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On the Differential Model of a Mitsubishi Robot

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Abstract

The paper analyses the differential model of the Mitsubishi RV-1A robot system. The steps for the calculus of the preferential Jacobean matrix corresponding to the Mitsubishi robot mechanism are emphasized. Finally, the expression of the preferential Jacobean matrix is presented.

Key words: *robot, differential model, preferential Jacobean matrix*

Introduction

The differential model of a robotic mechanism makes the connection between the variation of the operational coordinates and the variation of the generalized coordinates of the robot. This connection is realized using the Jacobean matrix [1,2]. The calculus of the Jacobean matrix for a robot mechanism with *n* modules is based on the calculus of the base Jacobean matrix J_n which verifies the relation [2]:

$$\begin{bmatrix} d_n \\ \delta_n \end{bmatrix} = J_n \cdot \mathrm{d}q \tag{1}$$

where: d_n and δ_n are the differential translation and rotation vectors corresponding to the system of coordinates attached to the last module of the robot and $\mathrm{d}q$ is the vector that contains the differential variations of generalized coordinates of the robot mechanism. The base Jacobean matrix can be decomposed in three matrices [2]: the first two matrices are of full rank and their inverses are immediate and the third matrix (called the preferential Jacobean matrix) has a very simple form so that its inverse can be very easy calculated (which ensures a rapid use in the control process). In the paper the steps for the calculus of the preferential Jacobean matrix corresponding to the Mitsubishi RV-1A robot mechanism are emphasized. Finally, the expression of the preferential Jacobean matrix is presented.

Theoretical Considerations and Simulation Results

In figure 1, the cinematic scheme of the mechanism of the Mitsubishi RV-1A robot system is presented. The systems of coordinates $(O_i x_i y_i z_i), i = \overline{0,6}$, have been attached to each component module *i*, $i = \overline{0,6}$, using the Khalil-Kleinfinger method [1,2].

