# Loads of Tracks Dippers which Lead to Cracks in the Structure During Quarrying 

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#### Abstract

The structure of a transportation great capacity dipper, used to supply crushed mineral aggregates was modelled. The demanding plastically and elastically deformation of structure was set under the conditions of the impact of a boulder delivered in the dipper, by passing the structure through resonance transmission and brake car. Graphs were drawn corresponding to rotating components of mechanical models calculations suggested for metalic structure deformation, subjected to shock and free and forced oscillations in longitudinal and transverse plan. Conditions of energetic equilibrium for the propagation of fissures in the structure and stress distribution around a hole were indicated.


Key words: dipper metal structure, dipper request of dumpe, fissures propagation, tension distribution in plates with holes

## Physical Model of Construction Transverse Request

## Metal structure loading of a transportation dipper for mining aggregate is observed

In the case of the dipper (carrying bucket), the construction of this model took into account the accidents suffered by the structure, which resulted in fissures and breakings in the area of the joint between the horizontal plate and the vertical walls - especially when great loads are placed in the dipper (carrying bucket) by pieces (fig. $1, \mathrm{a}, \mathrm{b}$ and c ).

The general computing scheme in Figure 2, structurally represent the overall dimensions for the structure of a transportation bucket, as well (fig.1, a).
Binding and inertial forces, as well as moments of inertia transversally act on the horizontal plate and side walls, in their load centres, under the action of the deformations caused by the carried loads. The weight of the plates and material, which was specified in the calculation of the plate deformation $W_{\max }$, is added to the above mentioned (these values are not specified in fig. 2).

The link between the horizontal plate and the walls is ensured by the horizontal stresses $X_{1}$ and $X_{2}$, the vertical stresses $Y_{1}$ and $Y_{2}$ and the embedding moments $M_{1}$ and $M_{2}$.


Fig. 1.
This computing scheme of the stress on the structure subjected to vibrations also includes the T stresses that act between the vertical side plates, in their upper part (see fig.2).


Fig. 2.
We admit that the general computing scheme in Figure 2 represent the structure of a dipper (carrying bucket) in a deformed state under stress. Here, the T link stress stands for the joint of the vertical walls with the frontal wall of the box-type carrying bucket.

High abrasion resistance makes this product Hardox steel to be used in a multitude of applications, such as those for dumpers, bucket containers, blind, rock crushers, stone or coal, conveyor troughs [4].

For the benefit of the kind of Figure 1,b is used Hardox 400, 5 mm thick.
Large pieces of rock thrown into a dump truck on site can weigh up to 5 tons; therefore, due to resistance to impact, Hardox 300 can be used effectively as part of consolidation on the lower tip-over.

In the above mentioned computing scheme (fig. 2), the following notations have been made: $\ddot{y}_{i}, \ddot{\varphi}_{2}$ - linear and angular accelerations of the plates, due to the structure deformation ( $i=1,2,3$ ); $\beta$ - linear rigidity coefficient of the transportation chute or the elasticity coefficient of the material of the carrying bucket horizontal plate; $y_{i}, \varphi_{i}$ - small movements and rotations of the plates, generated by the structure deformation $(i=1,2,3)$

The inertial forces and moments of inertia that act on the elements of the transversally deformed structure subjected to bending are the following:

$$
\begin{array}{rlrl}
F_{i 1} & =m_{1} \ddot{y}_{1} ; & F_{i 2}=m_{2} \ddot{y}_{2} ; & \\
M_{i 3}=m_{3} \ddot{y}_{3} ; \\
M_{i 1} & =J_{1} \ddot{\varphi}_{1} ; & M_{i 2}=J_{2} \ddot{\varphi}_{2} ; & \\
i 3 & =J_{3} \ddot{\varphi}_{3} ;
\end{array}
$$

where: $m_{i}, J_{i}$ - masses and moments of inertia that act on the chute plates $(i=1,2,3)$.
Let's consider the transversal rotation of the chute (or box) base plate around an edge of the chute when big stones that have to be transported are loaded.
The elastic forces $F_{1}$ and $F_{2}$ are taken by the supporting frame:

$$
\begin{equation*}
F_{1}=S_{1} \bullet \beta ; \quad F_{2}=S_{2} \bullet \beta \tag{1}
\end{equation*}
$$

where: $S_{1}, S_{2}$ - liniar elastic deformations of the horizontal plate against the supporting frame; $\beta$-the rigidity of the props $[\mathrm{N} / \mathrm{m}]$.
The binding moments of the vertical plates of the chute or box with the horizontal plate depend on the static rotation of the horizontal plate around one of its edges (see fig. 1). The deformations are expressed by the following relations:

$$
\begin{array}{ll}
\mathrm{S}_{1}=\varphi_{1} \cdot\left(\mathrm{a}+\mathrm{t}_{0}\right) ; \mathrm{y}_{1}=\varphi_{1}\left(\mathrm{a} / 2+\mathrm{t}_{0}\right) ; \\
\mathrm{S}_{2}=\varphi_{1} \cdot \mathrm{t}_{0} ; & \ddot{y}_{1}=\ddot{\varphi}\left(\mathrm{a} / 2+\mathrm{t}_{0}\right) .
\end{array}
$$

