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On the Numerical Solving of the Inverse Geometric Model of a Robotic Mechanism with Six Degrees of Freedom

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Abstract

The paper presents a method that permits the numerical solving of the inverse geometric model in the case of a robotic mechanism with six degrees of freedom. The position and the orientation of the tool frame attached to the gripper of the robot are determined using the method of the homogeneous transformation matrices. A computer program that allows the calculus of the values of the generalized coordinates of the robot when the position and the orientation of the tool frame are imposed has been developed. Some simulation results are presented in the case when the tool frame follows a linear path with an imposed orientation.

Key words: *robot, position, orientation, geometric model*

Introduction

Many times the tasks realized by the industrial robots involve only the calculus of the generalized coordinates of their mechanism that ensure an imposed position and orientation of the tool frame attached to the gripper of the robot. So, in these cases it is necessary to solve the inverse geometric model corresponding to the robot mechanism [1,2]. The numerical solving of the inverse geometric model can represent a possibility for obtaining a fast and accurate solution [3,4].

In this paper a method that permits the numerical solving of the inverse geometric model in the case of a robotic mechanism with six degrees of freedom is presented. A computer program that allows the calculus of the values of the generalized coordinates of the robot when the position and the orientation of the tool frame are imposed has been developed.

Theoretical Considerations and Simulation Results

The positional analysis of the robotic mechanism (fig. 1) has been realized using the homogeneous transformation matrices [1]. The homogeneous transformation matrices corresponding to the relative position and orientation between the component modules have the following expressions:

$$
\begin{cases}
{}^{0}T_1 = Rot(z, q_1) \\
{}^{1}T_2 = Trans(z, q_2) \\
{}^{2}T_3 = Trans(z, l_2) \cdot Trans(y, q_3) \\
{}^{3}T_4 = Trans(y, l_3) \cdot Rot(y, q_4) \\
{}^{4}T_5 = Trans(y, l_4) \cdot Rot(z, q_5) \\
{}^{5}T_6 = Trans(y, l_5) \cdot Rot(y, q_6)
\end{cases} \tag{1}
$$

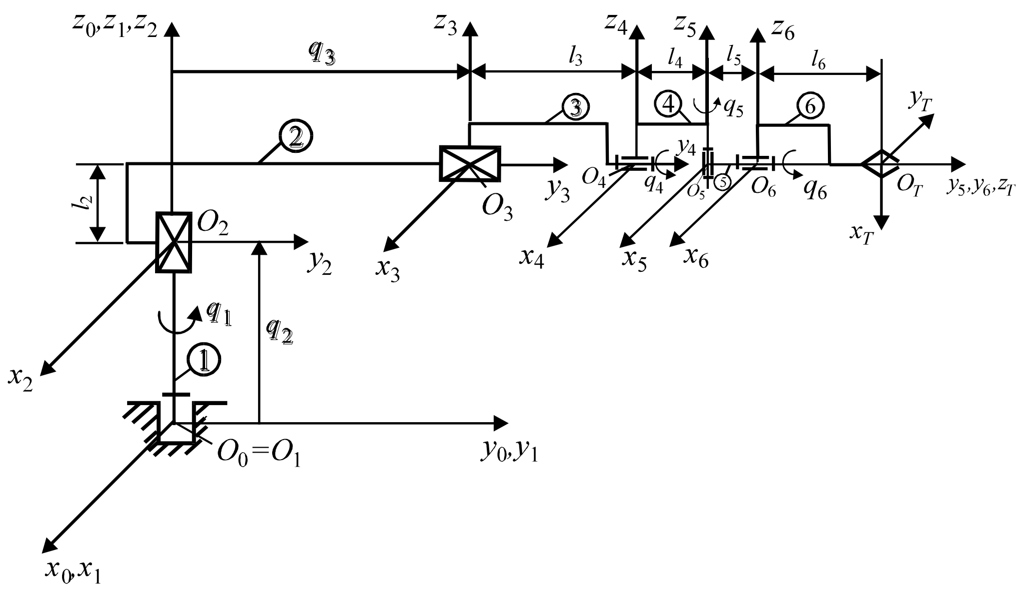

Fig. 1. Robotic mechanism with six degrees of freedom

The homogeneous matrices of type *Rot* and *Trans* [1] that appear in relation (1) have the following expressions:

$$
Rot(z, q_j) = \begin{bmatrix} \cos q_j & -\sin q_j & 0 & 0 \\ \sin q_j & \cos q_j & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};\ j \in \{1,5\}
$$

$$
Rot(y, q_j) = \begin{bmatrix} \cos q_j & 0 & \sin q_j & 0 \\ 0 & 1 & 0 & 0 \\ -\sin q_j & 0 & \cos q_j & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};\ j \in \{4,6\} \tag{2}
$$

$$
Trans(z, v) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & v \\ 0 & 0 & 0 & 1 \end{bmatrix};\ v \in \{q_2, l_2\};\quad Trans(y, v) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & v \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};\ v \in \{q_3, l_3, l_4, l_5\} \tag{3}
$$