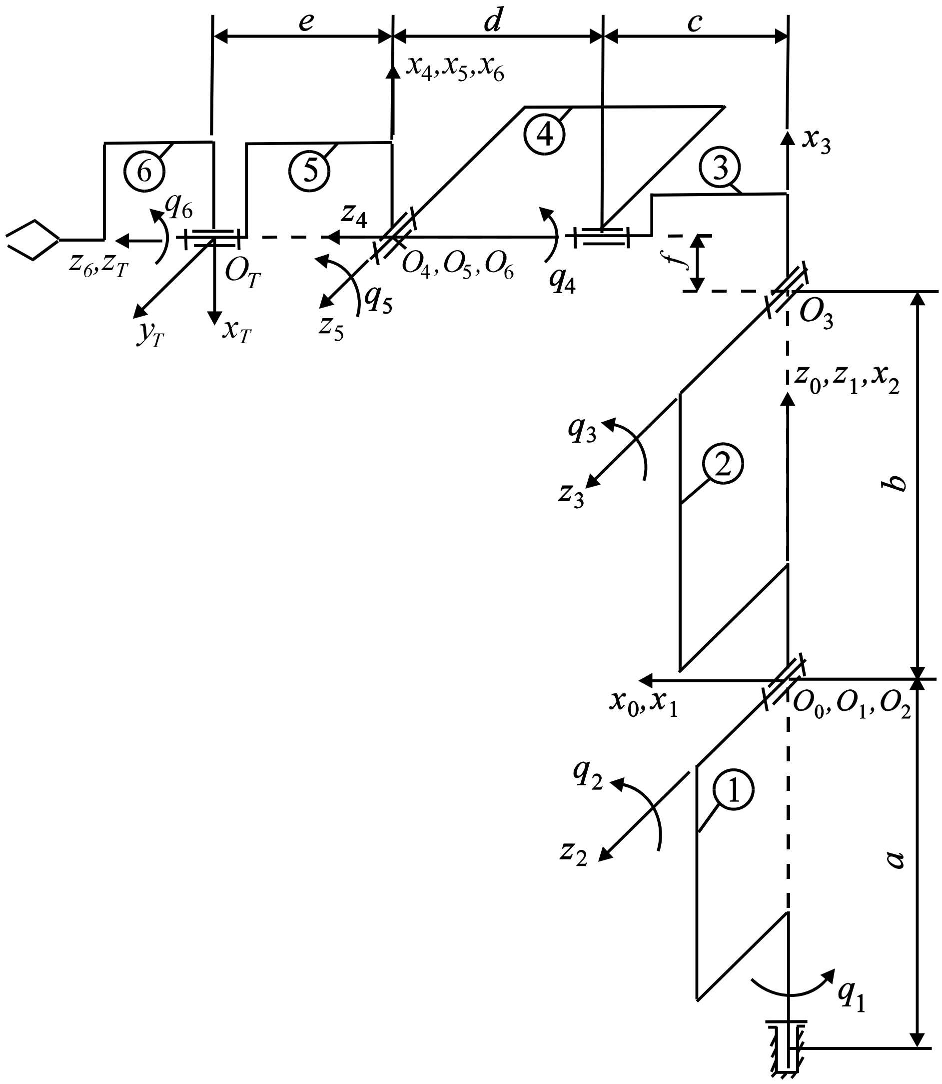

Fig. 1. Mitsubishi Melfa RV-1A robot mechanism

In figure 2 the parameters of this method are presented. The values corresponding to the Khalil-Kleinfinger parameters are given in table 1.

Table 1. The values of the Khalil-Kleinfinger parameters

i	α_i	d_i	θ_i	r_i
1	0	0	q_1	0
2	-90°	0	q_2	0
3	0	b	q_3	0
4	-90°	f	q_4	$d+c$
5	90°	0	q_5	0
6	-90°	0	q_6	0

The systems of coordinates attached to the consecutive modules i+1 and i are relatively positioned using four parameters [1,2] (fig. 2): the angle α_{i+1} between the axes $(O_i z_i)$ and $(O_{i+1} z_{i+1})$, the distance d_{i+1} between the same axes, the angle θ_{i+1} between the axes $(O_i x_i)$ and $(O_{i+1} x_{i+1})$ and the distance r_{i+1} between $(O_i x_i)$ and $(O_{i+1} x_{i+1})$, measured on the positive direction of the axis $(O_{i+1} z_{i+1})$.

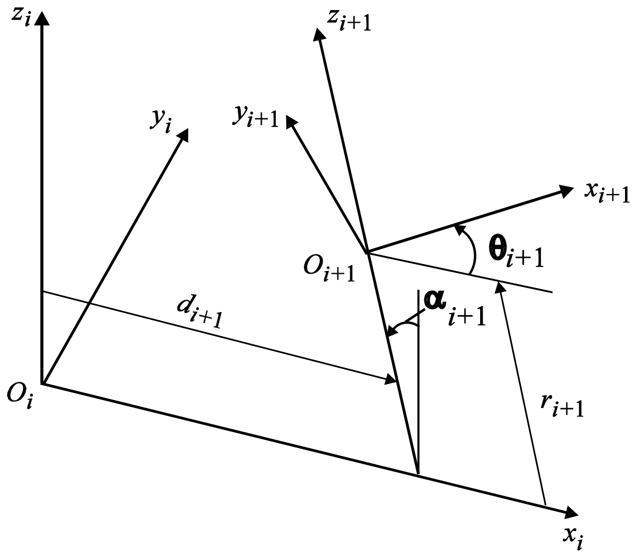

Fig. 2. Khalil-Kleinfinger parameters

The homogeneous matrix corresponding to the relative position and orientation of two consecutive modules i and i+1 has the following general form [1,2]:

$$ {}^{i}T_{i+1} = \begin{bmatrix} {}^{i}R_{i+1} & {}^{(i)}O_iO_{i+1} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_{i+1} & -\sin\theta_{i+1} & 0 & d_{i+1} \\ \cos\alpha_{i+1}\cdot\sin\theta_{i+1} & \cos\alpha_{i+1}\cdot\cos\theta_{i+1} & -\sin\alpha_{i+1} & -r_{i+1}\cdot\sin\alpha_{i+1} \\ \sin\alpha_{i+1}\cdot\sin\theta_{i+1} & \sin\alpha_{i+1}\cdot\cos\theta_{i+1} & \cos\alpha_{i+1} & r_{i+1}\cdot\cos\alpha_{i+1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

where: ${}^{i}R_{i+1}$ is the rotation matrix corresponding to the relative orientation of the modules.

