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# Internal Model Controller Design for Proportional-Type Processes

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### Abstract

Internal Model Control (IMC) algorithm was introduced as an alternative to the classical Proportional-Integral-Derivative (PID) control algorithm. The IMC algorithm is increasingly used in industry, especially for single input and single output (SISO) systems, because it has a simple form and the tuning parameters can be easily tuned online. The purpose of this paper is to design and implement a control system for a proportional-type process using the Internal Model Controller in two variants: standard and advanced.

Key words: Proportional-type processes, IMC algorithm, IMC controller design.

## Introduction

Internal Model Control concept was introduced by Garcia and Morari [5] but other researchers like Francis and Wonham [4], Zames [8], Arkun et al. [1] have studied and developed similar concepts. This algorithm was studied and developed ever more in the last 20 years because the algorithm is simple and effective, its advantages being exploited in many industrial applications [7].

Internal Model is a robust control method that aims a good setpoint tracking, even when disturbances appear, but the results strongly dependent on the precision of the process model. If the model is very well approximated, then the control system will have a very good performance and if the process model is exactly approximated, then the control system will act perfect [2].

The Internal Model Controller (IMC) design involves the following two steps:

- Process model identification;
- Controller design: finding the controller model using the process model.

The IMC controller can be used in two variants, standard or advanced, depending on the desired control system performance.

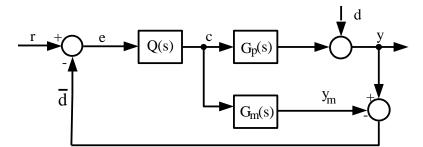
If the IMC controller is a standard one, it is necessary to know only the process proportional gain in order to design the IMC controller.

If the IMC controller is an advanced one, it is necessary to know the process dynamic model in order to design the IMC controller.

The objective of this paper is to design and implement an IMC controller for proportional-type process (which has the static gain finite and nonzero), using the two possible types: standard IMC or advanced IMC.

#### **Internal Model Controller**

The Internal Model Control system has the structure from Figure 1 [6].



**Fig. 1.** Internal Model Control structure: Q(s) – the primary controller transfer function, Gp(s) – the process transfer function, Gm(s) – the process model transfer function, r – setpoint, e – error, c – control variable, d – disturbance, ym – model output, y –process output.

In order to have a zero static, for a setpoint step change or for a disturbance step change, it is necessary that the control system to be stable and the controller static gain to be equal to the reverse of the model static gain [3]:

$$Q(0) = \frac{1}{G_{\rm m}(0)}.$$
 (1)

The simplest form of the IMC algorithm is to consider the transfer function as a zero order transfer function, equal to the reverse of the model gain:

$$Q(s) = \frac{1}{G_m(0)}.$$
 (2)

This is the case of the standard type of IMC controller.

In this case, the controller transfer function, which contains the primary controller Q and the model transfer function  $G_m$ , is:

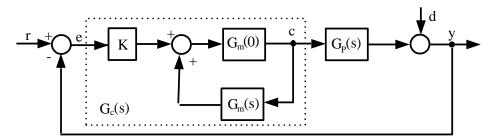
$$G_{\rm C}(s) = \frac{Q(s)}{1 - Q(s) \cdot G_{\rm m}(s)} = \frac{1}{G_{\rm m}(0) - G_{\rm m}(s)}.$$
(3)

In order to have a tunable controller a static gain K can be introduced in the control system structure, as in Figure 2. The standard value is 1. If the value of parameter K increases, we obtain an increase in the control variable power. The IMC controller will have the transfer function [3]:

$$G_{\rm C}(s) = \frac{K}{G_{\rm m}(0) - G_{\rm m}(s)}.$$
(4)

In the advanced case of the IMC algorithm, the primary controller transfer function is equal to the reverse of the model transfer function [3]:

$$Q(s) = \frac{1}{G_{\rm m}(s)}.$$
(5)



**Fig. 2.** Internal Model Control structure with tunable controller: K – the controller gain, Gp(s) – the process transfer function, Gm(s) – the process model transfer function, Gm(0) – the model static gain, Gc(s) – the internal model controller transfer function, r – setpoint, e – error, c – control variable, d – disturbance, y –process output [3].

#### IMC Controller Design for Proportional-Type Processes

For proportional-type stable processes with no over-damped step response, it is recommended that the process model to be chosen as a second-order transfer function with dead time, having the form:

$$G_{\rm m}({\rm s}) = \frac{k_m \cdot e^{-\pi {\rm s}}}{\left({\rm T}_m \cdot {\rm s} + 1\right)^2},\tag{6}$$

where  $k_m$  is the model static gain,  $\tau$  is the process model time delay and  $T_m$  is the process model time constant.

