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Research Concerning the Functional Constructive Optimization of the Sucker Rod Pumping Units

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Abstract

In this paper are presented some results concerning the optimum choice of the dimensions of the component elements of the mechanism of the sucker rod pumping units with conventional type geometry when it is imposed the reduction with a certain percentage of the extreme values of the acceleration at the polished rod. The optimization problem is solved by considering unchanged the value of the stroke of the pumping unit. A simulation computer program that uses the Optimization package in Maple program has been developed. Some simulation results obtained in the case of a C-640D-365-144 pumping unit are presented.

Key words: *pumping unit mechanism, acceleration, optimization*

Introduction

It is well known that most of oil production is obtained by pumping, the sucker rod pumping installations being the most used pumping systems for the wells that can not ensure an eruptive exploitation [1]. The prediction with a greater degree of precision of the performances of these installations during operation is extremely difficult due to complex interactions that exist between components [2, 3, 4]. Therefore the development of mathematical models that simulates accurately the dynamics of the sucker rod pumping installations can lead to the optimization of the working processes and production costs [5, 6].

The paper presents some results concerning the functional constructive optimization of the quadrilateral mechanism of the sucker rod pumping units. Some interesting results concerning the cinematic and dynamic analysis, synthesis and optimization of the quadrilateral mechanism are presented in [7, 8, 9].

In this paper is analyzed the optimum choice of the dimensions of the component elements of the mechanism of the pumping units with conventional type geometry when it is imposed the reduction with a certain percentage of the extreme values of the acceleration at the polished rod. The kinematic analysis of the pumping unit mechanism is realized using the method of the projection of the independent and closed vectorial contours [10, 11, 12, 13]. The optimization problem is solved by considering unchanged the value of the stroke of the pumping unit. A simulation computer program that uses the *Optimization* package in *Maple* [14] program has been developed. Some simulation results obtained in the case of a C-640D-365-144 pumping unit are presented.

Theoretical Considerations and Simulation Results

In Figure 1 the cinematic scheme of the mechanism of a sucker rod pumping unit with conventional type geometry is presented. There are also represented the extreme positions of the mechanism, corresponding to beginning and to the end of the upward movement of the polished rod. C_1, C_2 and C_3 are the mass centers of the cranks, connecting rods and of the rocker, respectively.

