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The Paris Formula for the Crack Propagation Rate, Approached as a Temperature and Asymmetry Coefficient Complex Function

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Abstract

The paper presents the propagation rate of the crack according to the Paris formula from a new point of view by taking into account the temperature ***T*** *variation and the asymmetry coefficient* ***R****. The fatigue loadings by axial-eccentric tensile test were made of CT specimens made by stainless steel V2A class, 10TiNiCr175 mark. The loading temperatures were: 213K, 253K and 273K, and the asymmetry factors were: R=0.1, R=0.3 and R=0.5. By using the Paris′ formula,* ***da/dN= C·(ΔK)*** $\mathbf{da/dN = C\cdot(\Delta K)^m}$ *with adequate mathematical programs,* ***C(R,T)*** *and* ***m(R,T)*** *parameters were determined as compound functions, using approximations by second degree parabolas. By simultaneously varying the loading temperature* ***T*** *and asymmetry factor* ***R****, there were drawn the variation surfaces, into a three orthogonal axes system, for the* ***C*** *and* ***m*** *parameters. Finally, with the* ***C*** *and* ***m*** *expressions, as functions of two variables (R and T), there was made a complex study for the propagation of the cracking speed* ***da/dN****, expressed by various drawn graphics.*

Key words: *crack, asymmetry coefficient, stress intensity factor, temperature, Paris formula*

Introduction

Inside certain products, there are defects due to products elaboration or their primal manufacturing. Sometimes, these can result through the respective elements long-term activity. The defects become micro-cracks primers that develop into the product, becoming cracks, with a particular propagation behavior. It is highlighted the notion of **crack growth rate** marked with d*a*/d*N* (or, sometimes, d*a*/d*t*), that represents a crack length increase during a loading cycle.

Regarding the study concepts from Fracture Mechanics, the existent stress state at a crack peak can be controlled by a complex quantity, that includes both the loading stress σ, and the defect length variation Δ*a*, and also the product shape or the loading type, and it is named **stress intensity factor** marked with *K*.

For a variable loading (at fatigue), it is marked with**:** $\Delta K = K_{max} - K_{min}$. Based on experimental testings and results methodical study, there have been set special analytics between the factor *K* and the cracking speed d*a*/d*N*, as: d*a*/d*N* = *f*(Δ*K*). One of the most used algebraic expressions is Paris formula, relation (1), where the coefficient ***C*** and the exponent *m* are material constants:

$$\frac{da}{dN} = C \cdot (\Delta K)^m \ . \tag{1}$$

The graphics, that show the da/dN cracking speed variation mode in relation to stress intensity factor (SIF) ΔK, are named **sigmoids**, drawn at bilogarithmically scale, [2]/p.222, [4]/p.205, [6]/p.37, [7]/p.42.

Paris formula best proves the second domain, that of defect stable propagation.

The Test

In order to achieve the suggested target, experimental tests have been made, applying on specimens a fatigue loading by oscillating tension [5, p.116]. The specimen type was CT model, with lateral slitting, [5, p.86], in Figure 1, manufactured with 10TiNiCr175 type stainless steel.