Generally, a first order model with dead time is too simple to describe the process dynamics, and a second order model with different time constants or an order higher than two is too complicated and does not usually gives significant advantages.

The three model parameters from (6),  $k_m$ ,  $\tau$  and  $T_m$ , can be found using the process step response trend.

If the process has the response as in Figure 3, then the following formulas can be used in order to compute the three model parameters:

• the model proportional gain  $(k_m)$  is equal to the process proportional gain  $(k_p)$  and it is obtained as the total variation value of the process output divided by the total variation of the process input, both being measured in percent:

$$k_m = k_p = \frac{\Delta y\%}{\Delta u\%},\tag{7}$$

- the model time delay  $(\tau)$  is equal to the process time delay as in fig. 3
- the model time constant  $(T_m)$  is obtained as the process transient time minus the time delay divided by 6:

$$T_m = \frac{T_{tr} - \tau}{6}.$$
(8)

In the standard case of the IMC controller, considering a proportional-type process (6), the IMC controller has the primary transfer function as

$$Q(s) = \frac{1}{G_{\rm m}(0)} = \frac{1}{k_m}.$$
(9)

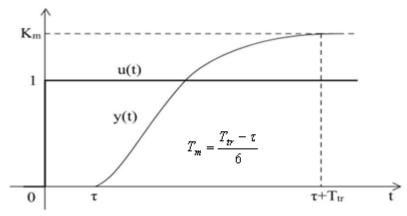


Fig. 3. A proportional-type process step response: u - process input variable, y - process output variable,  $k_m$  - the model static gain,  $\tau$  - the process model time delay,  $T_m$  - the process model time constant,  $T_{tr}$  - the process transient time.

In the advanced case of the IMC controller, considering a proportional-type model (6), the IMC controller has the primary transfer function as

$$Q(s) = \frac{1}{G_{\rm m}(s)} = \frac{({\rm T_m} \cdot s + 1)^2 \cdot e^{-\varpi}}{k_m}.$$
 (10)

Because the advanced IMC primary controller (10) is improper, in order to have a semi-proper controller, the transfer function (10) becomes

$$Q(s) = \frac{(T_{m} \cdot s + 1)^{2} \cdot e^{-s}}{k_{m}(T_{\varepsilon} \cdot s + 1)^{2}},$$
(11)

where  $T_{\varepsilon}$  is called filter time constant and it is a controller tuning parameter. This constant is chosen depending on the magnitude factor ( $f_m$ ), so that  $f_m \le 10$ , that is

$$\frac{Q(j\infty)}{Q(0)} \le 10. \tag{12}$$

In conclusion, the IMC controller can be as standard type or advanced type. In the standard variant we have as tuning parameter the controller gain K (fig. 2). In the advanced variant we have as tuning parameters the controller gain K (fig. 2) and the filter time constant,  $T_{\varepsilon}$ . Together with this parameters, the three process model parameters (6) also influence the performance of the control system.

Further the performance of the control system is investigated for both IMC variants, in case of changing the tuning parameters. It is tested also the influence of the modelling errors to the control system performance for a particular proportional-type process.

#### **Results**

The considered process has the step response shown in Figure 4.

Using the simulation data from Figure 4, and formulas (7) and (8), the process model parameters are found as:

$$k_m = \frac{\Delta y\%}{\Delta u\%} = \frac{2.5 - 0}{1 - 0} = 2.5, \ \tau = 3 \min, \quad T_m = \frac{T_{tr} - \tau}{6} = \frac{89.4}{6} = 14.9 \min.$$
 (13)

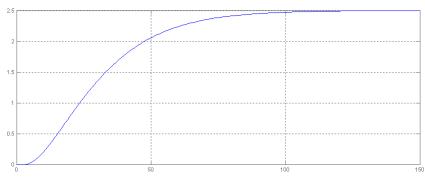


Fig. 4. The process time response to a step change of the input variable

Further, the process model (6) for the computed model parameters (13), has the following form

$$G_{\rm m}(s) = \frac{2.5 \cdot e^{-3s}}{(14.9 \cdot s + 1)^2}.$$
 (14)

Considering the standard IMC type, we have the primary controller transfer function (9) as

$$Q(s) = \frac{1}{k_m} = \frac{1}{2.5}.$$
(15)

Considering the advanced IMC type, and choosing the magnitude factor (fm) equal to 4.5, we have the primary controller transfer function (11) as

$$Q(s) = \frac{(14.9 \cdot s + 1)^2 \cdot e^{-3s}}{2.5(7 \cdot s + 1)^2}.$$
 (16)

Further, the control system performance was tested for the considered process, using both IMC standard and advanced variants, for different values of the controller tuning parameters.