The equation system that expresses the transversal oscillation of the structure is:

$$
\left\{\begin{array}{l}
s_{1} \beta \cdot a+x_{2}\left(s_{2}-s_{1}\right)+y_{2}\left(a+t_{0}\right)-y_{1} \cdot t_{0}+m_{1} \ddot{\varphi}_{1}\left(\frac{a}{2}+t_{0}\right) \cdot \frac{a}{2}+J_{1} \ddot{\varphi}_{1}  \tag{2}\\
-k \cdot \varphi_{1} \cdot t_{0}-k \cdot \varphi_{1}\left(a+t_{0}\right)=0 \\
-k \varphi_{1}\left(a+t_{0}\right)+x_{1} \cdot h+m_{1} \frac{h^{2}}{4} \ddot{\varphi}_{2}+J_{2} \ddot{\varphi}_{2}=0 \\
-k \varphi_{1} \cdot t_{0}+x_{2} \cdot h-m_{3} \frac{h^{2}}{4} \ddot{\varphi}_{3}+J_{3} \ddot{\varphi}_{3}=0 \\
T-m_{2} \ddot{y}_{2}+x_{1}=0 \\
T+m_{3} \ddot{y}_{3}-x_{2}=0
\end{array}\right.
$$

The horizontal plate of the machine is subjected to the rotation in a tranversal plane with the $\varphi_{1}$ angle (fig. 2) under the action of the liniar deformations $\mathrm{S}_{1}$ and $\mathrm{S}_{2}$. Depending on these liniar deformations, accepted by the structure of the construction, the moments $M_{1}$ and $M_{2}$ can be calculated.

## Modelling of the structure box transportation

The box has the following characteristics: volume $16-20 \mathrm{~m}^{3}$; lenght $A=4798 \mathrm{~mm}$; width $B=$ 2500 mm ; wall height $C=1600 \mathrm{~mm}$; box mass $m=5000 \mathrm{~kg}$; total possible mass of the loaded working element -38000 kg .

In order to calculate the maximum pressure $\sigma_{\max }$ and deformation $W_{\max }$ at the horizontal rectangular plates, the accepted elementary hypothesis is that the elastic plate in question has the short side built-in, and the other three sides simply propped. For this hypothesis, there is the following relation [5]:

$$
\begin{gather*}
\sigma_{\max }=\beta_{1} \frac{p \cdot B^{2}}{h_{1}^{2}} ; \text { the ratio } B / A=\alpha<1 \\
W_{\max }=-\alpha \frac{p \cdot B^{4}}{E \cdot h_{1}^{3}} \tag{3}
\end{gather*}
$$

where: $p$ - unitary pressure; $h_{1}$-the thickness of the base plate with its reinforcements; $B$ - box or chute width; $A$ - lenght; $a, t_{0}$ - the distance between the side bars of the propping frame of the chute and the corner bracket of the chute at the side bar in a transversal plane (fig. 1)

For [5]:

$$
\frac{A}{B}=2 ; \beta_{1}=0,73 ; \alpha=0,101
$$

The high amplitude in the resonance regime leads to a supplementary stress on the machine, especially on the elastic suspension.

Due to the big deformations of the elastic elements, the spring loops can touch each other, thus causing stresses by shock and noises that far exceed the admissible limits.
When starting the vibrating machine, the equation of the vibrator shaft movement is as follows [2]:

$$
\begin{equation*}
\frac{d^{2} \varphi}{d t^{2}} \cdot J=M m-M r \tag{4}
\end{equation*}
$$

where: $\varphi$ is the rotation angle of the vibrator shaft; $J$ - the moment of inertia reduced at the vibrator shaft of all the elements of the machine, including the motor rotor; $M_{m}$ - the motor moment, reduced at the vibrator shaft; $M_{r}$ - the moment of the resistant forces reduced at the vibrator shaft.

We write $\dot{\varphi}=\omega$, supposing that $M_{m}$ and $M_{r}$ do not depend on the rotation angle or time. In this case:

$$
\begin{equation*}
\omega_{n}=\frac{1}{J} \int_{0}^{\tau}(M m-M r) d t=\frac{M m-M r}{J} \cdot \tau \tag{5}
\end{equation*}
$$

where: $\omega_{n}$ - the nominal angular speed of the vibrator shaft; $\tau$ - the start up time.
From (5), it results that:

$$
\begin{equation*}
\tau=\frac{\omega_{n} \cdot J}{M m-M r} \tag{6}
\end{equation*}
$$

Studying the moment of the vibrating machine in the resonance regime, it is noticed that the vibration amplitude depends on time.

The forced vibration amplitude in the resonance regime depends on the time span in which the mechanical system is in resonance. The forced vibration amplitude in the resonance regime slightly increases if the mechanical system is maintained in this regime for a short time.

When starting the machine, the start-up time $\tau$ is reduced. In the relation (6), it is noticed that $\tau$ decreases if the moment of inertia of the vibrator shaft is reduced by using electric motors with a high starting moment - a tendency confirmed during the construction of this type of machines.