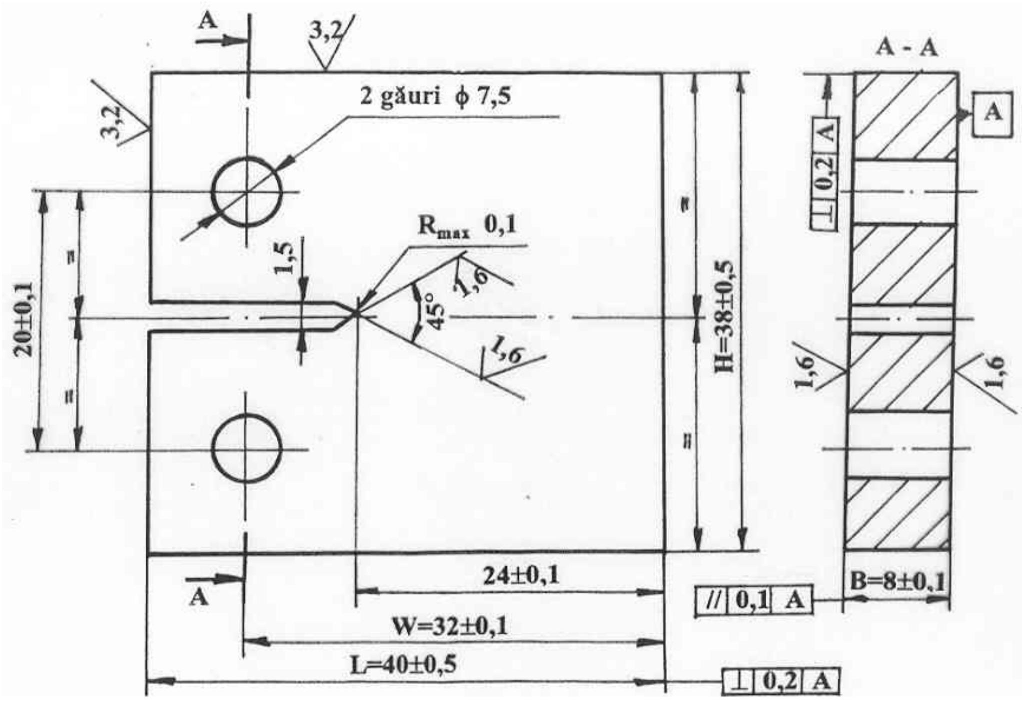

Fig. 1. CT specimen

The testing equipment consisted of SCHENK type plant, with hydro drive, in Figure 2, [5]/p.92, effectuating axial-eccentrical loading oscillatory positive cycles.

The loading factor was highlighted by three asymmetry factors: R=0.1, R=0.3 and R=0.5. The tests were made at room temperature (+20°C), and at low temperatures (-20°C and -60°C), namely: 293K, 253K and 213K. For negative temperatures, the machine was equipped with a refrigerating plant, Figure 2, using as cooling medium the liquid nitrogen (N_2L).

During testing, the a_i crack length variation was read at distances of 0.25 mm with an optical microscope, and also there was read the corresponding loading cycles number, N_i.

At low temperatures, for retaining the data, there was used the elastic compliance, [3], [5]/p.96, with the usage of lamella extensometer, Figure 3, [5]/p.97. The extensometer effectuates a mechanical quantities conversion in electrical parameters.