The homogeneous transformation matrix ${}^6T_T$ corresponding to the relative position and orientation between the tool frame $(O_T x_T y_T z_T)$ attached to the gripper of the robot and the system of coordinates $(O_6 x_6 y_6 z_6)$ attached to the module *6* has the following expression:

$$ {}^6T_T = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & l_6 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4} $$

The position of the origin O_T, given by ${}^{(0)}O_0O_T$ and the orientation of the tool frame $(O_T x_T y_T z_T)$, given by the rotation matrix ${}^0R_T$, can be established with the following relation:

$$ {}^0T_T = \begin{bmatrix} {}^0R_T & {}^{(0)}O_0O_T \\ 0 & 1 \end{bmatrix} = {}^0T_6 \cdot {}^6T_T \tag{5} $$

where:

$$ {}^0T_6 = {}^0T_1 \cdot {}^1T_2 \cdot {}^2T_3 \cdot {}^3T_4 \cdot {}^4T_5 \cdot {}^5T_6 \tag{6} $$

The orientation between the tool frame $(O_T x_T y_T z_T)$ and the fixed system of coordinates $(O_0 x_0 y_0 z_0)$ can be expressed using the roll-pitch-yaw angles: α, β and γ, respectively [4]. In this case the rotation matrix ${}^0R_T$ has the following expression:

$$ {}^0R_T = \begin{bmatrix} \cos\alpha \cdot \cos\beta & \cos\alpha \cdot \sin\beta \cdot \sin\gamma - \sin\alpha \cdot \cos\gamma & \cos\alpha \cdot \sin\beta \cdot \cos\gamma + \sin\alpha \cdot \sin\gamma \\ \sin\alpha \cdot \cos\beta & \sin\alpha \cdot \sin\beta \cdot \sin\gamma + \cos\alpha \cdot \cos\gamma & \sin\alpha \cdot \sin\beta \cdot \cos\gamma - \cos\alpha \cdot \sin\gamma \\ -\sin\beta & \cos\beta \cdot \sin\gamma & \cos\beta \cdot \cos\gamma \end{bmatrix} \tag{7} $$

Having imposed the values of the components of the rotation matrix ${}^0R_T$, the pitch angle β can be calculated from the equation:

$$ \sin\beta = -{}^0R_T(3,1) \tag{8} $$

and then the roll and yaw angles can be calculated from the following relations:

$$ \begin{cases} \alpha = ATAN2\left(\dfrac{{}^0R_T(2,1)}{\cos\beta}, \dfrac{{}^0R_T(1,1)}{\cos\beta} \right) \\ \gamma = ATAN2\left(\dfrac{{}^0R_T(3,2)}{\cos\beta}, \dfrac{{}^0R_T(3,3)}{\cos\beta} \right) \end{cases} \tag{9} $$

where: the function *ATAN*2(*y*,*x*) calculates the arctg(*y/x*), by taking into account the signs of *x* and *y*.

The relations above have been transposed into a computer program using Maple language [5]. The computer program allows to calculate the generalized coordinates of the robot mechanism necessary for obtaining a demanded position and orientation between the tool frame and the fixed system of coordinates $(O_0 x_0 y_0 z_0)$.

The geometric parameters corresponding to the robot mechanism have the following values: $l_2 = 0.6\,\text{m}$; $l_3 = 0.45\,\text{m}$; $l_4 = 0.25\,\text{m}$; $l_5 = 0.2\,\text{m}$; $l_6 = 0.3\,\text{m}$.

Some simulation results obtained with the computer program are presented further. It has been considered that O_T moves along a linear trajectory. The coordinates of the starting point are: $x_s = 0.2\,\text{m}$; $y_s = 0.8\,\text{m}$ and $z_s = 1.2\,\text{m}$, and the coordinates of the end point of the trajectory are: $x_e = 0.5\,\text{m}$; $y_e = 1.1\,\text{m}$ and $z_e = 1.5\,\text{m}$.