In [2], there is the analysis of the case of the vibrating machine with a unidirectional vertical disturbing force, which operates in a post-resonance regime, namely $\omega \gg P_{y}$.
The differential equation of the machine movement has the following form:

$$
\begin{equation*}
\ddot{y}+p_{y}^{2} \cdot y=q r \omega^{2} \cos \omega t \tag{7}
\end{equation*}
$$

where $q$ - the mass of the eccentric.
At $\omega>P_{y}$, for the equation (7), a particular solution is found:

$$
\begin{align*}
& y_{f}=A_{y} \cos (\omega t+\pi)  \tag{8}\\
& \dot{y}_{f}=\omega A_{y} \sin (\omega t+\pi)=\omega A_{y} \cos (\omega t+3 \pi / 2) \tag{9}
\end{align*}
$$

The relation (8) represents the forced vibration of the machine and the relation (9) represents the forced vibration speed in a steadied post-resonance regime. If the resistance forces are not taken into consideration, the equation of the machine arrest following the shaft arrest is written as follows:

$$
\begin{equation*}
\ddot{y}+p_{y}^{2} \cdot y=0 \tag{10}
\end{equation*}
$$

with the general solution:

$$
\begin{equation*}
y=C_{1} \cos p_{y} t+C_{2} \sin p_{y} t \tag{11}
\end{equation*}
$$

The constants $C_{1}, C_{2}$ are determined by using the initial conditions.

$$
\begin{equation*}
\text { at } t=0, y(0)=0 \text { and } \dot{y}(0)=\dot{y}_{f \max }=A_{y}^{\prime} \tag{12}
\end{equation*}
$$

Using the initial conditions, the solution is as follows:

$$
\begin{equation*}
y=A_{0} \sin p_{y} t \tag{13}
\end{equation*}
$$

in which:

$$
\begin{equation*}
A_{0}=A_{y} \frac{\omega}{p_{y}}=\frac{q r \omega^{3}}{p_{y}\left(\omega^{2}-p_{y}^{2}\right)} \tag{14}
\end{equation*}
$$

The expression (13) shows the law of the vibrating machine movement after the shaft arrest, which shows that the machine makes free vibrations with the pulsation $\mathrm{p}_{\mathrm{y}}$ and the amplitude $\mathrm{A}_{0}$.
From the expression (14) it results that the amplitude of the free vibration $\mathrm{A}_{0}$ is as many times higher than the amplitude of the forced vibration $\mathrm{A}_{\mathrm{y}}$ as the pulsation of the disturbing force is bigger than the pulsation of the free vibration.

From the values that annul the derivative of the function $A_{0}(\omega), \omega=\omega_{0}$ has a practical value.
For $\omega=\omega_{0}$, the function $A_{0}(\omega)$ has a minimum value if the arrest takes place at the moment when the angular speed of the vibrator shaft has the value $\omega_{0}$.

## Maximum stress regime of the bucket for transportation trucks

If we take into consideration the influence of the general pattern of the vibrating machine in transient regime on the deformed elastic pattern of the feeder construction, it results that the latter receives an impulse by the elastic force $F_{2}$ :

$$
\begin{equation*}
F_{2}=S_{2} \cdot \beta=t_{0} \cdot \beta \cdot \varphi_{1} \quad \text { meaning } \quad y=S_{2} \tag{15}
\end{equation*}
$$

There is:
or

$$
\begin{align*}
& F_{2}=y \cdot \beta=\beta \cdot A_{0} \sin p_{y} t=\beta \cdot \frac{q r \omega^{2}}{p_{y}\left(\omega^{2}-p_{y}^{2}\right)} \sin p_{y} \cdot t  \tag{16}\\
& \varphi_{1}^{x}=\frac{\beta \cdot A_{0} \sin p_{y} t}{\beta \cdot t_{0}^{2}}=\frac{q r \omega^{2}}{p_{y}\left(\omega^{2}-p_{y}^{2}\right) \cdot t_{0}^{2}} \sin p_{y} t \tag{17}
\end{align*}
$$

There is a particular solution of the forced rotation $\varphi_{1}^{x}$ generated by the transversal vibration of the vibrating machine structure given by the relation (17).

Solving the differential equation system (2) in $\varphi_{1}$ together and taking into account that:

$$
\begin{equation*}
Y_{1}+Y_{2}+\left(a+t_{0}\right) \beta \cdot \varphi_{1}-m_{1}\left(\frac{a}{2}+t_{0}\right) \ddot{\varphi}_{1}+\frac{\beta \cdot q r \omega^{2}}{p_{y}\left(\omega^{2}-p_{y}^{2}\right)} \sin p_{y} t=0 \tag{18}
\end{equation*}
$$

the differential equation of the rotation in $\varphi_{1}$ is as follows:

$$
\begin{align*}
& \ddot{\varphi}_{1}\left[\frac{13 a^{2}}{12}+\frac{10 a \cdot t_{0}}{3}+\frac{7}{3} t_{0}^{2}\right] m_{1}+\left[\frac{x_{1}+x_{2}}{6}\left(a+t_{0}\right)-\left(a+t_{0}\right)^{2} \cdot \beta\right] \cdot \varphi_{1}=  \tag{19}\\
& =\left(a+2 t_{0}\right) y_{1}-\frac{\beta\left(a+t_{0}\right)}{t_{0}} \cdot \frac{q r \omega^{2}}{p_{y}\left(\omega^{2}-p_{y}^{2}\right)} \sin p_{y} t
\end{align*}
$$

