can be written under the forms:

$$v(x) = v_o + \varphi_o \cdot x + \left(\frac{M_o \cdot l}{EI_{zo}} - \frac{2 \cdot T_o \cdot l^2}{EI_{zo}} \right) \cdot x + \frac{2 \cdot M_o \cdot l^2}{EI_{zo}} \left(1 - \frac{1}{1 - \frac{x}{2l}} \right) + \frac{4T_o \cdot l^3}{EI_{zo}} \left[1 - 2 \cdot \ln\left(1 - \frac{x}{2l}\right) - \frac{1}{1 - \frac{x}{2l}} \right] \quad (8)$$

$$\varphi(x) = \varphi_o + \frac{M_o \cdot l}{EI_{zo}} \left[1 - \frac{1}{\left(1 - \frac{x}{2l}\right)^2} \right] - \frac{2T_o \cdot l^2}{EI_{zo}} \left[1 - \frac{1}{1 - \frac{x}{2l}} \right]^2 \quad (9)$$

It is important to specify that the critical load expressed by (5) can be calculated with (8) and (9) relations, where $v(x)$ and $\varphi(x)$ can be be produced by any transversal loads that act on the beam. If it is considered that the deflection of the cantilever presented in figure 1 is produced by a concentrated force (F) that acts in the free end, the origin parameters are :

$$v_o = 0;\ \varphi_o = 0;\ M_o = -F \cdot l;\ T_o = F \quad (10)$$

With the (10) limit conditions the maximum deflection of the beam (from the free end) can be written under the form :

$$v_{\max} = 0.544 \frac{F \cdot l^3}{EI_{zo}} \quad (11)$$

It can be noticed that the (8) and (9) functions contain the origin parameters and can be used for any others limit conditions.

A Numerical Example

In order to evaluate the critical load for the cantilever presented in figure 1, the above (5), (8) and (11) relations have been used. A specialized programme has been developed in order to evaluate the integrals that appear in (5) relation.

The supports of the cantilevers presented in figure 1 allow the buckling of the beam only in xOy plane. Using the above relations and calculating all the integrals from relation (5) the critical load becomes:

$$P_{cr} = 1.344 \frac{EI_{zo}}{l^2} \quad (12)$$

It can be noticed that the above relation is nearly the same with those presented in [1], where a trigonometric function for the deflection of the beam has been adopted.

Conclusions

In the paper is presented a origin parameters method used for calculation of the deflections and the slopes for a beam with variable cross sectional area. The method can be used for any beam with different limit conditions and can be also used in the energetical method for the evaluation of the critical loads for beams axially compressed. The results obtained are analysed in a calculus example and it can be noticed that the critical load obtained using the presented methodology is nearly the same with those obtained in [1] (where a trigonometric function has been used for the deflection of the beam).

References

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O metodă a parametrilor în origine utilizată la determinarea sarcinii critice pentru bare cu secțiunea variabilă

Rezumat

În lucrare se prezintã o metodã a parametrilor in origine ce poate fi utilizatã pentru determinarea sarcinii critice a barelor cu secțiune variabilã, comprimate axial. Rezultatele obținute sunt analizate pe un exemplu de calcul.