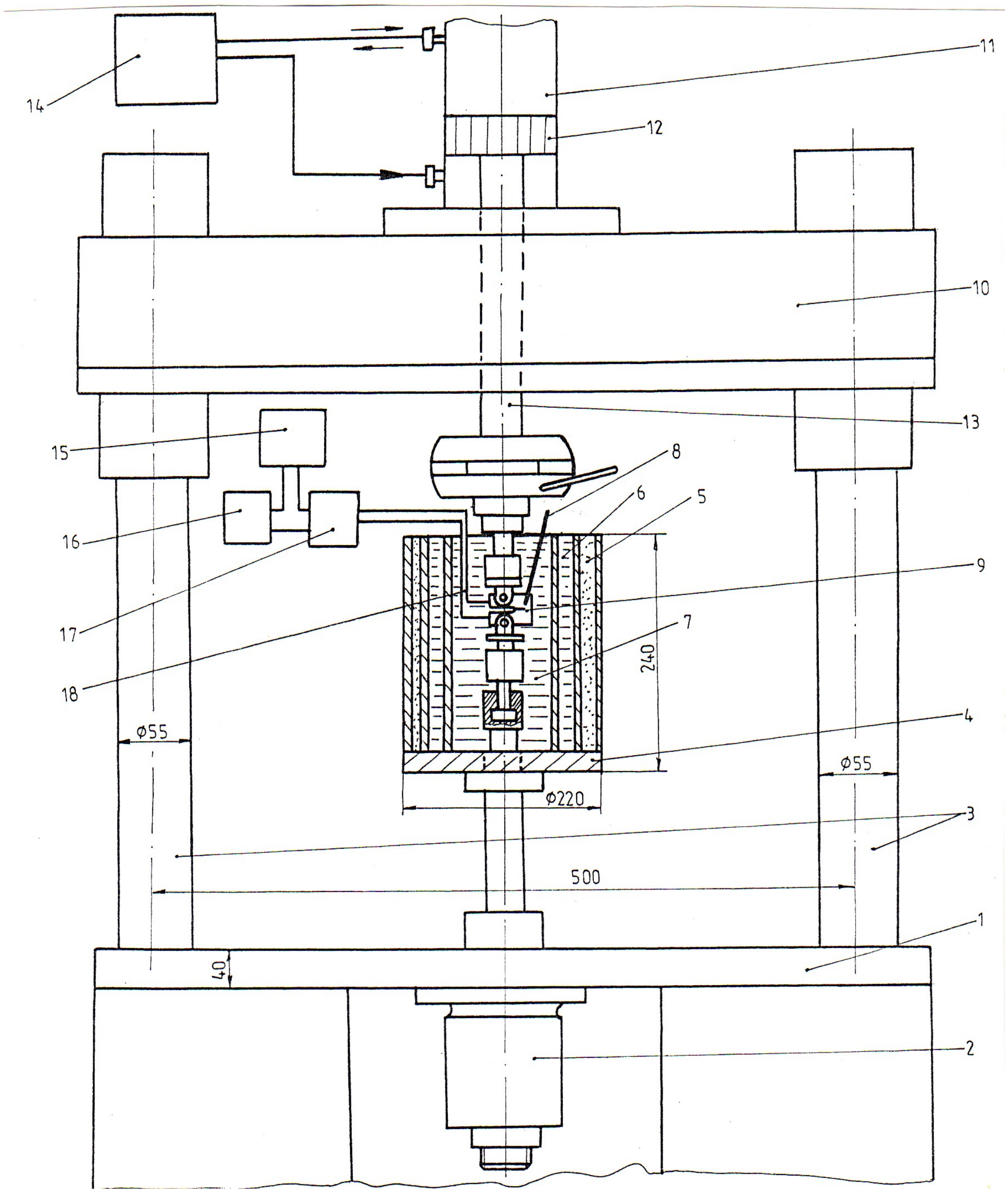

Fig. 2. Mechanical loadings equipment

The Study Methodology

From the experimental stage, for each loaded specimen, the primary data pairs (N_i, a_i) were retained. With these, a crack length approximation function was realized, in relation with the loading cycles number $a = a(N)$. By a statistical study, it was obtained the conclusion that the optimal approximation is given by a second degree function:

$$a(N) = a_2 + a_1 \cdot N + a_0 \cdot N^2 , \tag{2}$$

where the coefficients: a_0, a_1 and a_2 were determined from the approximation conditions. With the obtained relation, from a mathematical perspective, the crack propagation rate, da/dN, was computed:

$$\frac{da}{dN} = a_1 + 2a_0 \cdot N \,. \tag{3}$$

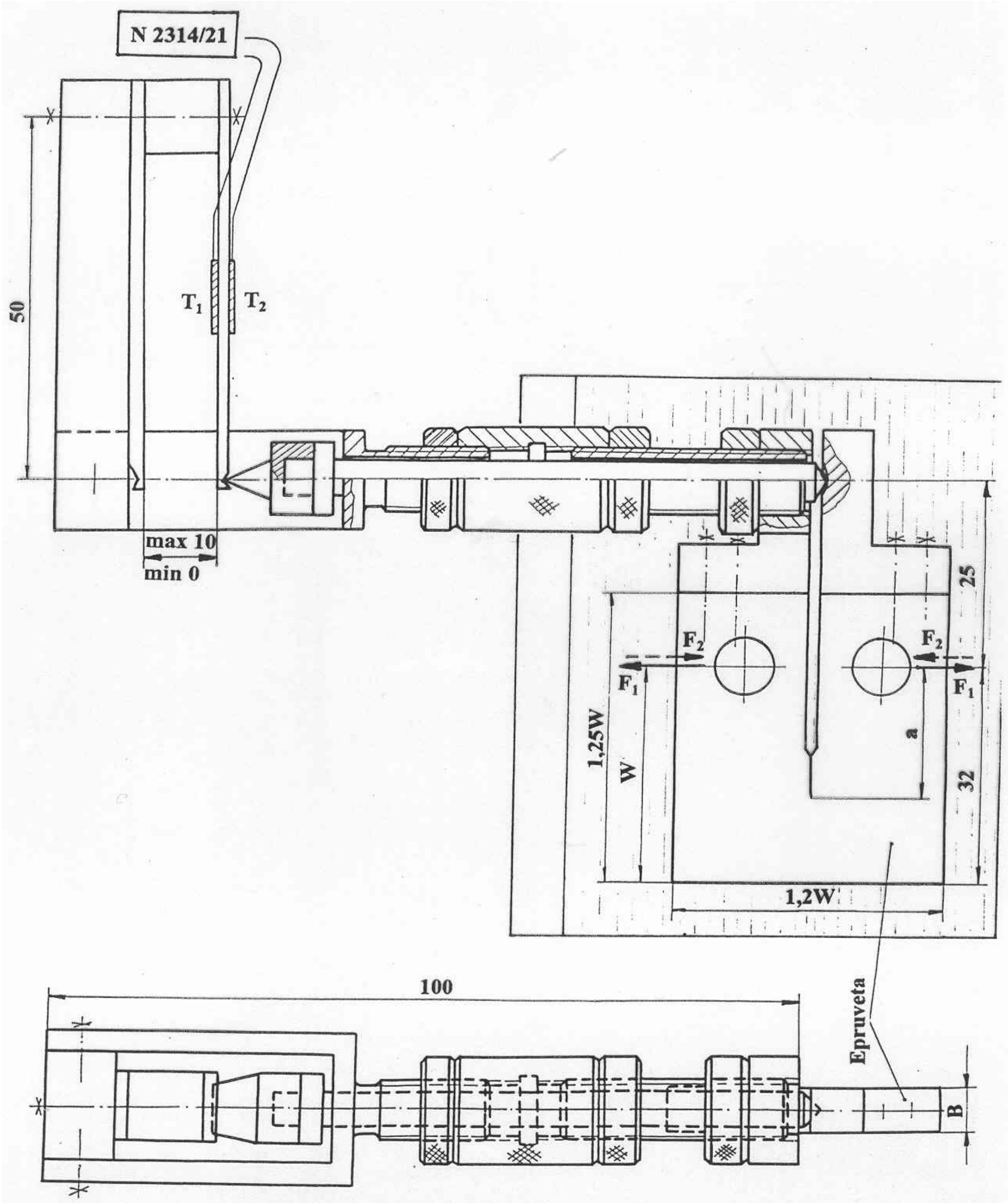

Fig. 3. Elastic lamella extensometer

Given the fact that the loaded specimens were CT model, and the loading was of eccentrical tensile type, with the experimental and computed data there were determined the tensile intensity factor variation ΔK, with the expression (4), [4]/p.66, [5]/p.83, [6]/p.36, [7]/p.146.

$$\Delta K = \frac{\Delta F}{B\sqrt{W}} \cdot \frac{2+\frac{a}{W}}{\left(1-\frac{a}{W}\right)^{\frac{3}{2}}} \cdot \left[-5{,}6 \cdot \left(\frac{a}{W}\right)^4 + 14{,}72\left(\frac{a}{W}\right)^3 - 13{,}32\left(\frac{a}{W}\right)^2 + 4{,}64\frac{a}{W} + 0{,}886\right], \tag{4}$$

where: $\Delta F = F_{max} - F_{min}$ is the loading forces variation, in N;

B – the specimen thickness, Figure 1, in mm;
W – the specimen active width, Figure 1, in mm;
T = *a/W* is the crack length relative variation.

