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About the Determination of the Direct Contact Ratio of the Spur-Gears

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Abstract

Determining the direct contact ratio is an important step in establishing and constructing the spur gears. This paper aims to present the manner in which the direct contact ratio for the spur gears is being determined. We shall refer here in a general and unitary manner to the spur gears with straight and helical teeth, modified or unmodified toothing.

Key words: *gear, gear wheel, contact ratio, pitch diameter, head or foot diameter, distance between the axes.*

General Considerations

The direct contact ratio is a fundamental concept in determining and constructing a gear. Defined as the proportion between the length of the arc of engagement and the step on the rolling wheel (divided circle) [2], the value of the contact ratio is defined by the medium number of teeth pairs which are put into gear simultaneously and which describe the gear teeth process.

In terms of kinematic transmission of continuous rotating movement from one wheel to another and in terms of dynamic force distribution on the gear teeth of wheels engaged. Coinciding with fig. 1, it has been obtained [2]:

$$\varepsilon_{\alpha w} = \frac{\overline{AE}}{p\cos\alpha} = \frac{\overline{AE}+\overline{CE}}{p\cos\alpha} = \frac{\left(\overline{AK_2}-\overline{CK_2}\right)+\left(\overline{EK_1}-\overline{CK_1}\right)}{p\cos\alpha} =$$

$$= \frac{\sqrt{r_{a2}^2 - r_{b2}^2} - r_{w2}.\sin\alpha_w + \sqrt{r_{a1}^2 - r_{b1}^2} - r_{w1}.\sin\alpha_w}{\pi m\cos\alpha}; \qquad (1)$$

$$\varepsilon_{\alpha w} = \frac{\sqrt{d_{a1}^2 - d_{b1}^2} + \sqrt{d_{a2}^2 - d_{b2}^2} - 2a_w\sin\alpha_w}{2\pi m\cos\alpha} .$$

The following situations have been analyzed.

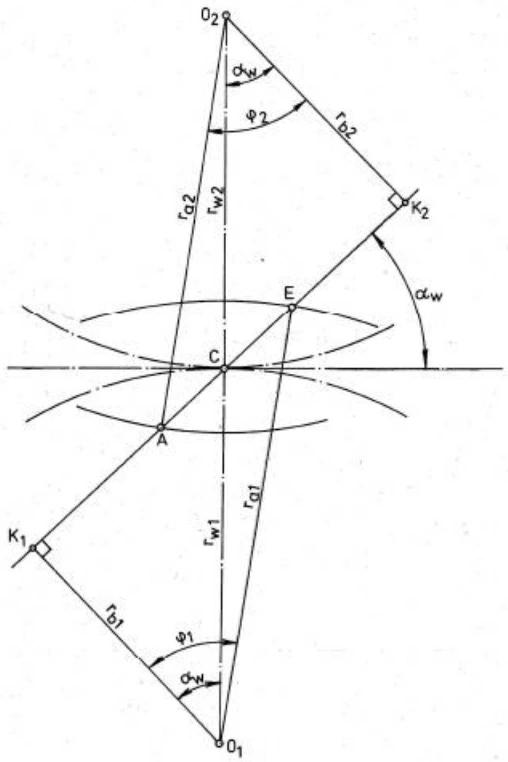

Fig.1. The schema of determining the direct contact ratio for a spur gear with straight teeth, involute profile, modified toothing

Spur Gear with Straight Teeth and Unmodified Toothing

The geometrical elements of the wheels and of the gears are being determined according to fig. 2, using the following ratios:

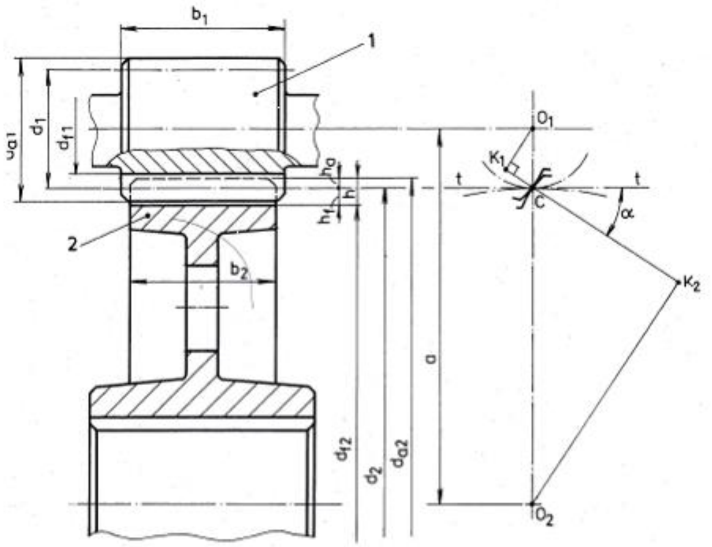

Fig.2. Geometrical elements of the straight teeth gear wheel, involute profile, and unmodified toothing

$$
\begin{aligned}
d_{a1} &= d_1 + 2h_a^* m = m z_1 + 2h_a^* m = m(z_1 + 2h_a^*)\,; \\
d_{a2} &= d_2 + 2h_a^* m = m z_2 + 2h_a^* m = m(z_2 + 2h_a^*)\,; \\
d_{b1} &= d_1 \cos\alpha = m z_1 \cdot \cos\alpha\,; \\
d_{b2} &= d_2 \cos\alpha = m z_2 \cdot \cos\alpha\,; \\
a_w &= a; \\
\alpha_w &= \alpha.
\end{aligned} \tag{2}
$$

According to ratio (1) the following result is being obtained:

$$
\varepsilon_\alpha = \frac{1}{2\pi}\left[\sqrt{\left(\frac{z_1+2h_a^*}{\cos\alpha}\right)^2 - z_1^2} + \sqrt{\left(\frac{z_2+2h_a^*}{\cos\alpha}\right)^2 - z_2^2} - (z_1+z_2)tg\alpha\right], \tag{3}
$$

where α is the pressure angle, z_1 and z_2 are the teeth numbers for the pinion and for the driven wheel, and ha* is the coefficient of the standard head.

Spur Gear with Straight Teeth and Modified Teeth

According to fig.3 and using the following ratios the following results are being obtained:

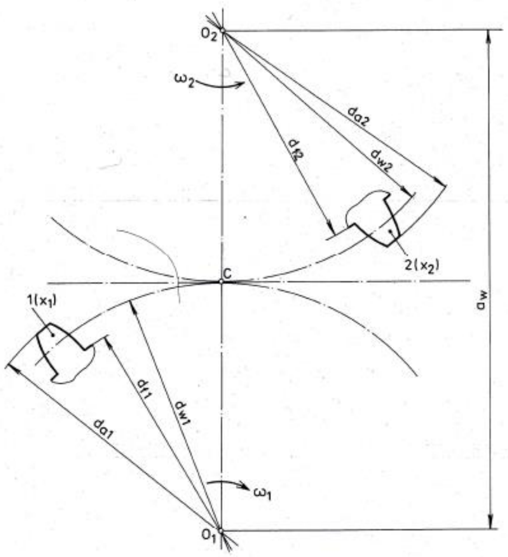

Fig.3. The diagram of some geometrical elements of the spur gear wheels with straight teeth, involute profile, and unmodified toothing

$$
\begin{aligned}
d_{a1} &= d_1 + 2h_a^* = m z_1 + 2(h_a^* + x_1)m\,; \\
d_{a2} &= d_2 + 2h_a^* = m z_2 + 2(h_a^* + x_2)m\,; \\
d_{b1,2} &= d_{1,2} \cdot \cos\alpha\,; \\
a_w &= a \cdot \frac{\cos\alpha}{\cos\alpha_w}\,;
\end{aligned} \tag{4}
$$

a being the distance between the reference axis.

