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Analysis of Stresses to the Carcase of Water Filter

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Abstract

The paper presents the calculation of stresses produced in the water carcase filter due to the inside water pressure. The stresses are determined at the end of the carcase between the lock ring and cylindrical shell. In this area are produced maximum stresses.

Key words: *thin-walled vessel, pressure.*

Introduction

At the operation of the water filter were frequent cases when the filter carcase was cracked due to water pressure. Therefore the paper presents a study of stresses in the carcase wall filter. Stresses study was performed by using the analytical method of calculation. Water filter assembly consists of a cover 1, filter shell 2, nut 3, filter element 4, as shown in figure 1.

At the filter operation, his carcase was broken several times due to water pressure. In figure 2 is shows the filter carcase cracked.

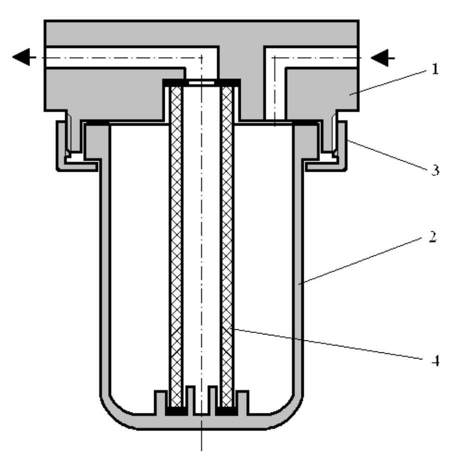

Fig. 1 Water filter assembly

Fig. 2 The filter body cracked

Assumptions for Calculation

Due to the dimensional characteristics of the body, we can be considered a revolution body with thin wall vessel, loaded with internal pressure.

Analytical Calculation of Stresses

For study of the stresses, we considered the top of the filter carcase. In figure 3 is represented the area considered for analysis.

In this case we have a static indeterminate system which is represented in figure 4.

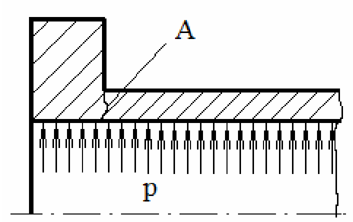

Fig. 3 End of the carcase area

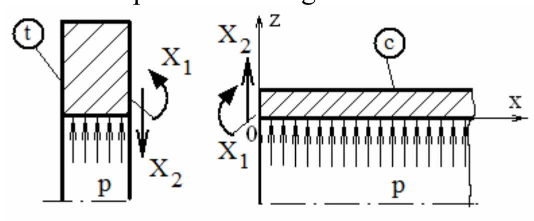

Fig. 4 The basic system

The basic system consists of a ring and a cylinder which have internal pressure. The ring can be considered as a portion of a thick wall tube which have internal pressure and the cylinder a revolution thin wall vessel with internal pressure.

The system equations will be:

$$\begin{aligned} X_1 \cdot \delta_{11} + X_2 \cdot \delta_{12} + \Delta_{10} &= 0 \\ X_1 \cdot \delta_{21} + X_2 \cdot \delta_{22} + \Delta_{20} &= 0 \end{aligned} \quad (1)$$

Where unknowns X1, X2 are respectively the bending moment and shear force. Calculation of coefficients δ_{11}, δ_{12}, δ_{22}, Δ_{10}, Δ_{20} for thin-walled cylindrical shell and thick wall tube corresponding ring, was made taking into account the loads and deformations shown in Figure 5 and account relationships 2..5.

