

# Reliability Model for Describing the Knowledge on the Failure Behavior of Tested and/or Diagnosed Systems Components

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## Abstract

*The present paper describes a reliability model, designed for reliability testing and or fault diagnosis of systems components. The reliability model takes into account the components failures and their interactions with the incorrect failure detection and recovery (repair) processes. Since the models parameters are not directly observable, they were calculated via particular systems properties of the operational experience, which (in their turn) represent specific functions of the reliability models parameters.*

**Key words:** *reliability models, state-space methods, incorrect failure detection, systems components.*

## Introduction

The goal of the reliability predictions is to make an adequate prognosis of systems failure behavior and recovery – [1], [5], [8]. In order to achieve a really good reliability predictions, as well as to enhance the accuracy of such predictions, the effects of all kinds of systems uncertainties should be assessed. In general, considerable quantities of components faults and failures are frequently treated in a inconsistent way, due to the uncertainties, existing in the data bases, derived from systems operational experience – [1], [4], [5]. In order, to obtain maximum accuracy and to avoid inadequate and/or simplified assumptions, specialized reliability tests, and/or diagnostic procedures are developed over the systems components. These tests (and respectively the diagnostic procedures) are done to find out whether the systems components are able to perform their functions adequately – [1], [4], [5].

So far as the reliability tests and diagnostic procedures are considered to be correct, it should not be necessary to make a distinction between the *actual state* of systems components (i.e., their behavior in a real demand), and *the test results*, i.e., the *knowledge*, obtained about the actual state – [4].

Since the reliability tests and the diagnostic procedures could never be *entirely correct* (i.e., *perfect*), there always exist possibilities for the generation of the following options:

- *Option 1 (marked as P1)*. A possible failure/fault of systems component is falsely detected, which leads to an unnecessary repair action and/or unavailability of the system;
- *Option 2 (marked as P2)*. An actual components fault or failure remains undetected;

- *Option 3 (marked as P3)*. An actually good (i.e., intact) component is failed as a result of reliability tests and/or diagnostic procedures.

The relevance (the adequateness) of the test *results* (and/or the diagnostic procedures) over the prediction of the components behavior, could be influenced by two general sources of inaccuracy, of the following type:

- An inaccurate designed (and therefore a potentially inadequate) test and/or diagnostic procedures – the sequence of the scheduled actions does not completely lead to success;
- A potentially incorrect test and/or diagnostic performance – deviations from the preliminary designed procedures.

The components abilities to perform in adequate manner *can be restored via the repair procedures*, which can respectively be developed as a *re-calibration* of the existing components and/or their *substitution* by new ones – [1], [4], [5], [7]. The *incorrect repair* can therefore be described by the following *features* (events):

- *Feature 1*. A detected actual failure/fault in a systems component is not entirely removed, or if it is removed – then, either a new fault is again incorporated and as a result the defective systems state can not be detected (during the final test and/or diagnostic procedures), either the failure is not entirely removed and remaining components degradation is not detected (always during the final tests);
- *Feature 2*. A new fault is incorporated in the system, during an unnecessary repair and, either the new fault is not detected (recognized) during the final test procedures, either a new but unrecognized degraded state is created during the incorrect repair procedures.

The present paper describes a reliability model [2], [3], [6], designed for reliability testing and or fault diagnosis of systems components. The reliability model takes into account the components failures and their interactions with the incorrect failure detection and recovery (repair) processes. Since the models parameters are not directly observable, they were calculated via particular systems properties of the operational experience, which (in their turn) represent specific functions of the reliability models parameters.

## Development of the interaction reliability model

### Definition of the systems states and design of the state-space reliability graph

Since the reliability tests/diagnostic procedures can be considered as incorrect, then with respect to a certain type of failure mode, any components state must be described by a pair of Boolean variables  $[S_1, S_2]$ , where the first variable  $S_1$  refers to the *actual state*, and the second variable  $S_2$  – to the *recognized state* (i.e., the test and/or diagnostic result). The values of both variables can be as follows:

- for the variable  $S_1$ : the logical value “0”, i.e., the success state  $S^S$ ;
- for the variable  $S_2$ : the logical value “1”, i.e., the failure state  $S^F$ .

The Boolean pair of the type  $[S^F, S^S]$  for example, expresses an actually failure state, which is *not recognized by the reliability tests and/or diagnostics*. There exist two more additional intermediate states of the type  $[S^S, Un]$ , and  $[S^F, Un]$ , that possess *unknown (unspecified) knowledge Un*.