The trajectory has been divided into $n = 10$ equal parts. The coordinates of the points on the trajectory have been calculated with the following relations:

$$\begin{cases} x_k = x_s + \dfrac{(k-1)}{n}\cdot(x_e - x_s) \\ y_k = y_s + \dfrac{(k-1)}{n}\cdot(y_e - y_s) \qquad k = \overline{1, n+1} \\ z_k = z_s + \dfrac{(k-1)}{n}\cdot(z_e - z_s) \end{cases} \tag{10}$$

The rotation matrix ${}^0R_T$ corresponding to each point on the trajectory has been considered to have the following expression:

$${}^0R_T = R(x, \theta_{1k}) \cdot R(y, \theta_{2k}) \tag{11}$$

where:

$$\begin{cases} \theta_{1k} = \theta_{1s} + \dfrac{(k-1)}{n}\cdot(\theta_{1e} - \theta_{1s}) \\ \theta_{2k} = \theta_{2s} + \dfrac{(k-1)}{n}\cdot(\theta_{2e} - \theta_{2s}) \end{cases} \qquad k = \overline{1, n+1} \tag{12}$$

$$\begin{cases} R(x, \theta_{1k}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{1k} & -\sin\theta_{1k} \\ 0 & \sin\theta_{1k} & \cos\theta_{1k} \end{bmatrix} \\ R(y, \theta_{2k}) = \begin{bmatrix} \cos\theta_{2k} & 0 & \sin\theta_{2k} \\ 0 & 1 & 0 \\ -\sin\theta_{2k} & 0 & \cos\theta_{2k} \end{bmatrix} \end{cases} \tag{13}$$

For simulations it has been considered that: $\theta_{1s} = 15^\circ$; $\theta_{1e} = 70^\circ$; $\theta_{2s} = 10^\circ$ and $\theta_{2e} = 60^\circ$.

So, by using the relations (8) and (9) can be calculated the imposed values of the roll, pitch and yaw angles: α_k, β_k and $\gamma_k, k = \overline{1, n+1}$, corresponding to each point on the trajectory.

The generalized coordinates of the robot mechanism: $q_{ik}, i = \overline{1,6}, k = \overline{1, n+1}$, necessary for obtaining the demanded position and orientation between the tool frame and the fixed system of coordinates $(O_0 x_0 y_0 z_0)$ in each point of the trajectory have been determined by numerical solving of the following system of equations:

$$\begin{cases} \alpha = \alpha_k;\ \beta = \beta_k;\ \gamma = \gamma_k \\ {}^0T_T(1,4) = x_k \\ {}^0T_T(2,4) = y_k \\ {}^0T_T(3,4) = z_k \end{cases} \tag{14}$$

For numerical solving the system of equations (14) it has been used the function *fsolve* from Maple program [5]. The initial values of the generalized coordinates used in the function *fsolve* were: $q_1 = \pi/3$; $q_2 = 0.2\,\mathrm{m}$; $q_3 = 0.2\,\mathrm{m}$; $q_4 = \pi/3$; $q_5 = \pi/3$; $q_6 = \pi/3$.

In figures 2÷7 there are presented the variation curves of the generalized coordinates $q_i, i = \overline{1,6}$.

The cross sign has been used to represent the values of the generalized coordinates for the points where the position and orientation between the tool frame $(O_T x_T y_T z_T)$ and the fixed system of coordinates $(O_0 x_0 y_0 z_0)$ have been imposed. Between these points a linear variation of the generalized coordinates has been considered.