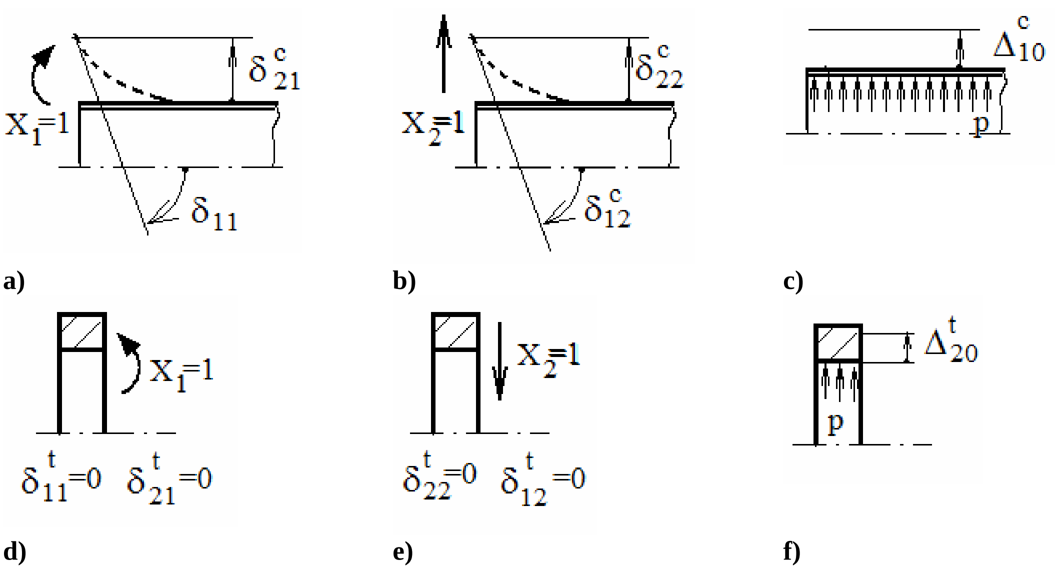

Fig. 5 Loads and strains on the basic system

Deformation and rotation of the cylindrical shell at one end loaded a bending moment and shear force are [1], [2], [3]:

$$w^c = \frac{1}{2 \cdot \beta^2 \cdot D} \cdot \left\{ M_0 \cdot e^{-\beta \cdot x} \cdot [\cos(\beta \cdot x) - \sin(\beta \cdot x)] + \frac{T_0}{\beta} \cdot e^{-\beta \cdot x} \cdot \cos(\beta \cdot x) \right\} + \bar{w} \quad (2)$$

$$\varphi^c = \frac{-1}{2 \cdot \beta \cdot D} \cdot \left\{ 2 \cdot M_0 \cdot e^{-\beta \cdot x} \cdot \cos(\beta \cdot x) + \frac{T_0}{\beta} \cdot e^{-\beta \cdot x} \cdot [\cos(\beta \cdot x) + \sin(\beta \cdot x)] \right\} + \bar{w}' \quad (3)$$

- for cylindrical shell whith inner pressure are:

$$w^c = \frac{p_i \cdot r^2}{2 \cdot E \cdot h} \cdot (2 - \mu) \quad (4)$$

- for thick-walled tube whith inner pressure are:

$$w^t = \frac{p_i}{E \cdot \left(\left(\frac{R_e}{R_i} \right)^2 - 1 \right)} \cdot \left[(1 - \mu) \cdot r + (1 + \mu) \frac{R_e^2}{r} \right] \quad (5)$$

Where:

- $\beta = \sqrt{\frac{\sqrt{3 \cdot (1 - \mu^2)}}{r \cdot h}}$, $D = \frac{E \cdot h^2}{12 \cdot (1 - \mu^2)}$ sizes that are involved in computing and are calculated according to dimensional characteristics of carcase and their material;
- E is the longitudinal modulus;-
- p_i is inner pressure;
- μ is Poisson coefficient;
- h is casing wall thickness;
- r current range;
- R_i, R_e, are the inner and outer radius of the tube with thick wall;
- x is abscissa;
- w is radial movement;
- φ is angle of rotation of the sections considered.

Coefficients for the Cylindrical Part

If it is considered that X1 = 1 corresponds to $M_0 = 1$ and $T_0 = 0$ of equation (2), (3) we obtaine:

$$\delta_{11}^c = -\varphi_{(x=0)} = \frac{1}{\beta \cdot D} \quad (6)$$

$$\delta_{21}^c = w_{(x=0)} = \frac{1}{2 \cdot \beta^2 \cdot D} \quad (7)$$

Internal pressure does not cause rotation of the cylinder cross section, we have in this case:

$$\Delta_{10}^c = 0 \quad (8)$$

If it is considered that X2 = 1 corresponds to $M_0 = 0$ and $T_0 = 1$ of equation (2), (3) we obtaine:

$$\delta_{12}^c = -\varphi_{(x=0)} = \frac{1}{2 \cdot \beta^2 \cdot D} \quad (9)$$

$$\delta_{22}^{c} = w_{(x=0)} = \frac{1}{2\cdot\beta^{3}\cdot D} \tag{10}$$

Internal pressure will cause movement of the cylinder cross section, the relation (4) we get:

$$\Delta_{20}^{c} = \frac{p_i\cdot r^{2}}{2\cdot E\cdot h}\cdot(2-\mu) \tag{11}$$