For this a general solution $\varphi_{1}=e^{\bar{k} \cdot t}$, at $\mathrm{t}=0$, it results that:

$$
\begin{equation*}
\bar{k}_{1,2}= \pm \sqrt{\frac{\frac{x_{1}+x_{2}}{6}\left(a+t_{0}\right)-\left(a+t_{0}\right)^{2} \cdot \beta}{m_{1}\left[\frac{13}{12} a^{2}+\frac{10}{3} a t_{0}+\frac{7}{3} t_{0}^{2}\right]}} \tag{20}
\end{equation*}
$$

For the equation of the rotation of the structure base plate $\varphi_{1}$ given by (20) the chosen solution is as follows:

$$
\begin{equation*}
\varphi_{1}=A_{1} \cos \bar{k}_{11} t+\beta_{1} \sin \bar{k}_{11} t+C+\varphi_{10}^{x} \tag{21}
\end{equation*}
$$

where the particular solution C represents the static rotation given by the relation:

$$
\begin{equation*}
C=\frac{Y_{1}\left(a+2 t_{0}\right)}{\frac{X_{1}+X_{2}}{6}\left(a+t_{0}\right)+\left(a+t_{0}\right)^{2} \cdot \beta} \tag{22}
\end{equation*}
$$

and $\varphi_{10}^{x}$ is the forced rotation given by the vibrating machine oscillation on the feeder structure. For $X_{1}=-X_{2}$, there is:

$$
\begin{equation*}
\varphi_{10}^{x}=\left[-\frac{\beta\left(a+t_{0}\right)}{t_{0}} \cdot \frac{q r \omega^{2}}{p_{y}\left(\omega^{2}-p_{y}^{2}\right)} \sin p_{y} \cdot t\right]:\left(a+t_{0}\right)^{2} \cdot \beta \tag{23}
\end{equation*}
$$

The following condition is accepted:

$$
\begin{equation*}
\varphi_{1}(t)=-\frac{X_{1} \cdot h}{\beta\left(a+t_{0}\right)^{2}}-\frac{\frac{7}{12} h^{2} m_{2}}{\beta\left(a+t_{0}\right)^{2}} \ddot{\varphi}_{2} \tag{24}
\end{equation*}
$$

According to the elasto-plastic model construction of the cross section of the structure of the elastically propped carrier consisting of three plates linked together, from solving the system (2) the following condition results:

$$
\begin{equation*}
X_{1}=m_{2} \frac{3}{2} h \ddot{\varphi}_{2}+m_{3} \frac{h}{2} \ddot{\varphi}_{3}+\frac{t_{0}^{2} \beta}{h} \varphi_{1} \tag{25}
\end{equation*}
$$

It ensures the link between the rotation $\varphi_{1}$, of the carrier (or carrying bucket) horizontal plate and the rotation accelerations of the two side walls $\ddot{\varphi}_{2}$ or $\ddot{\varphi}_{3}$.

For the equation (25), general solutions are chosen, as follows:

$$
\begin{equation*}
\varphi_{1}=e^{\bar{k}_{1} t}, \varphi_{2}=e^{\bar{k}_{2} t}, \varphi_{3}=e^{\bar{k}_{3} t} \tag{26}
\end{equation*}
$$

If they are introduced into (26) together with their derivatives, the following relation is obtained:

$$
\begin{equation*}
X_{1}=m_{2} \frac{3 h}{2} \bar{k}_{2}^{2}+m_{3} \frac{h}{2} \bar{k}_{3}^{2}+\frac{t^{2} \beta}{h} \tag{26}
\end{equation*}
$$

At $t=0$, it results that: $\quad X_{1}=m_{2} \frac{3 h}{2} \bar{k}_{2}^{2}+m_{3} \frac{h}{2} \bar{k}_{3}^{2} \quad\left(\right.$ for $\left.\varphi_{1}=0\right)$
Applying in (24) the general solutions for $t=0$, there is:

$$
\begin{gather*}
\bar{k}_{2}^{2}=-\frac{\beta\left(a+t_{0}\right)^{2}+X_{1} h_{1}}{\frac{7}{12} h^{2} m_{2}}  \tag{28}\\
\bar{k}_{3}^{2}=\frac{\frac{22}{7} x_{1}+\frac{18}{7 h} \beta\left(a+t_{0}\right)^{2}-\frac{t_{0}^{2} \cdot \beta}{h}}{\frac{h}{2} m_{3}} \tag{29}
\end{gather*}
$$

Using the values of the coefficients (28) and (29), at $t=0$, the value of the effort $X_{1}$ can be determined (see fig. 3).


Fig. 3.


Fig. 4.
For the bucket of the truck (fig.2): $m_{2}=m_{3}=930 \mathrm{~kg} ; a=850 \mathrm{~mm} ; t_{0}=725 \mathrm{~mm} ; h=1500 \mathrm{~mm}$ (bucket wall height); $\beta=1.57610^{6} \mathrm{~N} / \mathrm{mm}$.