The state-space graph of the interaction reliability model is developed, and respectively presented in **Fig.1**. The graphical representation of the model is similar to a Markov-like state graph. Since the test/diagnostic and repair procedures are considered to be incorrect (not perfect), the following conditional probabilities are incorporated to the state-to-state transitions in the reliability graph:

- $P^F$  – failure probability (in general a probability per demand, or the function of the failure rate);
- $A_{S1}$  – a probability, that, a component is actually in state  $S^S$ , but is recognized (falsely) as a state  $S^F$  ( $P_1$ );
- $A_{S2}$  – a probability, that, a component is actually in state  $S^F$ , but is recognized (falsely) as a state  $S^S$  ( $P_2$ );
- $F$  – a probability, that, a component is actually in a state  $S^S$ , which is failed by test/diagnostic ( $P_3$ );
- $D$  – a probability, that, a component is actually in a state  $S^F$ , which remains failed *after* repair ( $R_1$ );
- $E$  – a probability, that a component is falsely recognized as a state  $S^F$ , which is failed *by* the repair ( $R_2$ ).

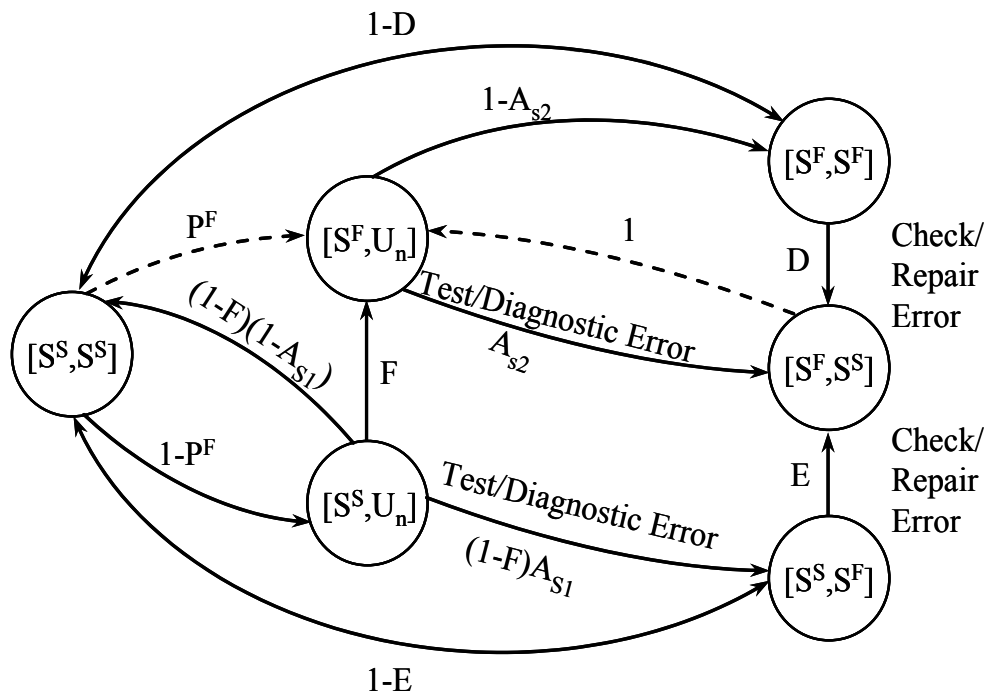


Fig.1. State-space graph of the interaction reliability model.

In the so-developed reliability graph the model parameters, that describe the transitions are not rates, but condition probabilities (like in a Markov graph). The dynamical effects are also not considered in the graph, i.e., only the *existence* of the stand-by, test/diagnosis and repair phases is included in the model, but not their *duration*.

### Mathematical description of the model equations

The model equations will be developed in a matrix-mode. The state-space relations are expressed by four different states (please see the developed graph) of the following type :  $[S^S, S^S]$ ,  $[S^S, S^F]$ ,  $[S^F, S^S]$  and  $[S^F, S^F]$ .

Thus, the state vector,  $\mathbf{S}$ , is defined as a *four-dimensional vector*, expressed by a sum of the four state probabilities, i.e.,  $\mathbf{S} = [\mathbf{S}_1, \mathbf{S}_2, \mathbf{S}_3, \mathbf{S}_4]$ . The sum of these probabilities is equalized to 1.