After these calculi, another processed data category was obtained, highlighting the pairs (ΔK,d*a*/d*N*), calculated with relation (4), respectively the relation (3).

Numerical Processing

Using bilogarithmical scale, log(ΔK) and log(d*a*/d*N*), the curves d*a*/d*N* = *f*(ΔK) named sigmoids were drawn for all tested specimens, with asymmetry factors *R*=0.1, *R*=0.3 and *R*=0.5, at the temperatures *T*=213K, *T*=253K and *T*=293K.

C and *m* parameters determination is made through simple mathematical calculi, on the equality condition between relation (1) and relation (3):

$$C\cdot\left(\Delta K\right)^{m} = a_1 + 2a_0\cdot N \quad . \tag{5}$$

This equality is looked up the logarithm and it is obtained:

$$m\cdot log\left(\Delta K\right) + logC = \log\left(a_1 + 2a_0\cdot N\right) . \tag{6}$$

For each two points from the left member straight line, the system is solved and the *m* and *C* values are obtained. These are influenced both of the asymmetry factor *R*, and the testing temperature *T*, namely they are functions, as: *m*(*R*,*T*) and *C*(*R*,*T*). By a statistic calculus for the determined values, we have obtained that the optimal approximation for *R* and *T* variables is also a second degree parabola shape with variable coefficients, [5]/p.132, p.133.

For the **10TiNiCr175 steel** tested material, *m*(*R*,*T*) and *C*(*R*,*T*) expressions are:

$$\begin{aligned} m\left(R,T\right) = {} & \left(-68.61\cdot R^2 + 37.087\cdot R - 3.0813\right)\cdot 10^{-4}\cdot T^2 + \left(3.384\cdot R^2 - 1.807\cdot R + 0.152\right)\cdot T + \\ & + \left(-397\cdot R^2 + 208.6\cdot R - 14.75\right) \end{aligned} ,(7)$$

$$\begin{aligned} C\left(R,T\right) = {} & \left(-19.35\cdot R^2 + 17.98\cdot R - 1.4825\right)\cdot 10^{-19}\cdot T^2 + \left(28.499\cdot R^2 - 20.549\cdot R + 1.69\right)\cdot 10^{-16}\cdot T + \\ & + \left(-66.8\cdot R^2 + 44.69\cdot R - 3.665\right)\cdot 10^{-14} \end{aligned} . (8)$$

Using a Matlab program, it was established a variation domain for the coefficient *R* within the limits (0,0.6), respectively for the temperature *T* within the limits of 200K and 300K.

The *m* and *C* values were calculated, then there were drawn the surfaces for the exponent *m* and coefficient *C* in a (*R*, *T*, *m*) and (*R*, *T*, *C*) tri-rectangular system. The resulted graphics are presented in Figure 4 and Figure 5.

Conclusions

From the *m*(*R*,*T*) and *C*(*R*,*T*) surface plotting graphics analysis, Figure 4 and 5, some conclusions can be extracted:

- The *m* index varies between **2.32** and **3.92**. The minimum values are recorded around the **0.3** value for the *R* asymmetry factor, respectively around the *T* = 230K temperature, Figure 4. **Maximum values** are highlighted by *R* = 0.1 and *R* = 0.7 asymmetry factors, having an increase tendency for the **200K** minimum temperature and also for the $T \approx$ 300K ambient temperature, Figure 4. In the general relation some customizations were made, obtaining the