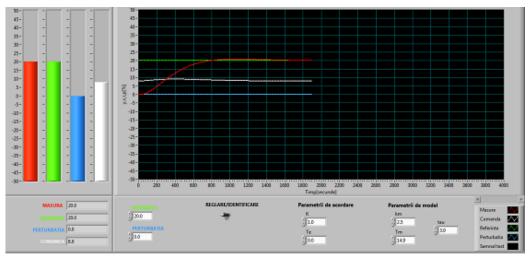


Fig. 5. Control system time response to a setpoint step change, in case of considering the standard IMC

In case of using the advanced IMC variant, the control system transient time is smaller than in the case of the standard IMC variant. Also, the control system output overshoot is bigger in case of using the standard variant.

If the controller gain K increases, the control variable power increases, the process output overshot increases but the control system transient time decreases.

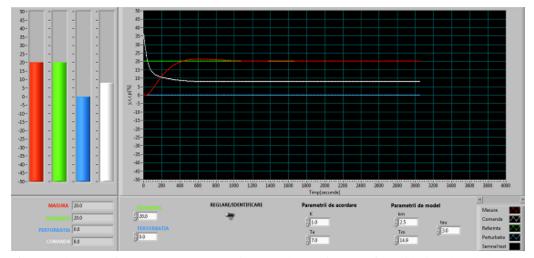
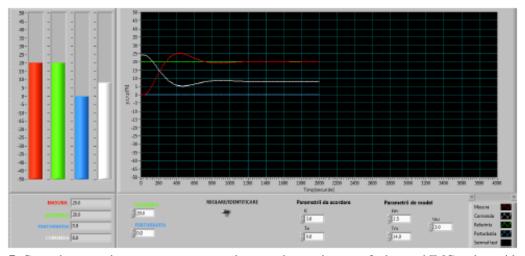


Fig. 6. Control system time response to a setpoint step change, in case of considering the advanced IMC.



**Fig. 7.** Control system time response to a setpoint step change, in case of advanced IMC and considering the controller gain *K*=1.5.

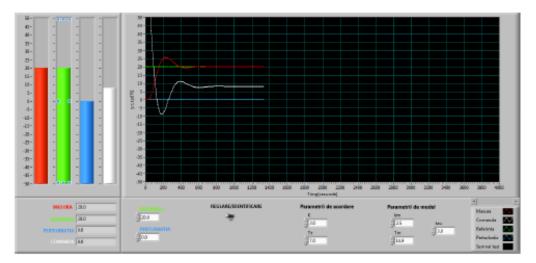


Fig. 8. Control system time response to a setpoint step change, in case of advanced IMC and considering the controller gain K=3.

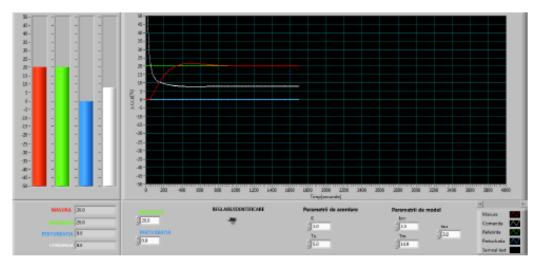


Fig. 9. Control system time response to a setpoint step change, in case of advanced IMC and considering the filter time constant  $T_{\varepsilon}$  =5 min.

In this case the controller output value it is stronger, the output overshoot increases and the transient time decreases.

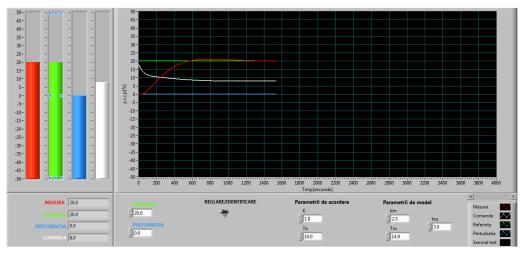


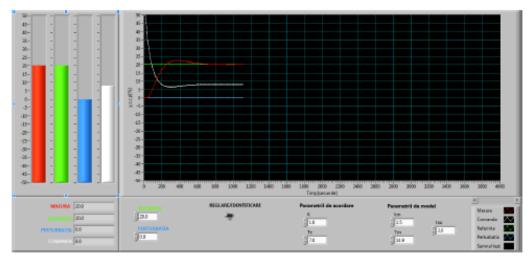
Fig. 10. Control system time response to a setpoint step change, in case of advanced IMC and considering the filter time constant  $T_{\varepsilon} = 10$  min.