Then the value of the effort $X_{1}$ is introduced into the law of the base plate rotation $\varphi_{1}$.
Two possible initial conditions result for the considered material system.
If $X_{1}=-X_{2}$ and $\ddot{\varphi}_{2}(0) \neq 0$ results: $\quad \ddot{\varphi}_{2}(0)=\frac{\frac{x_{2} h}{\beta}\left[\frac{1}{(a+b)^{2}}-\frac{1}{t_{0}^{2}}\right]}{m_{2} h^{2}\left[\frac{1}{\beta \cdot t_{0}^{2}}+\frac{\frac{7}{12}}{\beta\left(a+t_{0}\right)^{2}}\right]}$

Knowing the values of the wall rotation acceleration $\ddot{\varphi}_{2}(0)$, we can obtain the rotation value $\varphi_{1}(0)$. The rotation law $\varphi_{2}(t)$ of the side wall of the chute is shown in Figure 4.
If we take into account the structure of the carrying bucket subjected to free oscillations in a transversal plane, there are two possible cases, namely: load-free operation or maximum load operation (with a big stone taken out from the quarry), which deforms the structure. The graphs drawn for these two possible cases are shown in Figure 5, a and b. They represent the behaviour of the carrying bucket subjected to transversal oscillations when it runs empty or loaded for transport.



Fig. 5.
From the calculations, a maximum value $X_{1}=4,44 \times 10^{5} \mathrm{~N}$ resulted for $\ddot{\varphi}_{2}=0,319 \frac{1}{s^{2}}$ (corresponding to a carrying bucket of $18 \mathrm{~m}^{3}$ ). The general solution of the equation which makes the connection between the base plate rotation $\varphi_{1}$ and the rotation accelerations of the vertical walls $\ddot{\varphi}_{2}$ and $\ddot{\varphi}_{3}$, finally results in: $\varphi_{1}(t)=A_{1}\left(\cos \bar{k}_{11} t-1\right)$. This rotation was graphically drawn in Figures 5, a and b, for the two possible cases, resulting in the following calculations:
a) $A_{1}=-0,00185 ; \bar{k}_{11}=I 26,866$, for free transversal oscillations of the carrying bucket without load;
b) $A_{1}=-51,235 ; \bar{k}_{11}=I 26,866$, for free transversal oscillations of the carrying bucket with maximum load.

These expressions represent the rotation law for which the effort $X_{1}$ has been determined from the condition $X_{1}=X_{2}$, which leads to an extreme effort state. The values $X_{1}, X_{2}$ are determined from the expressions of the coefficients $\bar{k}_{11}, \bar{k}_{12}$ and $\bar{c}_{a}$ which give the general solution of rotation in $\varphi_{1}$

## Mechanical model for structure longitudinal plan request for dipper (carrying bucket) at quarry

The scheme for the elastic-plastic calculus model of main board of dipper construction in deformed state as it is shown in Figure 4:

- It is supposed that both the forces and moments of the inertia of the components with acts upon the masses $m_{4}, m_{5}, m_{6}$ to the elastic system that moves very little on the rugous road;
- Only the cylindrical rigidities at bending noted here with $K_{4}$ and $K_{5}$, depend on the rotations of the neighbour elements that form the elastic system;
- The action of the vertical forces $T_{4}$ represents here the static pressure of the material construction and $T_{5}$ pressure a big stone taken out from the quarry, which deforms the structure.
The differential equations for the elastic calculus consisting of three masses linked according to the calculus scheme in Figure 6 are as follows:


Fig. 6.

$$
\begin{align*}
& \beta_{4} \cdot a_{4} \cdot \varphi_{4}+m_{4} \frac{a_{4}}{2} \ddot{\varphi}_{4}-T_{4}=0 \\
& T_{4} a_{4}+m_{4}\left(\frac{a_{4}}{2}\right)^{2} \ddot{\varphi}_{4}+\frac{1}{3} m_{4}\left(\frac{a_{4}}{2}\right)^{2} \ddot{\varphi}_{4}+\beta_{4} \varphi_{4}^{2}\left(\varphi_{4}-\varphi_{5}\right)=0 \\
& T_{5} a_{5}+m_{5}\left(\frac{a_{5}}{2}\right)^{2} \ddot{\varphi}_{5}+\frac{1}{3} m_{5}\left(\frac{a_{5}}{2}\right)^{2} \ddot{\varphi}_{5}+\beta_{5} \varphi_{5}^{2}\left(\varphi_{6}-\varphi_{5}\right)=0  \tag{31}\\
& T_{5}+m_{6} \frac{a_{6}}{2} \ddot{\varphi}_{6}+\beta_{6} a_{6} \varphi_{6}=0 \\
& T_{5} a_{6}+m_{6}\left(\frac{a_{6}}{2}\right)^{2} \ddot{\varphi}_{6}+\frac{1}{3} m_{6}\left(\frac{a_{6}}{2}\right)^{2} \ddot{\varphi}_{6}+\beta_{6} \varphi_{6}^{2}\left(\varphi_{6}-\varphi_{5}\right)=0
\end{align*}
$$

