BULETINUL Universităţii Petrol – Gaze din Ploieşti	Vol. LXIV No. 3/2012	67 - 72	Seria Tehnică

Study of a Mathematical Model of Vehicle Vibration

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Abstract

In the paper we propose the analysis of the vehicle oscillations, through a mathematical model with a small number of degrees of freedom, the vehicle vibration problem being particularly important because the car without a proper suspension loses its value, even if it has a good dynamic performance, the speed of the vehicles, usually, not being limited to the engine power but to the quality of the suspension.

Key words: *vibrations, vehicles oscillations, mathematical model of the vibrations.*

Introduction

The vehicle is a vibrant system in which each element (masses, elastic elements, and depreciation) has a determined function and influences differently the behavior of the vehicle on the different vibration modes on which it is subjected during the movement.

The vehicle vibration problem is particularly important because the car without a proper suspension loses its value, even if it has good dynamic performance, the speed of the vehicles, usually, is not limited by the engine power but by the suspension quality.

Currently, the random oscillations of the vehicle are being studied, considering the function that describes the road bumps as being random and stationary with the null media. By STAS the study of the random oscillations of vehicles is required, the comfort degree being given by the square average value of the vertical acceleration.

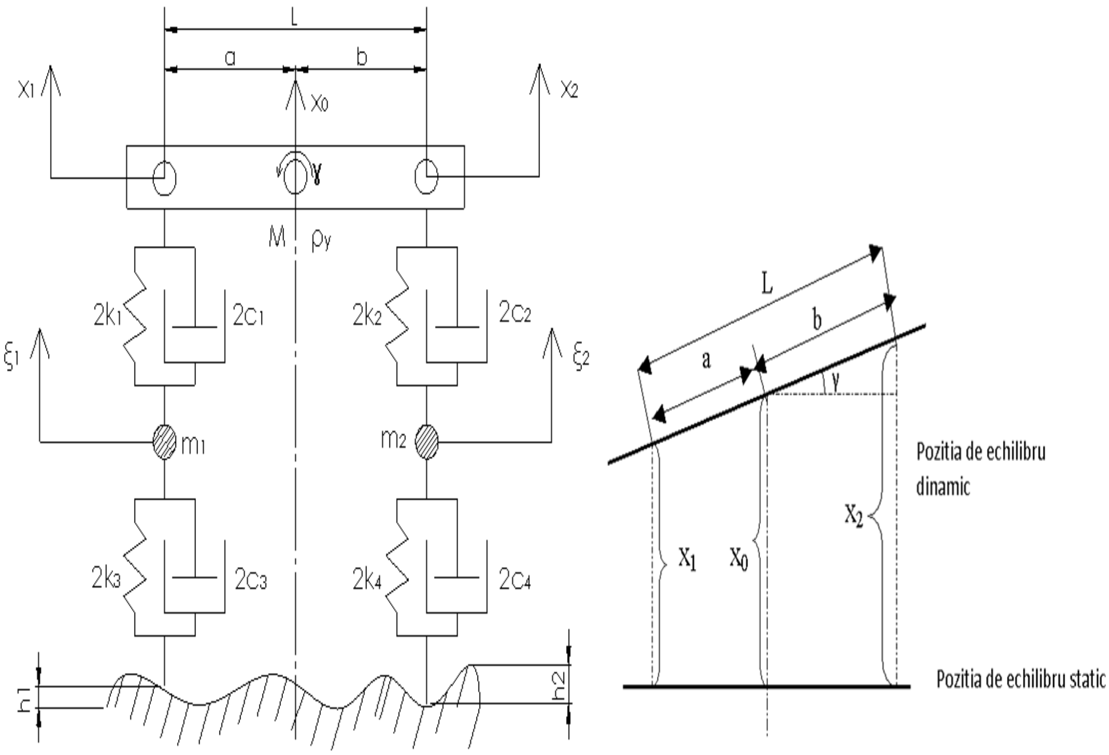
To analyze the car oscillations, the specialty literature studies relatively simple mathematical models, with a small number of degrees of freedom. A more complete model is the model that takes into consideration seven degrees of freedom, corresponding to the following parameters:

- three parameters for the housing (the two vertical displacements of the suspended mass in the face of the front and rear axles and the roll rotation of the suspended mass around the longitudinal axis of symmetry passing through the center of gravity of the vehicle);
- four parameters that describe the movement of the 4 wheels in vertical.