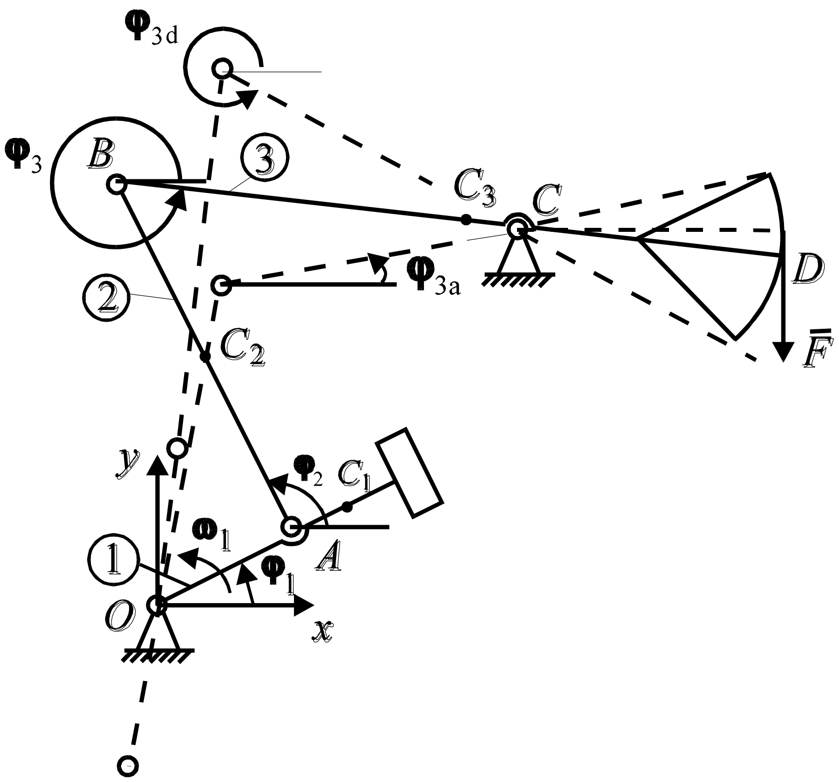

Fig. 1. Cinematic scheme of a pumping unit mechanism with conventional type geometry with the extreme positions corresponding to beginning and to the end of the upward movement of the polished rod

The kinematic analysis of the quadrilateral mechanism of the pumping unit is realized using the method of the projection of the independent and closed vectorial contours [10].

The values of the angles φ_2 and φ_3 (fig. 1) have been calculated from the following systems of equations obtained by projecting the contour $O-A-B-C-O$ on the *x* and *y* axes [3, 4]:

$$\begin{cases} l_1 \cdot \cos\varphi_1 + l_2 \cdot \cos\varphi_2 + l_3 \cdot \cos\varphi_3 - x_C = 0 \\ l_1 \cdot \sin\varphi_1 + l_2 \cdot \sin\varphi_2 + l_3 \cdot \sin\varphi_3 - y_C = 0 \end{cases} \quad (1)$$

where: $l_1 = OA$; $l_2 = AB$; $l_3 = BC$.

Then, the angular speeds and accelerations: $\omega_j, \varepsilon_j, j = 2,3$, of the links *2* and *3* have been calculated by deriving with time the variation functions corresponding to the angles φ_2 and φ_3 with the following relations [4]:

$$\omega_j = \dot{\varphi}_j = \frac{d\varphi_j}{d\varphi_1} \cdot \frac{d\varphi_1}{dt} = \omega_1 \cdot \frac{d\varphi_j}{d\varphi_1}; \quad j = 2,3 \tag{2}$$

$$\varepsilon_j = \ddot{\varphi}_j = \varepsilon_1 \cdot \frac{d\varphi_j}{d\varphi_1} + \omega_1^2 \cdot \frac{d^2\varphi_j}{d\varphi_1^2}; \quad j = 2,3 \tag{3}$$

where: ω_1 and ε_1 are the angular speed and the angular acceleration of the cranks.

The speed v_D and the acceleration a_D (fig. 1) of the end of the polished rod can be calculated with the following relations: $v_D = \omega_3 \cdot l_{3p}$; $a_D = \varepsilon_3 \cdot l_{3p}$, where $l_{3p} = CD$. The speed and the acceleration of the other points on the pumping unit mechanism can be calculated by applying *Euler* formula and *Rivals* formula, respectively [4, 7].

Further, it is analyzed the problem of the optimum choice of the dimensions of the component elements of the mechanism of the pumping units with conventional type geometry when it is imposed the reduction with a certain percentage of the extreme values of the acceleration a_D, by maintaining unchanged the value of the stroke.

The value of the stroke s_D of the pumping units with conventional type geometry can be calculated with the following relation [2]:

$$s_D = (2\pi - \varphi_{3d} + \varphi_{3a}) \cdot l_{3p} \tag{4}$$

where: φ_{3d} and φ_{3a} are the values of the angle φ_3 for the extreme positions of the rocker of the mechanism (fig. 1).

The values of the angles φ_{3d} and φ_{3a} and of the angles φ_{1d} and φ_{1a} corresponding to the crank angle φ_1 for the extreme positions of the rocker can be calculated from the following systems of equations obtained by projecting the contour $O-A-B-C-O$ on the *x* and *y* axes (fig. 1) for these two extreme positions of the rocker [2]:

$$\begin{cases} (l_1 + l_2) \cdot \cos\varphi_{1d} + l_3 \cdot \cos\varphi_{3d} - x_C = 0 \\ (l_1 + l_2) \cdot \sin\varphi_{1d} + l_3 \cdot \sin\varphi_{3d} - y_C = 0 \end{cases} \tag{5}$$

$$\begin{cases} (l_1 - l_2) \cdot \cos\varphi_{1a} + l_3 \cdot \cos\varphi_{3a} - x_C = 0 \\ (l_1 - l_2) \cdot \sin\varphi_{1a} + l_3 \cdot \sin\varphi_{3a} - y_C = 0 \end{cases} \tag{6}$$

The relations above have been transposed into a computer program using *Maple* programming language [14]. For solving the optimization problem the function *NLPSolve* included in *Optimization* package has been used.