By applying the relation (2) and by taking into account the values of the Khalil-Kleinfinger parameters in table 1, the following expressions for the homogeneous matrices ${}^{i}T_{i+1}, i = \overline{0,5}$, are obtained:

$$ {}^{0}T_1 = \begin{bmatrix} c1 & -s1 & 0 & 0 \\ s1 & c1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

$$ {}^{1}T_2 = \begin{bmatrix} c2 & -s2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s2 & -c2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

$$ {}^{2}T_{3} = \begin{bmatrix} c3 & -s3 & 0 & b \\ s3 & c3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5} $$

$$ {}^{3}T_{4} = \begin{bmatrix} c4 & -s4 & 0 & f \\ 0 & 0 & 1 & d+c \\ -s4 & -c4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6} $$

$$ {}^{4}T_{5} = \begin{bmatrix} c5 & -s5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s5 & c5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7} $$

$$ {}^{5}T_{6} = \begin{bmatrix} c6 & -s6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s6 & -c6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8} $$

where:

$$ \begin{cases} si = \sin q_i \\ ci = \cos q_i \end{cases} \quad i = \overline{1,6} \tag{9} $$

For a robotic mechanism with *n* modules the base Jacobean matrix J_n, projected on the system of coordinates attached to the last module is given by the following relation [2]:

$$ {}^{(n)}J_n = \begin{bmatrix} {}^{n}R_i & 0 \\ 0 & {}^{n}R_i \end{bmatrix} \cdot \begin{bmatrix} I_3 & -{}^{i}R_j \cdot ({}^{(j)}O_jO_n)^v \cdot {}^{j}R_i \\ 0 & I_3 \end{bmatrix} . {}^{(i)}J_j \tag{10} $$

where: I_3 is the identity matrix of rank three; $j = [n/2]+1$; $i = [n/2]$; ${}^{(i)}J_j$ is the preferential Jacobean matrix; the matrix $({}^{(j)}O_jO_n)^v$ is obtained by taking into account the following rule: if $f = \begin{bmatrix} f_x & f_y & f_z \end{bmatrix}^T$ is a vector, then:

$$ f^v = \begin{bmatrix} 0 & -f_z & f_y \\ f_z & 0 & -f_x \\ -f_y & f_x & 0 \end{bmatrix} \tag{11} $$

In the case of the Mitsubishi RV-1A robot mechanism: $n = 6$; $i = 3$; $j = 4$ and $O_4O_6 = 0$ (fig. 1), so that the relation (10) becomes:

$$ {}^{(6)}J_6 = \begin{bmatrix} {}^{6}R_3 & 0 \\ 0 & {}^{6}R_3 \end{bmatrix} . {}^{(3)}J_4 \tag{12} $$

The preferential Jacobean matrix ${}^{(i)}J_j$, when the robotic system has only rotation modules, has the following expression [2]:

$$ {}^{(i)}J_j = \left[\begin{array}{cc|c} {}^iR_1 \cdot {}^{(1)}k_1^v \cdot {}^{(1)}O_1O_j & {}^iR_2 \cdot {}^{(2)}k_2^v \cdot {}^{(2)}O_2O_j & {}^iR_n \cdot {}^{(n)}k_n^v \cdot {}^{(n)}O_nO_j \\ {}^iR_1 \cdot {}^{(1)}k_1 & {}^iR_2 \cdot {}^{(2)}k_2 & {}^iR_n \cdot {}^{(n)}k_n \end{array} \right] \quad (13) $$

where: ${}^{(l)}k_l = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$ is the unit vector corresponding to the z_l axis of the system of coordinates attached to the l module of the robot and the matrix ${}^{(l)}k_l^v$ is obtained by taking into account the rule explained using the relation (11).