In this case, the controller output power decreases, we have a smaller output overshoot but also a smaller transient time.

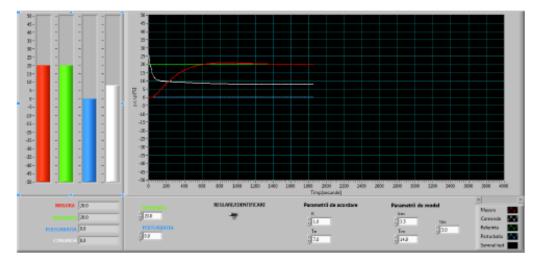
If the process model static gain  $(k_m)$  has a smaller value than the value obtained by process model identification, the controller output will have the initial value greater than the final value, the process output increasing speed will increase and we will have output overshoot.

If the process model static gain  $(k_m)$  has a bigger value than the value obtained by process model identification, the controller output will have the initial value smaller than the final value, the process output increasing speed will decrease and we will have a greater transient time.

If the process model time constant  $(T_m)$  has a smaller value than the value obtained by process model identification, the controller output begin to increase at  $t = \tau$  moment, the process output increasing speed will increase and we will have output overshoot.



**Fig. 11.** Control system time response to a setpoint step change, in case of advanced IMC and considering  $k_m = 1.5$ .



**Fig. 12.** Control system time response to a setpoint step change, in case of advanced IMC and considering  $k_m = 3.5$ .

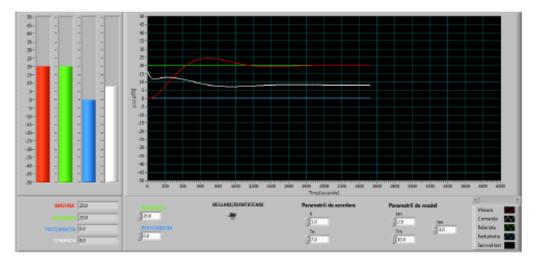


Fig. 13. Control system time response to a setpoint step change, in case of advanced IMC and considering  $T_m = 10$  min.

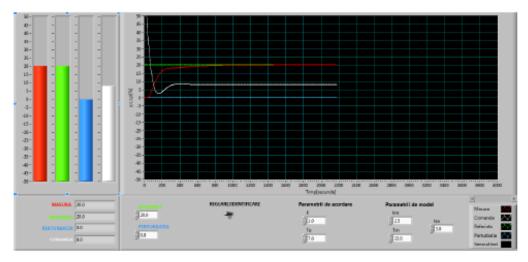


Fig. 14. Control system time response to a setpoint step change, in case of advanced IMC and considering  $T_m = 22$  min.

If the process model time constant  $(T_m)$  has a bigger value than the value obtained by process model identification, the controller output will begin to decrease at  $t = \tau$  moment, and the output overshoot will be approximately zero. In the advanced variant, because the  $T_m/T_t$  ratio is bigger, the controller output will have an initial value bigger, the control system transient time will decrease and the output overshoot will have a smaller value.

By testing the control system performance, can be observed that the obtained model with  $k_m = 2.5$ ,  $T_m = 14.9$  min and  $\tau = 3$  min describes with good precision the process dynamic behaviour, and the control system response has a similar shape with process step response, that means that they were well found.

### Conclusions

From the simulation results we can see that when the process mathematical model is well known, the IMC control system has very good performance, but when we have modelling errors, the control system output has overshoot and the control system transient increases.

The advantages and disadvantages of the two control variants were outlined, making a comparison of the performances of the two control systems.

In conclusion, an automatic control system with internal model controller has the advantage of being simple and robust and having very good performance when the process model it is very well known. A disadvantages is that if we have modelling errors, the control system performance are not so well and another disadvantage is that these two types of IMC controller can only be used for stable proportional-type processes with no over damped response.

In case of processes that have an over-damped response, integral-type or unstable using the two types of IMC control algorithm becomes complicated, complex methods are needed to reverse the process model.

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# Proiectarea regulatorului cu model intern pentru procese de tip proporțional

### Rezumat

Algoritmul de reglare cu model intern (IMC) a fost introdus ca o alternativă la algoritmul de reglare clasic Proporțional-Integral-Derivativ (PID). Algoritmul IMC este folosit în industrie, în special pentru sistemele monovariabile cu o singură intrare și o singură ieșire (SISO), datorită faptului că are o formă simplă, iar parametrii de acordare pot fi ușor modificați on-line. Scopul acestei lucrări este de a proiecta și implementa un sistem de reglare pentru un proces de tip proporțional, cu ajutorul regulatorului cu model intern în două variante: standard și avansată.