where: $S_{4}, S_{5}$ and $S_{6}$ - elastic movement on vertical; $F_{4 \mathrm{i}}, F_{5 \mathrm{i}}, F_{6 \mathrm{i}}$ - forces of inertia, that act upon the masses; $K_{4}, K_{5}, K_{6}$ cylindrical rigidities at bending of the elastic models; $\beta_{4}$ and $\beta_{6}$ are the rigidity coefficients at the ends of the elastic model. $\varphi_{4}, \varphi_{5}, \varphi_{6}$ - the rotations of the linked masses $m_{4}, m_{5}$ and $m_{6}$. For instance: $S_{4}=a_{4} \cdot \varphi_{4} ; F_{4 i}=m_{4} \frac{a_{4}}{2} \ddot{\varphi}_{4} ; K_{4}=\beta_{4} \cdot a_{4}\left(\varphi_{4} \cdot \varphi_{5}\right)$, etc.

Computing together the differential equation system (31) it results a bi-quadratic differential equation $\varphi_{4}(t)$ that represents the rotation of the elastic modulus at mass $\mathrm{m}_{4}$ level, having the following form:

$$
\begin{equation*}
\frac{m_{4} m_{5} a_{5}}{6 \beta_{4}} \varphi_{4}+\frac{m_{4} \bar{a}_{4}+m_{5} a_{5}}{2} \ddot{\varphi}_{4}+\beta_{4} a_{4} \varphi_{4}=m_{6} \frac{a_{6}}{2} \ddot{\varphi}_{6}+\beta_{6} \cdot a_{6} \cdot \varphi_{1} \tag{32}
\end{equation*}
$$

We take the general solutions having the form $\varphi_{i}=e^{U_{i} t}$. We have:

$$
\begin{equation*}
U^{4} \frac{m_{4} m_{5} a_{5}}{6 \beta_{4}}+\frac{m_{4} a_{4}+m_{5} a_{5}}{2} U^{2}+\beta_{4} a_{4}=m_{6} \frac{a_{6}}{2} V^{2}+\beta_{6} a_{6} \tag{33}
\end{equation*}
$$

The general solution of the rotation $\varphi_{4}(t)$ reduced at mass $\mathrm{m}_{4}$ of the elastic model is as follows:

$$
\begin{align*}
& \varphi_{4}(t)=A \cos i U_{1} t+B \sin i U_{2} t+C \cos i U_{3} t+D \sin i U_{4} t+E \cos i V_{1} t+  \tag{34}\\
& +F \sin i V_{2} t+G
\end{align*}
$$

The roots of the characteristic bi-quadratic equation are as follows:

$$
\begin{align*}
& \theta_{1,2}=\left[-\frac{m_{4} a_{4}+m_{5} a_{5}}{2} \pm \sqrt{\frac{m_{4} a_{5}^{2}}{2}+\frac{m_{5} a_{5}^{2}}{2}+\frac{m_{4} m_{5} a_{4} a_{5}}{3}}\right] \frac{3 \beta_{4}}{m_{4} m_{5} a_{5}} ; \\
& V_{1,2}= \pm \sqrt{-\frac{2 \beta_{6} a_{6}}{m_{6} a_{6}} ; \text { where } U_{1}, \cdots, U_{4}= \pm \sqrt{\theta_{1,2}} ; \text { and }} \\
& \beta_{4}=-\frac{\ddot{\varphi}_{6} \frac{m_{6} a_{6}^{2}}{m_{5} a_{5}^{2}}\left(\frac{m_{4} a_{4}+m_{5} a_{5}}{2}\right)}{T_{5} a_{4}} \tag{35}
\end{align*}
$$

If we admit the initial determined conditions in the equation system at $\mathrm{t}=0$;

$$
\begin{align*}
& \varphi_{4}(0)=0 ; \ddot{\varphi}_{4}(0)=\frac{2 T_{4}}{m_{4} a_{4}} ; \\
& \varphi_{6}(0)=\frac{-T_{5}\left[1-\frac{m_{5} a_{5}^{2}}{a_{6}\left(m_{4} a_{4}+m_{5} a_{5}\right)}\right]}{\beta_{6} a_{6}} ;  \tag{36}\\
& \ddot{\varphi}_{6}(0)=-\frac{T_{5}}{\frac{m_{6} a_{6}^{2}}{m_{5} a_{5}^{2}}\left(\frac{m_{4} a_{4}+m_{5} a_{5}}{2}\right)}
\end{align*}
$$

That is $\ddot{\varphi}_{4}(0)=f\left(T_{4}\right) ; \varphi_{6}(0)$ and $\ddot{\varphi}_{6}(0)=f\left(T_{5}\right)$, at the initial moment the caterpillar fulfil these conditions of rotation on the rugous soil.

The vertical forces $\mathrm{T}_{4}$ and $\mathrm{T}_{5}$ that interfere into the presupposed elastic model digitization fulfil the following condition: $T_{4}+T_{5}=\frac{G_{m}}{2}$, where $G_{m}$ represents the total machine weight together with the working equipment.