There are presented below some results of the simulations performed on a pumping unit C-640D-365-144. The geometric elements corresponding to this pumping unit are presented in table 1.

Table 1. Geometric elements of a C-640D-365-144 pumping unit

l_1 [m]	l_2 [m]	l_3 [m]	l_{3p} [m]	x_C [m]	y_C [m]	φ_{1d} [rad]	s_D [m]
1.19	3.72	3.05	4.55	3.05	3.72	1.522	3,657

The variation on a cinematic cycle of a_D / ω_1^2 beginning with the value of the crank angle φ_{1d}, corresponding to the starting of the upward movement of the polished rod, is presented in fig. 2.

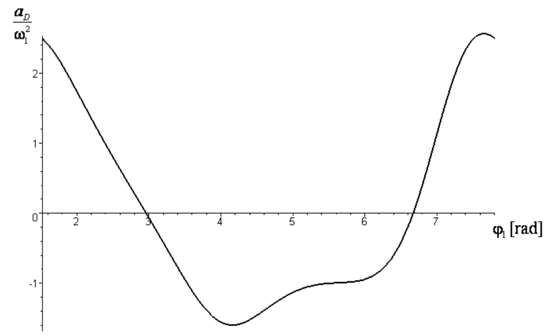

Fig. 2. The variation on a cinematic cycle of a_D / ω_1^2 for a C-640D-365-144 pumping unit

The extreme values for a_D / ω_1^2 are: -1.6 m at $\varphi_1 = 4.15\,\text{rad}$ and 2.58 m at $\varphi_1 = 7.65\,\text{rad}$.

By applying *NLPSolve* function it was aimed to determine the dimensions of the component elements of the analyzed pumping unit which lead to minimizing the next function:

$$F_{acc}(l_1, l_2, l_3, l_{3p}, x_C, y_C) = \left(\left(\frac{a_D}{\omega_1^2}\right)_{\varphi_1=7.65} - k_1\right)^2 + \left(\left(\frac{a_D}{\omega_1^2}\right)_{\varphi_1=4.15} - k_2\right)^2 \qquad (7)$$

Further, there are presented some simulations results when: $k_1 = 0.8 \cdot 2.58$ and $k_2 = 1.15 \cdot (-1.6)$, which means a decrease of 20% of the maximum value of a_D / ω_1^2 and a decrease of 15% of its minimum value. The following constraints have been considered by applying *NLPSolve* function: the values of $l_1, l_2, l_3, l_{3p}, x_C$ and y_C can vary by ± 15% of their initial values; maintaining unchanged the value of the stroke s_D; the solution must verify *Grashof* condition: $l_{\min} + l_{\max} \le p$, where: $p = (l_1 + l_2 + l_3 + l_0)/2$; $l_0 = \sqrt{x_C^2 + y_C^2}$ and $l_{\min}$ and $l_{\max}$ represent the minimum and the maximum value, respectively, of l_1, l_2, l_3 and l_0.

The following results have been obtained after simulation: $l_1 = 1.149\,\text{m}$; $l_2 = 4.278\,\text{m}$; $l_3 = 2.783\,\text{m}$; $l_{3p} = 4.174\,\text{m}$; $x_C = 3.507\,\text{m}$ and $y_C = 4.124\,\text{m}$.