Coefficients for the Stiffening Ring

Bending moment and shear force will produce very small rotations and displacement as a result of stiffening ring coefficients δ_{11}, δ_{12}, δ_{22}, will be neglected. We have:

$$\delta_{11}^{t} = \delta_{22}^{t} = \delta_{12}^{t} = 0 \tag{12}$$

Internal pressure does not cause rotation of the ring cross section, in this case we have:

$$\Delta_{10}^{t} = 0 \tag{13}$$

Internal pressure will produce displacement of the ring section, the relation (5) will give:

$$\Delta_{20}^{t} = \frac{p_i}{E\cdot\left(\left(\frac{R_e}{R_i}\right)^{2}-1\right)}\cdot\left[(1-\mu)\cdot r+(1+\mu)\frac{R_e^{2}}{r}\right] \tag{14}$$

Coefficients of the system of equations (1) are obtained by applying the superposition principle we have in this case:

$$\delta_{11} = \delta_{11}^{c} + \delta_{11}^{t} = \frac{1}{\beta\cdot D} \tag{15}$$

$$\delta_{12} = \delta_{21} = \delta_{12}^{c} + \delta_{12}^{t} = \frac{1}{2\cdot\beta^{2}\cdot D} \tag{16}$$

$$\delta_{22} = \delta_{22}^{c} + \delta_{22}^{t} = \frac{1}{2\cdot\beta^{3}\cdot D} \tag{17}$$

$$\Delta_{20} = \Delta_{20}^{c} + \Delta_{20}^{t} = \frac{p_i\cdot r^{2}}{2\cdot E\cdot h}\cdot(2-\mu) + \frac{p_i}{E\cdot\left(\left(\frac{R_e}{R_i}\right)^{2}-1\right)}\cdot\left[(1-\mu)\cdot r+(1+\mu)\frac{R_e^{2}}{r}\right] \tag{18}$$

$$\Delta_{10} = \Delta_{10}^{c} + \Delta_{10}^{t} = 0 \tag{19}$$

Solution of Equations

The system of equations (1) becomes:

$$\frac{1}{\beta \cdot D} \cdot X_1 + \frac{1}{2 \cdot \beta^2 \cdot D} \cdot X_2 = 0$$

$$\frac{1}{2 \cdot \beta^2 \cdot D} \cdot X_1 + \frac{1}{2 \cdot \beta^3 \cdot D} \cdot X_2 = -\frac{p_i \cdot r^2}{2 \cdot E \cdot h} \cdot (2-\mu) + \frac{p_i}{E \cdot \left(\left(\frac{R_e}{R_i}\right)^2 - 1\right)} \cdot \left[(1-\mu) \cdot r + (1+\mu)\frac{R_e^2}{r}\right]$$

(20)

The system of equations (20) will have the following solutions:

$$X_1 = 2 \cdot \beta^2 \cdot D \cdot k \text{ , .. } X_2 = -4\beta^3 \cdot D \cdot k \qquad (21)$$

Where:

- $k = \frac{p_i \cdot r^2}{2 \cdot E \cdot h} \cdot (2-\mu) + \frac{p_i}{E \cdot \left(\left(\frac{R_e}{R_i}\right)^2 - 1\right)} \cdot \left[(1-\mu) \cdot r + (1+\mu)\frac{R_e^2}{r}\right]$ is a constant value.