We can use the general theorems of dynamics for writing the equations of motion. This work method attracts laborious calculations in order to eliminate the reactions and then to arrange the equations in a symmetrical form. The obtained system does not take into account the drift and shake movements of the vehicle and neglects seat suspension.

The seven degrees of liberty model is solved only in particular cases[1] (when the result is a model with two degrees of freedom). The most common are the models with four and with two degrees of freedom.

The mathematical model with four degrees of freedom (fig. 1) is determined by the following parameters: the vertical movements x_1, x_2 of the suspended mass M (in the face of the front-rear axles) and the vertical displacements ξ_1, ξ_2 of the unsuspended masses m_1 or m_2, respectively.



a) b)

Fig. 1. Oscillating system with four degrees of freedom

The study model takes into account the equivalent elements for the spring and damper for suspension (reduced to the tread) k_1, k_2, c_1, c_2 and the tire stiffness k_3 and k_4. Regarding the tire damping c_3 and c_4, we neglect it.

By isolating the system bodies, the scheme of the forces acting on the suspended masses and the unsuspended masses is shown in Figure 2.

By applying the general theorems of the dynamics (for the rigid M with motion in parallel plan and for the two material points m_1 and m_2) results:

$$
\begin{aligned}
M\ddot{x}_0 &= -2k_1(x_1-\xi_1)-2c_1(\dot{x}_1-\dot{\xi}_1)-2k_2(x_2-\xi_2)-2c_2(\dot{x}_2-\dot{\xi}_2)\\
M\rho_y^{2\ddot{\gamma}} &= 2k_1(x_1-\xi_1)a+2c_1(\dot{x}_1-\dot{\xi}_1)a-2k_2(x_2-\xi_2)b-2c_2(\dot{x}_2-\dot{\xi}_2)b\\
m_1\ddot{\xi}_1 &= 2k_1(x_1-\xi_1)+2c_1(\dot{x}_1-\dot{\xi}_1)-2k_3(\xi_1-h_1)-2c_3(\dot{\xi}_1-\dot{h}_1)\\
m_2\ddot{\xi}_2 &= 2k_2(x_2-\xi_2)+2c_2(\dot{x}_2-\dot{\xi}_2)-2k_4(\xi_2-h_2)-2c_4(\dot{\xi}_2-\dot{h}_2)
\end{aligned}
\qquad (1)
$$

[1] The study is made for the case when the following two conditions are respected:

- the vehicle has stiff suspension on both axles (and is obtained a decoupling of the vertical oscillations of the roll);
- if the oscillations in the front of the car are decoupled from those in the back.

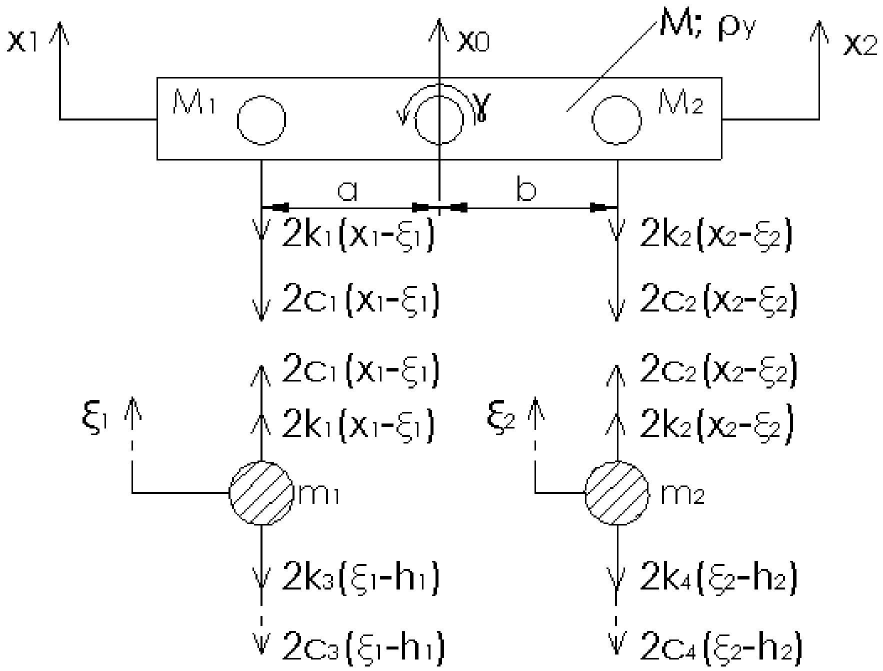


Fig. 2. Forces that act on the suspended mass and the unsuspended mass

The change of variables is made:

$$x_0 = \frac{x_1 b + x_2 a}{L} \quad \text{and} \quad \gamma = \frac{x_2 - x_1}{L} \tag{2}$$