## Transmitted shock of all of a boulder in the bucket truck

For the model of the plate simply propped against the contour, the destortion in the centre of the plate is [2]:

$$
W_{\max }=\frac{0,1422 p b^{4}}{E h^{2}\left(1+2,21 \alpha^{3}\right)}, c u \quad \mu=0,3
$$

the sides ratio $\frac{b}{a}=\alpha<1$ and $\mathrm{p}-$ and p represent the load evenly allotted.
The initial conditions for the elastic model of the caterpillar stressed at the vertical shock at $\mathrm{t}=0$, $\varphi_{4}(0)=0$, for which a movement at mass $m_{6}$ appears:

$$
\begin{equation*}
y_{6}(0)=\varphi_{6} \cdot a_{6}=\delta_{s t}\left(1+\sqrt{1+\frac{2 h}{\delta_{s t}}}\right)=\frac{G}{k}\left(1+\sqrt{1+\frac{2 h \cdot k}{G}}\right) \tag{37}
\end{equation*}
$$

In phase II:

$$
\begin{equation*}
T_{4}=\frac{G_{m}}{2}-T_{5} ; T_{5}=\frac{-\frac{G}{k}\left(1+\sqrt{1+\frac{2 h \cdot k}{G}}\right) \cdot \beta_{6}}{1-\frac{m_{5} a_{5}^{2}}{a_{6}\left(m_{4} a_{4}+m_{5} a_{5}\right)}} \tag{38}
\end{equation*}
$$

The base of the bucket is constrained as a plate simply propped on the contour pressed by a load evenly allotted or by a plate constrained to a cylindrical bending loaded by a load uniform linearly allotted only on the width. The plate is propped against to a parallel sides and the surface deformed in the middle is [2]:

$$
\begin{equation*}
W_{\max }=\frac{5 p l^{4}}{384 D} \tag{39}
\end{equation*}
$$

where
considered

$$
D=\frac{E h^{2}}{12\left(1-\mu^{2}\right)}
$$

$$
\begin{equation*}
\beta_{6}=\frac{T_{5}\left[\frac{m_{5} a_{5}^{2}}{a_{6}\left(m_{4} a_{4}+m_{5} a_{5}\right)}-1\right]}{\frac{G}{k}\left(1+\sqrt{1+\frac{2 k h}{G}}\right)} \tag{40}
\end{equation*}
$$

where: $G$ is the weight of the stone; $h$ - verticat stroke $h=10 \mathrm{~mm}$; $T_{5}$ - vertical loading on the bucket taken by a plate.

From computing it results, $T_{5}=13917 \mathrm{~N} ; \beta_{6}=|-33867,2| \mathrm{N} / \mathrm{m} ; K=17567.3 \mathrm{~N} / \mathrm{m}$.
The rotation $\varphi_{6}=\frac{G}{k}\left(1+\sqrt{\frac{2 h \cdot k}{G}}\right) \frac{1}{a_{6}}$, has a computed value $\varphi_{6}=0.546$, corresponding to the shock produced by the weight. $T_{4}=G_{m} / 2-T_{5}=176083 \mathrm{~N}$, takes almost the whole loading allotted on the plate
The duration of the shock produced during a hole is:

$$
\begin{equation*}
T_{1}^{\text {soc }}=\sqrt{\frac{G}{g \cdot k}}\left[\pi+\operatorname{arctg} \sqrt{\frac{g \cdot G}{V_{0}^{2} \cdot k}}\right] \text {, for } V_{0}=0.2777 \mathrm{~m} / \mathrm{s}, T_{1}^{\text {Soc }}=0,27 \mathrm{~s} . \tag{41}
\end{equation*}
$$

From the studies done upon the behaviour of the dynamic models using the proposed computing schemes of the system represented under Figure 7 by the curves a and b .

## Energetic Equilibrium Conditions for Firuress Propagation inside the Elastic Structure [2]

The study of the energetic balance conditions clears into a general context, the crack propagation. By passing from the state 1 to the state 2 , the whole volume $V$ it will increase with $\Delta V$; the surface $\Sigma$ will change with $\Delta \Sigma$.


Fig. 7. Oscillations of the elastic system representing the behaviour of the assembly plate
The balance equation will base the following form:

$$
\begin{equation*}
\Delta U+\Delta K=\Delta A+\Delta Q-\Delta \pi \tag{42}
\end{equation*}
$$

where: $\Delta U$ - elastic energy variations; $\Delta K$ - kinetic energy variation; $\Delta \mathrm{A}$ - mechanical work variation of external forces; $\Delta Q$ - heat flux; $\Delta \pi$ - energy flux provoked by some thing else but only the energy spent for breaking is taken into account.