All changes in the state probabilities are represented by four  $[4 \times 4]$  transition matrixes of the following type:

- *Failure Matrix* – **[FM]**, is defined via the components failure probability, i.e.,

$$\mathbf{[FM]} = \begin{bmatrix} 1 - P^F & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ P^F & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{1}$$

The **[FM]**-matrix concerns only the real components, reduces the state probability  $S_1$  with the factor  $(1 - P^F)$ , but also adds the difference to the third state probability  $S_3$  of the state, with a really defective component. The effects on the second and fourth probabilities, (that express an imaginary failed components) are irrelevant, and therefore are ignored, since these states are not occupied (available) in the beginning of the operation phase of the industrial system. In fact, this is not a restriction of the model, and an eventual error could be included in the two repair errors  $R_1$  and  $R_2$ .

- *Transition Matrix* – **[TM]**, expresses the test/diagnostic transitions of the state probabilities, and describes the effects of the  $P_3$  probability over the same matrix structure, but this time the  $P^F$  probability is replaced with **F**-probability, i.e.,

$$\mathbf{[TM]} = \begin{bmatrix} 1 - F & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ F & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{2}$$

- *Cognitive Matrix* – **[CM]**. This matrix has a different structure (compared to the structures of **[FM]** and **[TM]** matrixes), since it affects only the imaginary parts of the states and its relation with  $A_{S1}$  and  $A_{S2}$ . Here the effects of  $P_1$  take place, and as a result – only a systems component in the first state  $S_1$ , can be *wrongly recognized as failed*, thus reducing  $S_1$ , and at the same time creating an occupation of the second state ( $S^S, S^F$ ).

$$\mathbf{[CM]} = \begin{bmatrix} 1 - A_{S1} & 0 & 0 & 0 \\ A_{S1} & 0 & 0 & 0 \\ 0 & 0 & A_{S2} & 0 \\ 0 & 0 & 1 - A_{S2} & 0 \end{bmatrix} \tag{3}$$

Since the effect of  $P_3$  probability also takes place, only a component in a state  $S_3$ , could be wrongly recognized as operational.

- *Repair matrix* – **[RM]**. The results of the repair, which is applied to components in states, expressing only *imaginary failures*, are, that these state probabilities can be unified, i.e.,  $S_2 = S_4 = 0$ .

$$\mathbf{[RM]} = \begin{bmatrix} 1 & 1 - E & 0 & 1 - D \\ 0 & 0 & 0 & 0 \\ 0 & E & 1 & D \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{4}$$

Since, the [RM] is function only of the probabilities **E** and **D**, and as consequence – the state probabilities  $S_3$  and  $S_4$  are increased. It follows also, that only the states  $[S^S, S^S]$  and  $[S^F, S^S]$  shall remain occupied permanently, while the states  $[S^S, S^F]$ , shall be only temporary occupied (after test/diagnostic and until the completion of the repair actions).

## Interactions of the developed reliability model with the systems operational data bases.

The six general parameters of the developed reliability model (i.e.,  $P^F$ ;  $A_{S1}$ ;  $A_{S2}$ ;  $F$ ;  $D$  and  $E$ ) could also be determined via the systems operational data (at least approximately), since their observable and/or measured quantities may not always be adequate and/or representative. The measured and/or observed quantities can be determined from data bases, obtained during the test/diagnostic/repair records (in general such data are informative and representative).

The adequate quantities can also be derived (as specific functions) of the models parameters, as follows:

- The probability  $P_R^F$ , of *observed* failures (i.e., failures, that urge repair actions). Such probability defines the set of particular probabilities  $P^F(S^F, S^F)$  and  $P^F(S^S, S^F)$ , that express the transition of the systems components into particular states of the type  $[S^F, S^F]$  and  $[S^S, S^F]$ , respectively, as a result of test/diagnostic procedures, i.e.:

$$P_R^F = P^F(S^F, S^F) + P^F(S^S, S^F) = A_{S1}(1-F)(1-P^F)S_1 + F(1-A_{S2})(1-F)S_1 + FS_1 + S_3 \quad (5)$$

- The probability  $P_R^R$ , of *unnecessary repair actions*, (but performed anyway). Since no failure/fault was detected in a state  $[S^S, S^F]$  during the performance of the repair actions this probability is expressed as follows:

$$P_R^R = P^F(S^S, S^F) = A_{S1}(1-F)(1-P^F)S_1 \quad (6)$$

- The probability,  $P_R^{T,D}$ , that a component can fail during test/diagnostic procedures (as a consequence from a wrong test methods). Such a failure can be revealed during the repair actions, that should be developed in the state  $[S^F, S^F]$ , i.e.:

$$P_R^{T,D} = F(1-A_{S2})(1-P^F)S_1 \quad (7)$$

- The probability of a wrong (a faulty) repair  $P_R^{RF}$ , developed during the repair procedures. Such a probability can be defined as a sum of the probabilities for the particular states  $[S^F, S^F]$  and  $[S^S, S^F]$ , that should transit respectively to state  $[S^F, S^S]$ , i.e.:

$$P_R^{RF} = E(1-A_{S2})P^F(S^S, S^F) + D P^F(S^F, S^F) \quad (8)$$

In order to determine the entire set of model parameters it becomes necessary to analyze some of the features, related to the incorrect tests/diagnostics and incorrect repair actions. The analysis performed over the options (probabilities)  $P_1$ ,  $P_2$  and  $P_3$ , as well as over the two repair errors (features)  $R_1$  and  $R_2$ , reveals, that, although the two repair features  $R_1$  and  $R_2$  are different (in general), their basic nature expresses *the failure* of the test/diagnostic/repair actions. Since the test/diagnostic failures of the components, leading to the state  $[S^F, S^S]$  are already expressed by the option  $P_2$  (with error probability of  $A_{S2}$ ), then – it could be accepted, that all three options (i.e.,  $P_1$ ,  $P_2$ , and  $P_3$ ) could be referred approximately the same error development, i.e.,

$$E \approx D \approx A_{S2} \tag{9}$$

Thus, the remaining functional relations involve only the model parameters PF, AS1, AS2, and F, which can easily be determined by the equations (5), (6), (7) and (8).

### Obtained results

It became obvious, that for a given set of systems components, the number of actual faults and/or failures and the number of faults and/or failures, detected via test/diagnostic does not coincide (i.e., does not agree), due to an incorrect detection process, and in general corresponds to the basic modeling error. Such an error can always be overcome by the developed (i.e., the proposed) model.

One of the most interesting options of the developed model is represented by the generation of the state  $[S^F, S^S]$ , which is permanently occupied and could not be recognized via tests/diagnostic, but only during a real demand. The lowest degree of probability of occupation of this particular state can be of second order, if the observed quantities are rather small, i.e.,

$$P_R(S^F, S^S)_{\min} = A_{S2}(A_{S1} + 2F + 2P^F) \tag{10}$$

It must be emphasized, that, such an incorrectness in combination with the failure detection and recovery may provoke some real perturbations in the components failure behavior. There exists a correlation (a link) between the operational data bases, which are described in terms of the observable quantities  $P_R^F, P_R^R, P_R^{T,D}, P_R^{RF}$ , and the mathematical model, described by the modeling parameters  $P^F, A_{S1}, A_{S2}, F$ .

The model requires an enhanced understanding between the components failure and the incorrect process of failure detection. The interaction process is described by a modeling logical structure, developed as state-space Markov-like graph.

### Conclusions

An enhanced reliability model, designed to describe the knowledge on the failure behavior of tested and/or diagnosed systems components, as well as to reflect the possible incorrectness in the failure detection process was developed as a state-space Markov-like graph.

The developed reliability model takes into account the components failures and their interactions with the incorrect failure detection and recovery (repair) processes. Since the model parameters are not directly observable, they were calculated via particular systems properties of the operational data, that are expressed as specific functions of the reliability model parameters.

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## Model de fiabilitate pentru descrierea cedărilor componentelor sistemelor testate și/sau analizate

### Rezumat

*Acest articol descrie un model de fiabilitate, proiectat pentru testarea fiabilității și/sau diagnoza cedărilor componentelor sistemelor. Modelul de fiabilitate ia în considerare cedarea componentelor și interacțiunea lor cu detectarea incorectă a cedărilor și procesele de reparare. Deoarece parametrii sistemului nu sunt direct observabili, ei au fost calculați pe baza unor proprietăți particulare ale sistemelor legate de experiența operațională, care (la rândul lor) reprezintă funcții specifice ale parametrilor modelului de fiabilitate.*