Noting the reduced masses:

$$M_1 = \frac{b^2 + \rho_y^2}{L^2} M \ ; \quad M_2 = \frac{a^2 + \rho_y^2}{L^2} M \ ; \quad M_3 = \frac{ab - \rho_y^2}{L^2} M \tag{3}$$

and performing additional calculations in the system (1) to obtain symmetric equations, results:

$$\begin{aligned}
&M_1\ddot{x}_1 + 2c_1\dot{x}_1 + 2k_1x_1 + M_3\ddot{x}_2 - 2c_1\dot{\xi}_1 - 2k_1\xi_1 = 0 \\
&M_2\ddot{x}_2 + 2c_2\dot{x}_2 + 2k_2x_2 + M_3\ddot{x}_1 - 2c_2\dot{\xi}_2 - 2k_2\xi_2 = 0 \\
&m_1\ddot{\xi}_1 + 2(c_1 + c_3)\dot{\xi}_1 + 2(k_1 + k_3)\xi_1 - 2c_1\dot{x}_1 - 2k_1x_1 = 2k_3h_1 + 2c_3\dot{h}_1 \\
&m_2\ddot{\xi}_2 + 2(c_2 + c_4)\dot{\xi}_2 + 2(k_2 + k_4)\xi_2 - 2c_2\dot{x}_2 - 2k_2x_2 = 2k_4h_2 + 2c_4\dot{h}_2
\end{aligned} \tag{4}$$

If the suspended mass distribution is such that $\rho_y^2 = ab$, then $M_3 = 0$ and the system of equations (4) decomposes into two distinct parts. The equations with the coordinates x_1 and ξ_1 corresponding to the oscillations in front of the car cease to be linked with the equations of the coordinates x_2 and ξ_2 for the oscillations in the rear of the car. This enables to simplify the calculations.

Considering the equal structure of the equations (4), if $M_3 = 0$ we can study the oscillator system with two degrees of freedom (fig. 3) excited due to the h oscillation of the road. The equations (4) in this case become:

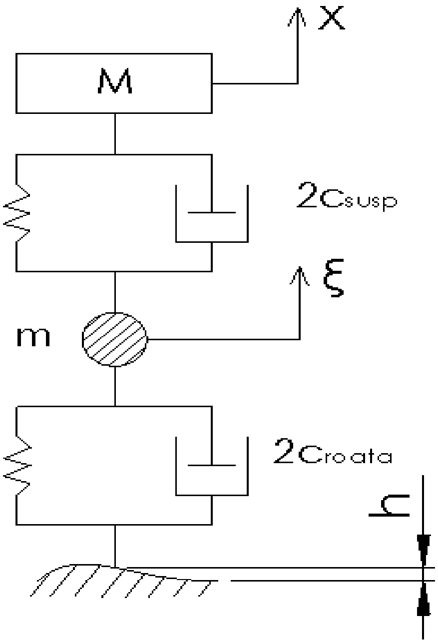


Fig. 3. Oscillating system with two degrees of freedom

$$\begin{aligned} &M\ddot{x}+2c_{susp}\dot{x}+2k_{susp}x-2c_{susp}\dot{\xi}-2k_{susp}\xi-2c_{susp}\dot{\xi}=0 \\ &m\ddot{\xi}+2(c_{susp}+c_{roata})\dot{\xi}+2(k_{susp}+k_{roata})\xi-2c_{susp}\dot{x}-2k_{susp}x=2k_{roata}h+2c_{roata}\dot{h} \end{aligned} \qquad (4')$$

The dynamic wheel charge is transmitted through the spring and damping elements of the wheel (fig. 3) and is expressed as:

$$P=2k_{roata}(h-\xi)+2c_{roata}(\dot{h}-\dot{\xi}) \qquad (4'')$$

Below is presented the solving of the motion equations using the equations method in complex. For the model with four degrees of freedom are denoted the complex coordinates z_1, z_2, z_3, z_4, corresponding to the real movements x_1, x_2, ξ_1, ξ_2.