On the hypothesis of small deforming (quasi-static) it may be neglected both the kinetic energy variation and the heat flux:

$$
\begin{equation*}
\Delta U-\Delta A=-\Delta \pi \tag{43}
\end{equation*}
$$

where $U-A=W$ denotes the potential energy of the body; so

$$
\begin{equation*}
\Delta W=-\Delta \pi \tag{44}
\end{equation*}
$$

It results that it is necessary to compute the potential energy variation of the body during the process of crack enlarging. Taking into account near the crack peak it is obtained:

- For slot plan traction:

$$
\begin{equation*}
\Delta W_{I}=-\frac{K_{I}^{2}}{8 G}(\Lambda+1) \Delta l \tag{45}
\end{equation*}
$$

- For cross wise shear:

$$
\begin{equation*}
\Delta W_{I I}=-\frac{K_{I I}^{2}}{8 G}(\Lambda+1) \Delta L \tag{46}
\end{equation*}
$$

- For longitudinal shear:

$$
\begin{equation*}
\Delta W_{I I I}=-\frac{K_{I I I}^{2}}{2 G} \Delta l \tag{47}
\end{equation*}
$$

For the crack to enlarge it is necessary to overcome the linking forces between the particles of the atom network in material both the normal and the lateral ones. According to Griffith the dislocation energy is equal to:

From the relations:

$$
\begin{equation*}
\Delta \pi=2 \gamma \cdot \Delta \sum \tag{48}
\end{equation*}
$$

For $\quad \gamma>0 \quad$ and $\quad d \pi=2 \gamma \cdot \Delta l$

$$
\begin{align*}
& \frac{K_{I}^{2}}{8 G}(\Lambda+1)=2 \gamma  \tag{49}\\
& \frac{K_{I I}^{2}}{8 G}(\Lambda+1)=2 \gamma  \tag{50}\\
& \frac{K_{I I}^{2}}{2 G}=2 \gamma
\end{align*}
$$

and $\mathrm{p}(\xi)=$ constant, it results Griffith's formula

$$
\begin{equation*}
p^{2}=\frac{2 E \gamma}{\pi l\left(1-v^{2}\right)} \tag{51}
\end{equation*}
$$

$\Delta W$ is calculated after stress state - deforming in the crack peak that has there a singularity.
Through the crack peak the system energy flows; this energy is spent in this area to destroy the material. In the crack peak the small deforming conditions and Hooke's law are not satisfied [2].

## Tensions Distribution around the Hole of Curved Plate [5]

For an exact image of demands developed inside a hole surrounded by $L$ contour of the fissure, for example, it is necessary to know the tension variation and moments around a point [5]. For a plate having the thickness $h(z=h / 2)$, with an elliptic hole inside and bending moments equally distributed applied on the four sides, it is situated at $\mathrm{n}=1$ values of the coefficient K distribution for $\gamma=0,3$ and that of $b=3, m=1 / 2$, set in Figure 8 [5]. The maximum takes place in point $\theta=0, \pi$ and it is for $K=3,576$. The coefficient K is:

$$
K=M_{\rho \theta} / M,
$$

where: $M$ is the bending moment, equally applied on the plate edge $M_{\rho \theta}$ - the moment developed around the hole. Coefficient $C$ corresponds to coefficient $K$ (because $=K+C$ ), for $z=+-h / 2$,

$$
C=16.45(h / R)^{2}, \quad \text { where } \quad R=(a+b) / 2 .
$$

For $\theta=+-90^{\circ}$ there is [5]: $K=1.475$, and coefficient $C$ is:

$$
C=-6 m(1-m) K(z) /(3+\gamma)(1+m)^{5}
$$

at $\theta=90^{\circ}$, or in $z$ point $=+-h / 2$ it reduces to:

$$
C=2(2-\gamma)(1-m) / 5(3+\gamma)(1+m)^{5} .
$$



Fig. 8. $K$ coefficient value distributions, $K=K+C$ having a plate with eliptic hole [5].

## General Conclusions

For the elastic structure of a dipper (transportation bucket) statically requested (fig. 2), the following facts can be noticed, transversal oscillation of basic plate (fig. 3), is directly influenced by lateral walls, considered during their rotation. Free oscillations of the dipper structure in transversal plan with and without being loaded are shown in Figure 5, a and b.

In calculations, effort $X_{1}$ variation is influenced by the frequency of lateral walls oscillations (see fig.4). They generate fissures through twisting material under the influence of superficial cuts of the surface that occur during exploitation, influencing the value of superficial energy. Fissures propagate on horizontal plan and along the structure. (see fig. $1, b$ and $c$ ).
Rotation $\varphi_{4}(\mathrm{t})$ of elastic model is represented on the dipper bottom on longitudinal plan, under the influence of the shock transmitted by the falling boulder in dipper (fig. $7, \mathrm{a}$ and b ), according to the two conditions of bearing of the plate. Finally the energetic ecquilibrum conditions are given for fissures propagation in elastic fracture models according to the efforts distribution around the hole Figure 8.

## References

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## Solicitări ale benelor de camion care conduc la apariția fisurilor în structură, la exploatarea în carieră

## Rezumat

S-a modelat structura unei bene de transport de mare capacitate, montată pe camion, care lucrează in exploatări miniere, pentru alimentarea instalațiilor de concasat agregate minerale. $S$-a determinat solicitarea structurii la deformare elasto - plastică, in condiții de exploatare la impactul unui bolovan descărcat în benă, sau la trecerea structurii prin rezonanță la frânarea transmisiei maşinii. Au fost trasate graficele corespunzătoare rotirii componentelor modelelor de calcul, ale structurii propuse pentru analiza deformărilor structurii mecanice supuse la şocuri la inncărcare, şi la oscilații libere şi forțate in transport, in plan longitudinal şi transversal.

