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On the Positional Synthesis of a Mechanism with Two Independent Contours

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Abstract

In the paper some results concerning a positional synthesis problem for a plane mechanism with two independent contours is analyzed. The positional analysis, transposed into a computer program using the Maple programming language, is realized using the independent contours projection method. By imposing the values of the stroke of the component plunger for different values of the crank angle and the values of the crank angle for the maximum and minimum values of that stroke the dimension of the component links of the mechanism are determined using the optimization function LSSolve included in Maple program.

Key words: *mechanism, positional synthesis, optimization*

Introduction

It is a well known fact that various industrial activities require very precise mechanisms in the component machines serving the working processes. In a first phase of the design of these mechanisms they must meet certain positional conditions regarding the values of some angles or strokes of the component links [1, 2, 3]. So, in these cases a positional synthesis problem needs to be solved for determining the dimensions of the component links.

In this paper some results concerning a positional synthesis problem for a plane mechanism with two independent contours is analyzed. The positional analysis, transposed into a computer program using the Maple programming language [6], is realized using the independent contours projection method. By imposing the values of the stroke of the component plunger for different values of the crank angle and the values of the crank angle for the maximum and minimum values of that stroke the dimensions of the component links of the mechanism are determined using the optimization function *LSSolve* from Maple program [6].

Theoretical Considerations and Simulation Results

In figure 1 the cinematic scheme of a mechanism with two independent contours is presented. The two independent contours are: *O-A-B-C-O* and *C-B-D-C'-C.* φ_1 is the driving crank angle ($\varphi_1 \in [0, 2\pi)$) and ω_1 is the angular speed of the driving crank.

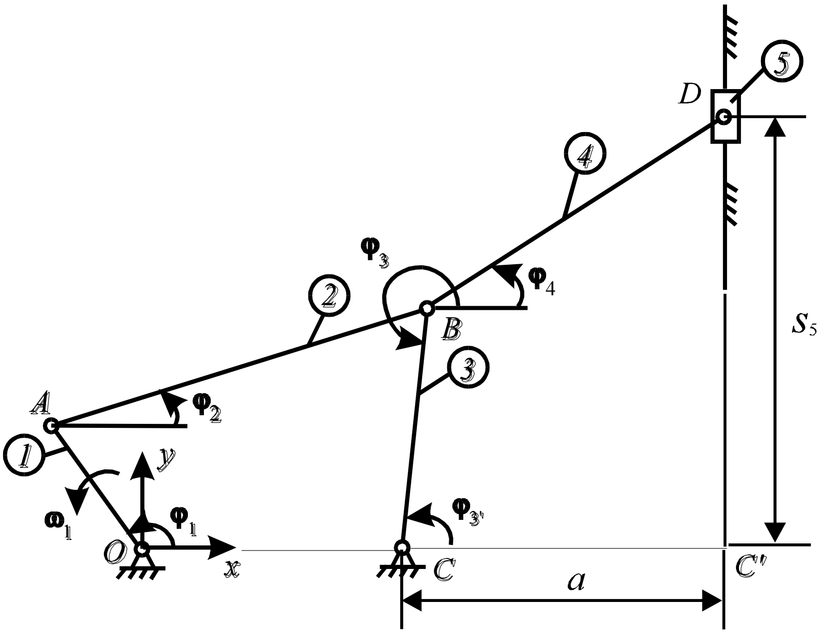

Fig. 1. Mechanism with two independent contours

The variation functions depending on φ_1 corresponding to the angles: $\varphi_2, \varphi_3, \varphi_4$ and to the stroke s_5 have been determined using the method of the projection of the independent vector contours [3,4].

The independent contour *O-A-B-C-O* has been projected on *x* and *y* axes (fig. 1) and the following system of equations was obtained:

$$\begin{cases} l_1 \cdot \cos\varphi_1 + l_2 \cdot \cos\varphi_2 + l_3 \cdot \cos\varphi_3 - l_0 = 0 \\ l_1 \cdot \sin\varphi_1 + l_2 \cdot \sin\varphi_2 + l_3 \cdot \sin\varphi_3 = 0 \end{cases} \tag{1}$$

where: $l_1 = OA$; $l_2 = AB$; $l_3 = BC$; $l_0 = OC$.