The equations (4) passed into complex become:

$$\begin{aligned} &[-M_1\omega^2+2(c_1\omega i+k_1)]z_1-M_3\omega^2 z_2-2(c_1\omega i+k_1)z_3=0; \\ &-M_3\omega^2 z_1+[-M_2\omega^2+2(c_2\omega i+k_2)]z_2-2(c_2\omega i+k_2)z_4=0; \\ &-2(c_1\omega i+k_1)z_1+[-m_1\omega^2+2(c_1+c_3)\omega i+2(k_1+k_3)]z_3= \\ &=2h_0e^{i\omega t}(k_3+c_3\omega i) \\ &-2(c_2\omega i+k_2)z_2+[-m_2\omega^2+2(c_2+c_4)\omega i+2(k_2+k_4)]z_4= \\ &=2h_0e^{i\omega(t-t_0)}(k_4+c_4\omega i) \end{aligned} \qquad (5)$$

By solving the complex algebraic equation system (5), the complex values z_1, z_2, z_3, z_4 result for displacement of the body in front of the axels and the displacements of the unsuspended masses. Corresponding to these displacements can be calculated the response functions in frequency of the system for the accelerations $|A_1(\omega)|$, $|A_2(\omega)|$, $|A_3(\omega)|$, $|A_4(\omega)|$ that represent the ratio between the accelerations and the oscillation amplitude.

In case of the model with two degrees of freedom (fig. 3), to the real coordinates x and ξ correspond the complex variables z_{caros}, respectively z_{roata}. Passing the relations (4') in complex we obtain the following system of complex algebraic equations:

$$\begin{aligned}&\left(-M\omega^2 + 2c_{susp}\omega i + 2k_{susp}\right)z_{caros} - \left(2c_{susp}\omega i + 2k_{susp}\right)z_{roata} = 0;\\ &-\left(2c_{susp}\omega i + 2k_{susp}\right)z_{caros} + \left[-m\omega^2 + 2\left(c_{susp} + c_{roata}\right)\omega i + 2\left(k_{susp} + k_{roata}\right)\right]z_{roata} =\\ &= 2h_0\left(k_{roata} + c_{roata}\omega i\right)e^{i\omega t}\end{aligned} \tag{6}$$

From the system of equations (6) can be calculated the response function in frequency for the acceleration $\left|A(\omega)\right|$, representing the ratio between the body acceleration and the oscillation amplitude. Comparing the wheel load P given by the relation (4") to the oscillation amplitude and taking the model of this report we obtain the response frequency function for the dynamic load $\left|\psi(\omega)\right|$.

Conclusions

Knowing the system transfer functions $\left|A(\omega)\right|$ and $\left|\Psi(\omega)\right|$ we can calculate the square medium values of the acceleration and the dynamic load. Solving the motion equations by moving to the complex variables is adequate, easily applied for the study of the oscillations.

Symbols

M, m_1, m_2 – masses [kg];
h – range of road irregularity [cm];
h_1, h_2 – height of road irregularities in the face of the front and rear axle [cm];
a – distance from the center of gravity of the vehicle at the front axle [cm];
b – distance from the center of gravity of the vehicle at the rear axle [cm];
$\left|A(\omega)\right|$ - Frequency response function of the system for acceleration (is the ratio between the acceleration and the amplitude of road undulations), $\left[\frac{\text{cm/s}^2}{\text{cm}}\right]$;
$\left|\psi(\omega)\right|$ - Frequency response function of the system for the dynamic wheel stem (is the ratio between the wheel load and the amplitude of road undulations) [daN/cm];
c – damping constant [daNs/cm];
k – spring stiffness [daN/cn];
x_0 – vertical movement of the center of gravity of the suspended mass;
ξ - vertical unsuspended mass displacement;
γ - rotation of the suspended mass around its transverse axis O_y passing through its center of gravity (suspended mass pitching) [rad].
ρ - radius of inertia [cm];
ω – pulsation of the disruptive force dependent on speed [s^{-1}];
P – dynamic charge on the wheel [daN];
z – the complex generalized coordinate relative to the road oscillation amplitude;

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Studiul unui model matematic al vibraţiilor autovehiculelor

Rezumat

În lucrare se propune analiza oscilaţiilor automobilelor, printr-un model matematic, cu un număr mic de grade de libertate, problema vibraţiilor autovehiculelor fiind deosebit de importantă, deoarece automobilul fără o suspensie corespunzătoare îşi pierde din valoare, chiar dacă are performante dinamice bune, viteza de deplasare a automobilelor, de obicei, nu este limitată de puterea motorului ci de calitatea suspensiei.