The following result is being obtained:

$$\varepsilon_{\alpha w}=\frac{\sqrt{d_{a1}^2-d_{b1}^2}+\sqrt{d_{a2}^2-d_{b2}^2}-2a_w\sin\alpha_w}{2\pi m\cos\alpha}=$$

$$=\frac{\sqrt{\left[mz_1+2(h_a^*+x_1)m\right]^2-(mz_1\cos\alpha)^2}+\sqrt{\left[mz_2+2(h_a^*+x_2)m\right]^2-(mz_2\cos\alpha)^2}-2\cdot a\cdot\frac{\cos\alpha}{\cos\alpha_w}\cdot\sin\alpha_w}{2\pi m\cos\alpha}$$

$$\varepsilon_{\alpha w}=\frac{1}{2\pi}\left[\sqrt{\left(\frac{z_1+2(h_a^*+x_1)}{\cos\alpha}\right)^2-z_1^2}+\sqrt{\left(\frac{z_2+2(h_a^*+x_2)}{\cos\alpha}\right)^2-z_2^2}-(z_1+z_2)tg\,\alpha_w\right]. \quad (5)$$

In this ratio: x_1 and x_2 are the travel coefficients for the first and second wheel, aw is the pressure angle on the rolling cylinder.

$$\alpha_w=f(\alpha,x_1,x_2,z_1,z_2), \quad (6)$$

It is determined using ratio [1].

$$inv\alpha_w=inv\alpha+2\cdot\frac{x_1+x_2}{z_1+z_2}\cdot tg\alpha\,. \quad (7)$$

Spur Gear with Helical Teeth and Unmodified Toothing

In this situation we have:

$$\varepsilon_{\alpha t}=\frac{\sqrt{d_{a1}^2-d_{b1}^2}+\sqrt{d_{a2}^2-d_{b2}^2}-2a_w\cdot\sin\alpha_{tw}}{2\pi\, m_t\cos\alpha_t}\,. \quad (8)$$

Some geometrical elements of the wheels and of the spur gear with helical teeth, involute profile, and unmodified toothing. It is being determined according to fig. 4, using the following ratios.

The following result is being obtained:

$$d_{a1}=d_1+2h_{an}^*m_n=m_n\left(\frac{z_1}{\cos_\beta}+2h_{an}^*\right),$$

$$d_{a2}=d_2+2h_{an}^*m_n=m_n\left(\frac{z_2}{\cos\beta}+2h_{an}^*\right), \quad (9)$$

$$d_{b1,2}=d_{1,2}\cos\alpha_t=\frac{m_n}{\cos\beta}z_{1,2}\cdot\cos\alpha_t,$$

$$\varepsilon_{\alpha t}=\frac{\sqrt{m_n^2\left(\frac{z_1}{\cos\beta}+2h_{an}^*\right)^2-\frac{m_n^2}{\cos^2\beta}z_1^2\cdot\cos^2\alpha_t}+\sqrt{m_n^2\left(\frac{z_2}{\cos\beta}+2h_{an}^*\right)^2-\frac{m_n^2}{\cos^2\beta}z_2^2\cos^2\alpha_t}-2a\sin\alpha_t}{2\pi\frac{m_n}{\cos\beta}\cdot\cos\alpha_t};$$

$$\varepsilon_{\alpha t}=\frac{1}{2\pi}\left[\sqrt{\left(\frac{z_1+2h_{an}^*\cos\beta}{\cos\alpha_t}\right)^2-z_1^2}+\sqrt{\left(\frac{z_2+2h_{an}^*\cos\beta}{\cos\alpha_t}\right)^2-z_2^2}-(z_1+z_2)tg\,\alpha_t\right]$$

where β is the rake of division, and α_t is the direct pressure angle for division. These are determined using ration [2]:

$$tg\,\alpha_t = \frac{tg\alpha_n}{\cos\beta}. \tag{10}$$

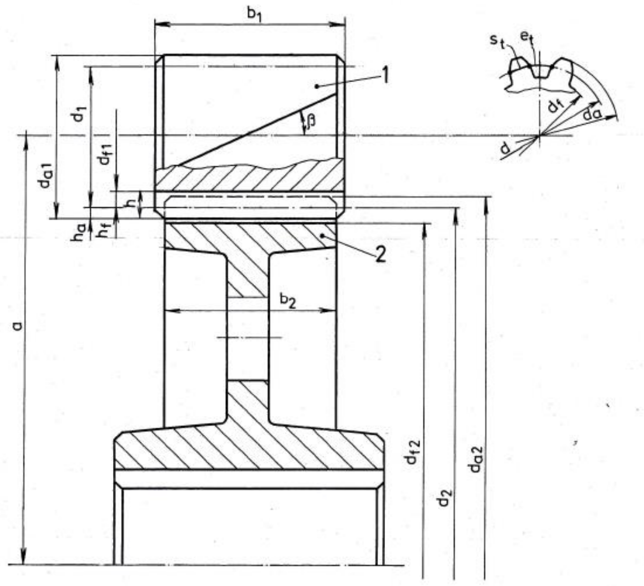

Fig.4. Some geometrical elements of the gear wheels with helical teeth, involute profile, and unmodified toothing

Spur Gear with Helical Teeth, and Modified Toothing

The ratio is the following (11) we have the following result:

$$\varepsilon_{\alpha tw} = \frac{\sqrt{d_{a1}^2 - d_{b1}^2} + \sqrt{d_{a2}^2 - d_{b2}^2} - 2a_w \sin\alpha_{tw}}{2\pi m_t \cos\alpha_t}, \tag{11}$$

where:

$$d_{a1} = d_1 + 2(h_{an}^* + x_{n1})m_n = \frac{m_n}{\cos\beta} z_1 + 2(h_{an}^* + x_{n1})m_n;$$

$$d_{a2} = d_2 + 2(h_{an}^* + x_{n2})m_n = \frac{m_n}{\cos\beta} z_2 + 2(h_{an}^* + x_{n2})m_n;$$

$$m_t = \frac{m_n}{\cos\beta}; \tag{12}$$

$$a_w = a \cdot \frac{\cos\alpha_t}{\cos\alpha_{tw}} = \frac{m_n}{2\cos\beta}(z_1 + z_2) \cdot \frac{\cos\alpha_t}{\cos\alpha_{tw}},$$

where:

$$inv\alpha_{tw} = inv\alpha_t + 2 \cdot \frac{x_{n1} + x_{n2}}{z_1 + z_2} \cdot tg\alpha_n, \tag{13}$$

and in (11):

$$\varepsilon_{\alpha tw}=\frac{1}{2\pi}\left[\sqrt{\left(\frac{z_1+2(h_{an}^*+x_{n1})\cos\beta}{\cos\alpha_t}\right)^2-z_1^2}+\sqrt{\left(\frac{z_2+2(h_{an}^*+x_{n2})\cos\beta}{\cos\alpha_t}\right)^2-z_2^2}-(z_1+z_2)tg\alpha_{tw}\right]. \quad (14)$$

Conclusions

This paper aims to present how the direct contact ratio is being determined. We refer here to the direct contact ratio for a spur gear with straight teeth, unmodified or modified toothing. Furthermore, we have extended our research in order to present how the direct contact ratio for a spur gear with helical teeth, modified or unmodified toothing is being determined. The ratios which we obtain are easily applied on object cases of spur gears which have a large applicability in the automotive engineering.

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Unele aspecte privind calculul gradului de acoperire frontal la angrenajele cilindrice

Rezumat

Definit ca raportul dintre lungimea arcului de angrenare si pasul pe cercul de rostogolire (de divizare), gradul de acoperire constituie un concept fundamental in calculul si constructia angrenajului. Concepută intr-o concepţie unitară si in acelasi timp generală, lucrarea prezintă intr-un mod riguros şi clar calculul gradului de acoperire frontal in cazul angrenajului cilindric cu dinţi drepţi, dantura nemodificată şi modificată. Lucrarea extinde calculul gradului de acoperire frontal şi pentru angrenajele cilindrice cu dinţi inclinaţi, dantura nemodificată şi dantura modificată.