In the case of the Mitsubishi RV-1A robot mechanism the relation (13) becomes:

$$ {}^{(3)}J_4 = \begin{bmatrix} {}^3R_1 \cdot {}^{(1)}k_1^v \cdot {}^{(1)}O_1O_4 & {}^3R_2 \cdot {}^{(2)}k_2^v \cdot {}^{(2)}O_2O_4 & {}^{(3)}k_3^v \cdot {}^{(3)}O_3O_4 & 0 & {}^3R_5 \cdot {}^{(5)}k_5^v \cdot {}^{(5)}O_5O_4 & {}^3R_6 \cdot {}^{(6)}k_6^v \cdot {}^{(6)}O_6O_4 \\ {}^3R_1 \cdot {}^{(1)}k_1 & {}^3R_2 \cdot {}^{(2)}k_2 & {}^{(3)}k_3 & {}^3R_4 \cdot {}^{(4)}k_4 & {}^3R_5 \cdot {}^{(5)}k_5 & {}^3R_6 \cdot {}^{(6)}k_6 \end{bmatrix} (14) $$

where:

$$ \begin{cases} {}^3R_1 = ({}^1R_3)^T = ({}^1R_2 \cdot {}^2R_3)^T \\ {}^3R_2 = ({}^2R_3)^T \\ {}^3R_5 = {}^3R_4 \cdot {}^4R_5 \\ {}^3R_6 = {}^3R_5 \cdot {}^5R_6 \end{cases} \quad (15) $$

The vectors: ${}^{(j)}O_jO_4, j = 1,2,3,5,6$; can be found on the last column of the matrices: ${}^jT_4, j = 1,2,3,5,6$; where:

$$ \begin{cases} {}^1T_4 = {}^1T_2 \cdot {}^2T_3 \cdot {}^3T_4 \\ {}^2T_4 = {}^2T_3 \cdot {}^3T_4 \\ {}^5T_4 = ({}^4T_5)^{-1} \\ {}^6T_4 = ({}^4T_5 \cdot {}^5T_6)^{-1} \end{cases} \quad (16) $$

By transposing the relations above in a computer program the following expression for the preferential Jacobean matrix of the Mitsubishi RV-1A robot has been obtained:

$$ {}^3J_4 = \begin{bmatrix} 0 & -d-c+s3\cdot b & -d-c & 0 & 0 & 0 \\ 0 & f+c3\cdot b & f & 0 & 0 & 0 \\ c23\cdot f - s23\cdot(d+c) + c2\cdot b & 0 & 0 & 0 & 0 & 0 \\ -s23 & 0 & 0 & 0 & s4 & c4\cdot s5 \\ -c23 & 0 & 0 & 1 & 0 & c5 \\ 0 & 1 & 1 & 0 & c4 & s4\cdot s5 \end{bmatrix} \quad (17) $$

where:

$$ \begin{cases} s23 = \sin(q_2 + q_3) \\ c23 = \cos(q_2 + q_3) \end{cases} \quad (18) $$

Conclusions

In this paper the differential model of the Mitsubishi RV-1A robot system is analyzed. The steps for the calculus of the base Jacobean matrix and of the preferential Jacobean matrix corresponding to the Mitsubishi robot mechanism are emphasized. Khalil-Kleinfinger

parameters have been used for obtaining the relative position and orientation between the component modules of the robot. Finally, the expression of the preferential Jacobean matrix is presented.

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Asupra modelului diferențial al unui robot Mitsubishi

Rezumat

Articolul analizează modelul diferențial al sistemului robot Mitsubishi RV-1A. Sunt evidențiate etapele de calcul ale matricei Jacobiene preferențiale corespunzătoare mecanismului robotului Mitsubishi. În final este prezentată expresia matricei Jacobiene preferențiale.