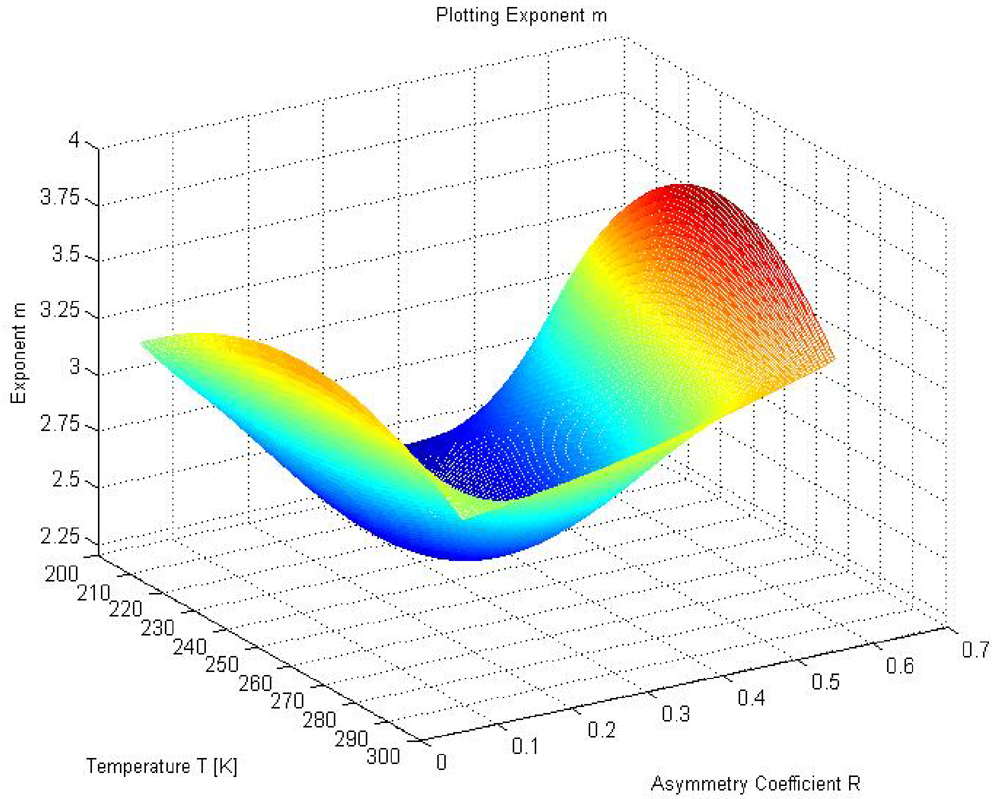

Fig. 4. The $m(R,T)$ surface

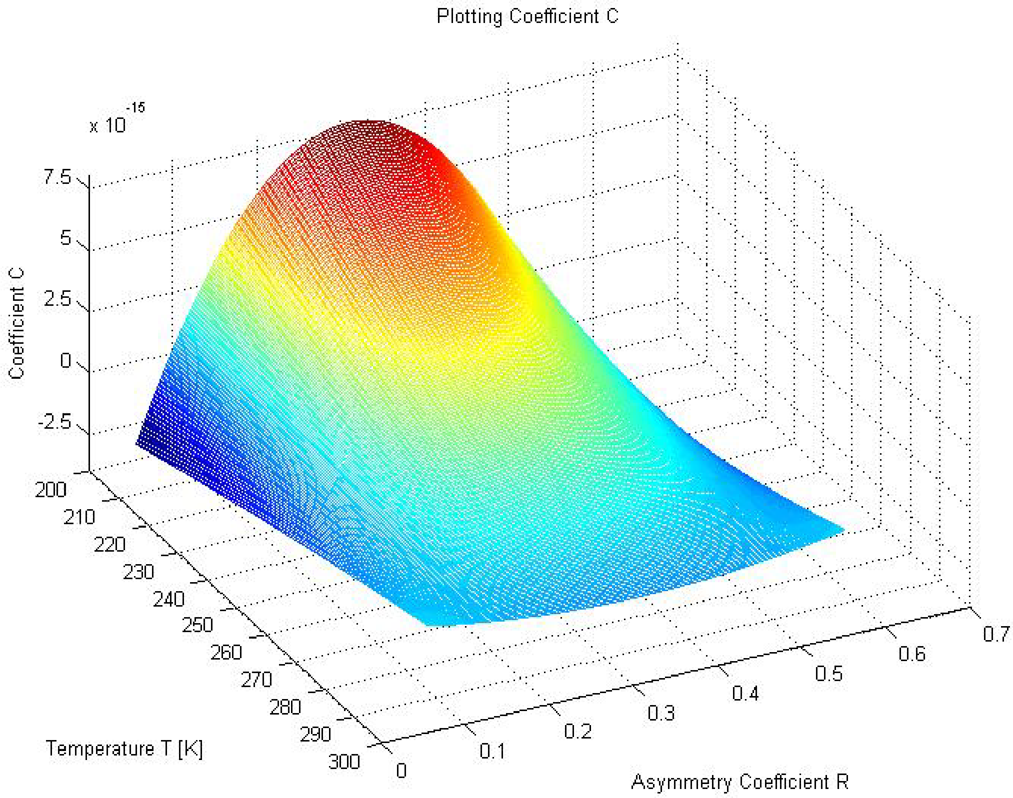

Fig. 5. The $C(R,T)$ surface

values $m(0.3, 253) = 2.43$, $m(0.1, 293) = 3.14$, respectively $m(0.5, 213) = 2.76$.

These are close to the ones obtained with Paris formula.

- Referring to the material factor **C**, the variation limits are obtained by the domain extrapolation between $\mathbf{-3.3592\cdot10^{-15}}$ and $\mathbf{7.74\cdot10^{-15}}$. $C(R,T)$ surface has a maximum for R=0.3 and $T \approx 210$K, Figure 5. For R tending to **0.1**, respectively to **0.6**, the *C* **factor** decreases, but the *T* temperature increase to the ambient one leads to a decrease of *C* values.

  By customization, in this case we also obtain the next values: $C(0.3, 213) = 6.4665\cdot10^{-15}$, $C(0.1, 293) = 0.66\cdot10^{-15}$ and $C(0.5, 253) = 2.114\cdot10^{-15}$.

  These values are also comparable to the ones obtained with Paris formula.

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Formula lui Paris pentru viteza de propagare a fisurii, abordare ca o funcţie complexă de temperatură şi coeficient de asimetrie

Rezumat