By solving the system of equations (1), the unknown angles φ_2 and φ_3 can be calculated from the following relations:

$$\begin{cases} A_2 \cdot \sin\varphi_2 + B_2 \cdot \cos\varphi_2 = C_2 \\ A_3 \cdot \sin\varphi_3 + B_3 \cdot \cos\varphi_3 = C_3 \end{cases} \tag{2}$$

where:

$$\begin{cases} A_2 = 2 \cdot l_1 \cdot l_2 \cdot \sin\varphi_1 \\ B_2 = 2 \cdot l_1 \cdot l_2 \cdot \cos\varphi_1 - 2 \cdot l_0 \cdot l_2 \\ C_2 = l_3^2 - l_1^2 - l_2^2 - l_0^2 + 2 \cdot l_0 \cdot l_1 \cdot \cos\varphi_1 \end{cases} \tag{3}$$

$$\begin{cases} A_3 = 2 \cdot l_1 \cdot l_3 \cdot \sin\varphi_1 \\ B_3 = 2 \cdot l_1 \cdot l_3 \cdot \cos\varphi_1 - 2 \cdot l_0 \cdot l_3 \\ C_3 = l_2^2 - l_1^2 - l_3^2 - l_0^2 + 2 \cdot l_0 \cdot l_1 \cdot \cos\varphi_1 \end{cases} \tag{4}$$

By projecting on *x* and *y* axes the second independent contour *C-B-D-C'-C* (fig. 1) the following system of equations was obtained:

$$\begin{cases} l_3 \cdot \cos\varphi_{3'} + l_4 \cdot \cos\varphi_4 - a = 0 \\ l_3 \cdot \sin\varphi_{3'} + l_4 \cdot \sin\varphi_4 - s_5 = 0 \end{cases} \tag{5}$$

where: $\varphi_{3'} = \varphi_3 - \pi$, $l_4 = BD$.

By solving the system of equations (5), the angle φ_4 and the stroke s_5 can be calculated from the following relations:

$$\begin{cases} \cos\varphi_4 = \dfrac{1}{l_4} \cdot \left(-l_3 \cdot \cos\varphi_{3'} + a\right) = \dfrac{1}{l_4} \cdot \left(l_3 \cdot \cos\varphi_3 + a\right) \\ s_5 = l_3 \cdot \sin\varphi_{3'} + l_4 \cdot \sin\varphi_4 = -l_3 \cdot \sin\varphi_3 + l_4 \cdot \sin\varphi_4 \end{cases} \tag{6}$$

It is denoted by φ_{1d} and φ_{1a} the values of the crank angle φ_1 corresponding to the extreme positions of the rocker *3*. φ_{1d} is the value of the crank angle φ_1 when the connecting rod *2* is in the prolongation of the driving crank *1* and φ_{1a} corresponds to the case when the connecting rod *2* overlaps the driving crank *1*.

By taking into account the mechanism structure (fig. 1), the angles φ_{1d} and φ_{1a} correspond to the extreme values of the stroke s_5.

By projecting the contour *O-A-B-C-O* on the *x* and *y* axes (fig. 1) for these two extreme positions of the rocker, the following systems of equations were obtained:

$$\begin{cases} (l_1 + l_2) \cdot \cos\varphi_{1d} + l_3 \cdot \cos\varphi_{3d} - l_0 = 0 \\ (l_1 + l_2) \cdot \sin\varphi_{1d} + l_3 \cdot \sin\varphi_{3d} = 0 \end{cases} \tag{7}$$

$$\begin{cases} (l_1 - l_2) \cdot \cos\varphi_{1a} + l_3 \cdot \cos\varphi_{3a} - l_0 = 0 \\ (l_1 - l_2) \cdot \sin\varphi_{1a} + l_3 \cdot \sin\varphi_{3a} = 0 \end{cases} \tag{8}$$

where: φ_{3d} and φ_{3a} are the values of the angle φ_3 for the extreme positions of the rocker *3* of the mechanism.