In figures 3 and 4 there are represented the variation curves on a cinematic cycle (beginning with the value of the crank angle φ_{1d}) of the displacement *s* of the end of the polished rod and of a_D / ω_1^2 for the initial dimensions of the elements and for the dimensions obtained after optimization. For the new dimensions of the elements, the angle φ_{1d} is equal to 1.385 rad.

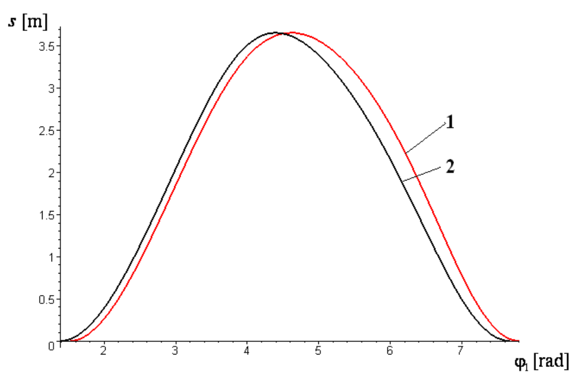

Fig. 3. The variation on a cinematic cycle of the displacement of the end of the polished rod for the initial dimensions of the elements (curve *1*) and for the dimensions obtained after optimization (curve *2*)

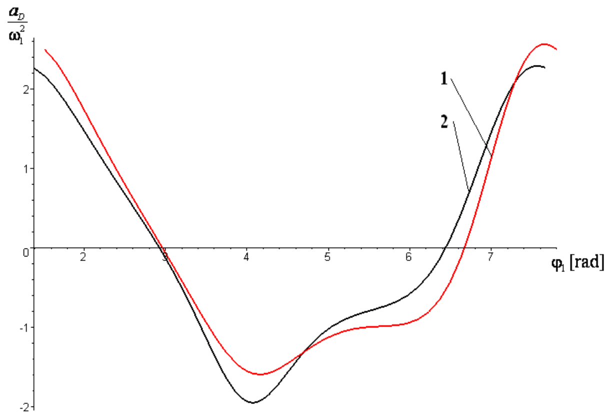

Fig. 4. The variation on a cinematic cycle of a_D / ω_1^2 for the initial dimensions of the elements (curve *1*) and for the dimensions obtained after optimization (curve *2*)

Fig. 3 shows that the condition of maintaining unchanged the value of the stroke have been respected and in addition it was kept the same aspect of the variation on a cinematic cycle of the displacement of the end of the polished rod.

From fig. 4 it results that the extreme values of a_D / ω_1^2 obtained after optimization are: -1.95 m at $\varphi_1 = 4.06\,\text{rad}$ and 2.28 m at $\varphi_1 = 7.56\,\text{rad}$. Thus after optimization it was obtained a decrease of the maximum value with aproximative 12% and a decrease of the minimum value with aproximative 21%.

Conclusions

In this paper has been presented some results concerning the functional constructive optimization of the mechanism of the sucker rod pumping units with conventional type geometry. It was analyzed the optimum choice of the dimensions of the component elements of this mechanism when it is imposed the reduction with a certain percentage of the extreme values of the acceleration at the polished rod that has an important positive effect on the variation of the force at the end of the polished rod. The optimization problem has been solved by considering unchanged the value of the stroke of the pumping unit. Some interesting simulation results have been presented in the case of a C-640D-365-144 pumping unit.

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Cercetări privind optimizarea constructiv funcţională a unităţilor de pompare cu prăjini

Rezumat

In articol sunt prezentate o serie de rezultate privind alegerea optimă a dimensiunilor elementelor componente ale mecanismului unităţilor de pompare cu prăjini cu geometrie convenţională când se impune reducerea cu un anumit procent a valorilor extreme ale acceleraţiei la prăjina lustruită. Problema de optimizare este rezolvată considerând nemodificată valoarea cursei unităţii de pompare. A fost realizat un program de calculator care utilizează pachetul Optimization din programul Maple. Sunt prezentate o serie de rezultate ale simulărilor obţinute în cazul unei unităţi de pompare C-640D-365-144.