Lucrarea îşi propune studiul vitezei de propagare a fisurii conform relaţiei lui Paris printr-o abordare nouă a analizei ţinând seama simultan de variaţia temperaturii ***T*** *şi a coeficientului de asimetrie* ***R****. Încercările de oboseală prin întindere axial- excentrică au fost efectuate pe epruvete tip CT executate din oţel inoxidabil clasa V2A, marca 10TiNiCr175. Temperaturile de încercare au fost 213K, 253K şi 293K, iar coeficienţii de asimetrie ai solicitării: R=0.1, R=0.3 şi R=0.5. Folosind formula lui Paris,* $\mathbf{da/dN=C\cdot(\Delta K)^m}$*, cu programe matematice adecvate, s-au determinat parametrii* ***C(R,T)*** *şi* ***m(R,T)****, ca funcţii compuse, utilizând aproximarea prin parabole de gradul al doilea. Variind simultan temperatura de încercare* ***T*** *şi coeficientul de asimetrie* ***R****, au fost trasate suprafeţele de variaţie, într-un sistem de axe triortogonale, pentru parametrii* ***C*** *şi* ***m****. În final, cu expresiile pentru C şi m, ca funcţii de două variabile, (R şi T), s-a făcut un studiu amplu pentru viteza de propagare a fisurii* ***da/dN****, materializat prin diverse grafice trasate.*