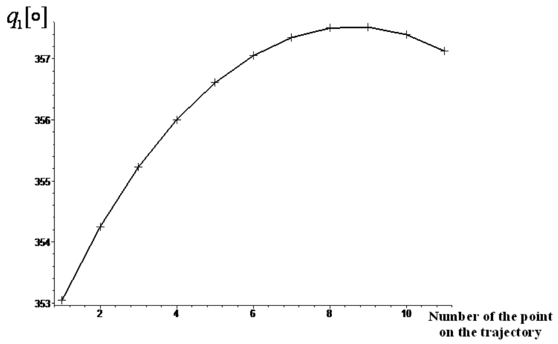

Fig. 2. The variation of q_1

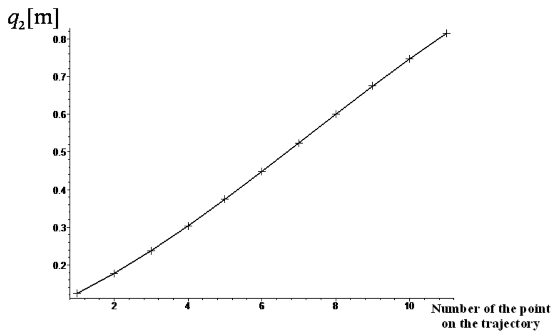

Fig. 3. The variation of q_2

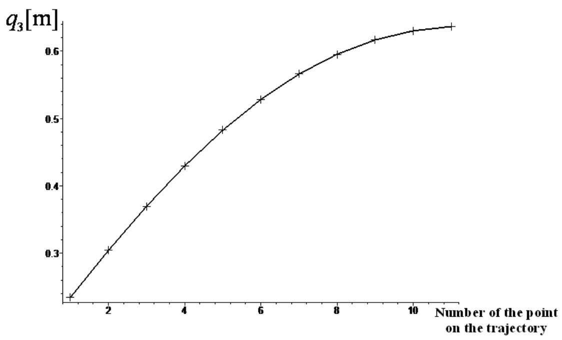

Fig. 4. The variation of q_3

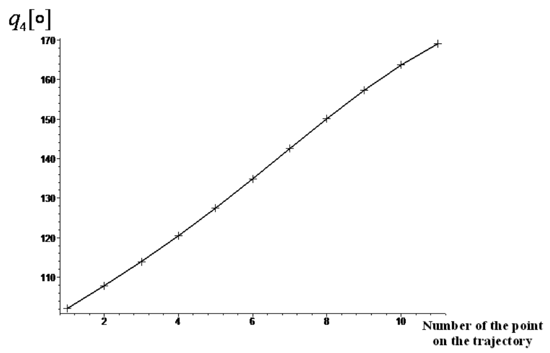

Fig. 5. The variation of q_4

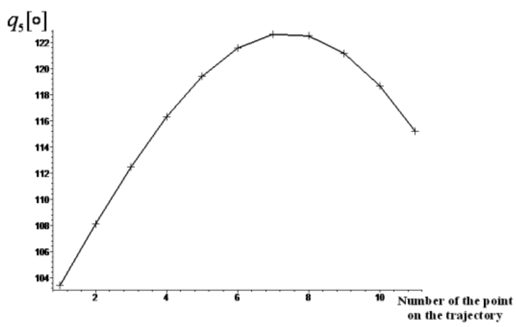

Fig. 6. The variation of q_5

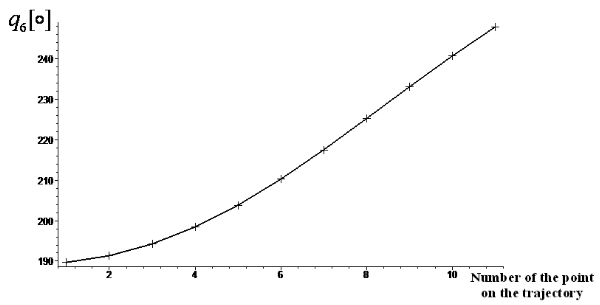

Fig. 7. The variation of q_6

Conclusions

In this paper a method that permits the numerical solving of the inverse geometric model in the case of a robotic mechanism with six degrees of freedom has been presented. The method of the homogeneous transformation matrices has been used to determine the relative position and orientation between the consecutive modules of the robot and to establish the position and the orientation of the tool frame attached to the gripper. A computer program that allows the calculus of the values of the generalized coordinates of the robot when the position and the orientation of the tool frame are imposed has been developed. Some interesting simulation results in the case when the tool frame follows a linear path with an imposed orientation have been presented. The method presented represents a possibility for obtaining a fast and accurate solution of the inverse geometric model of robotic mechanisms.

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Asupra rezolvării numerice a modelului geometric invers al unui mecanism robotic cu şase grade de libertate

Rezumat

Articolul prezintă o metodă care permite rezolvarea numerică a modelului geometric invers în cazul unui mecanism robotic cu şase grade de libertate. Poziţia şi orientarea reperului sculei ataşat griperului robotului sunt determinate folosind metoda matricelor de transformare omogenă. A fost dezvoltat un program de calculator care permite calculul coordonatelor generalizate ale robotului când poziţia şi orientarea reperului sculei sunt impuse. Sunt prezentate o serie de rezultate ale simulărilor când reperul sculei urmăreşte o traiectorie liniară, având o orientare impusă.