Stresses Determination

With the determined values for X1 and X2 we can calculate the tension in the wall of the cylindrical shell. In this case we have:

$$\sigma_x = \frac{N_x}{h} \pm \frac{6 \cdot M_x}{h^2} \qquad (22)$$

- is stress on the cylinder axis direction.

$$\sigma_\theta = \frac{N_\theta}{h} \pm \frac{6 \cdot M_\theta}{h^2} \qquad (23)$$

- is the stress on the circumferential direction.

In figure 4 are shown diagrams of axial and circumferential stresses.

Efforts M_x, M_θ, N_x, N_θ will be determined by relations:

$$M_x = M_0 \cdot e^{-\beta \cdot x} \cdot [\cos(\beta \cdot x) + \sin(\beta \cdot x)] + \frac{T_0}{\beta} \cdot e^{-\beta \cdot x} \cdot \sin(\beta \cdot x) \qquad (24)$$

$$N_\theta = 2 \cdot \beta^2 \cdot \left\{ M_0 \cdot e^{-\beta \cdot x} \cdot [\cos(\beta \cdot x) - \sin(\beta \cdot x)] + \frac{T_0}{\beta} \cdot e^{-\beta \cdot x} \cdot \cos(\beta \cdot x) \right\} + \frac{E \cdot h}{r} \cdot \bar{w} + \mu \cdot N_x$$

(25)

$$N_x = \frac{r_i^2 \cdot p_i}{2 \cdot (r_i + \frac{h}{2})} \qquad (26)$$

Tresca equivalent stress will be:

$$\sigma_{Tresca} = \sigma_x - \sigma_\theta \qquad (27)$$

In figure 4 is shown diagram of Tresca equivalent stresses.

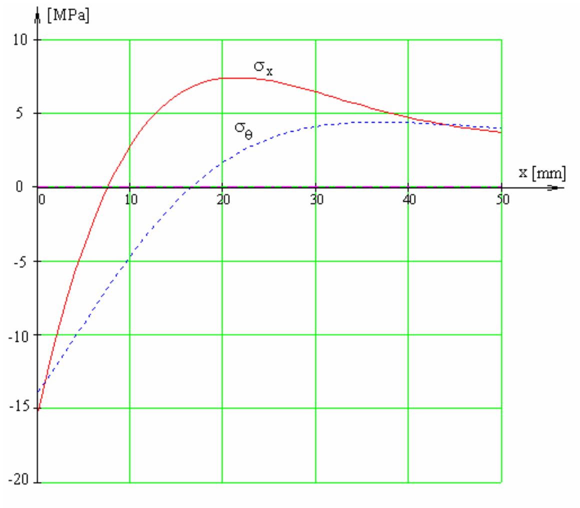

Fig. 6 Axial and circumferential stresses

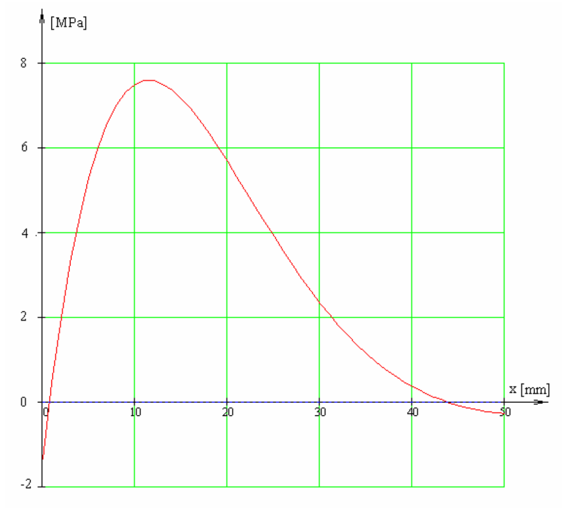

Fig. 7 Tresca equivalent stresses

Conclusions

Maximum equivalent stress is located at 12 mm distance of stiffening ring stiffening. In the same area carcase was cracking, see figure 2.

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Analiza tensiunilor la corpul filtrului de apa

Rezumat

In lucrare este prezentat calculul tensiunilor produse în carcasa filtrului de apă, de presiunea apei din interior. Tensiunile sunt determinate la capatul carcasei in zona de trecere de la inelul de rigidizare la invelisul cilindric. În acesta zona se produc tensiunile maxime.