By solving the systems of equations (7) and (8), the unknown parameters φ_{1d} and φ_{1a} were calculated from the following relations:

$$\begin{cases} A_{1d} \cdot \cos\varphi_{1d} = C_{1d} \\ A_{1a} \cdot \cos\varphi_{1a} = C_{1a} \end{cases} \tag{9}$$

where:

$$\begin{cases} A_{1d} = -2 \cdot l_0 \cdot (l_1 + l_2) \\ C_{1d} = l_3^2 - (l_1 + l_2)^2 - l_0^2 \end{cases} \tag{10}$$

$$\begin{cases} A_{1a} = -2 \cdot l_0 \cdot (l_1 - l_2) \\ C_{1a} = l_3^2 - (l_1 - l_2)^2 - l_0^2 \end{cases} \tag{11}$$

and then the unknown parameters φ_{3d} and φ_{3a} have been determined from the following relations:

$$\begin{cases} \varphi_{3d} = ATAN2(-(l_1 + l_2) \cdot \sin\varphi_{1d}, l_0 - (l_1 + l_2) \cdot \cos\varphi_{1d}) \\ \varphi_{3a} = ATAN2(-(l_1 - l_2) \cdot \sin\varphi_{1a}, l_0 - (l_1 - l_2) \cdot \cos\varphi_{1a}) \end{cases} \tag{12}$$

where: *ATAN*2(*y*,*x*) function calculates arctg(*y*/*x*) by taking into account the sign of the parameters *y* and *x* [3].

The relations above have been utilized to develop a computer program using Maple program that has integrated powerful functions for symbolical calculus. In this way, the analytical expressions of the angles $\varphi_2, \varphi_3, \varphi_4$ and of the stroke s_5, depending on the lengths of the component links and on the crank angle φ_1, have been obtained.

For simulations the following values for the dimensions of the component links have been considered: $OA = 0.2\,\mathrm{m}$, $AB = 0.5\,\mathrm{m}$, $BC = 0.45\,\mathrm{m}$, $OC = 0.6\,\mathrm{m}$, $BD = 0.75\,\mathrm{m}$, $a = 0.3\,\mathrm{m}$. In Figure 2 the variation of the angles $\varphi_2, \varphi_3, \varphi_4$ on a cinematic cycle is presented and in Figure 3 it is shown the variation on a cinematic cycle of the stroke s_5.

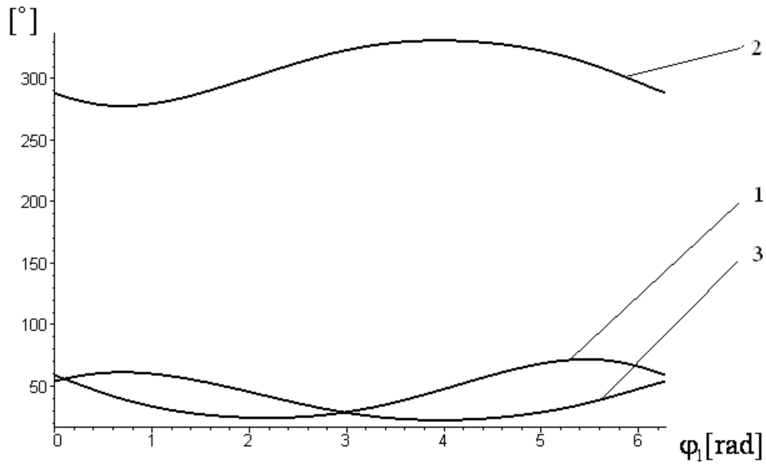

Fig. 2. The variation on a cinematic cycle of the angles φ_2 (curve *1*), φ_3 (curve *2*) and φ_4 (curve *3*)

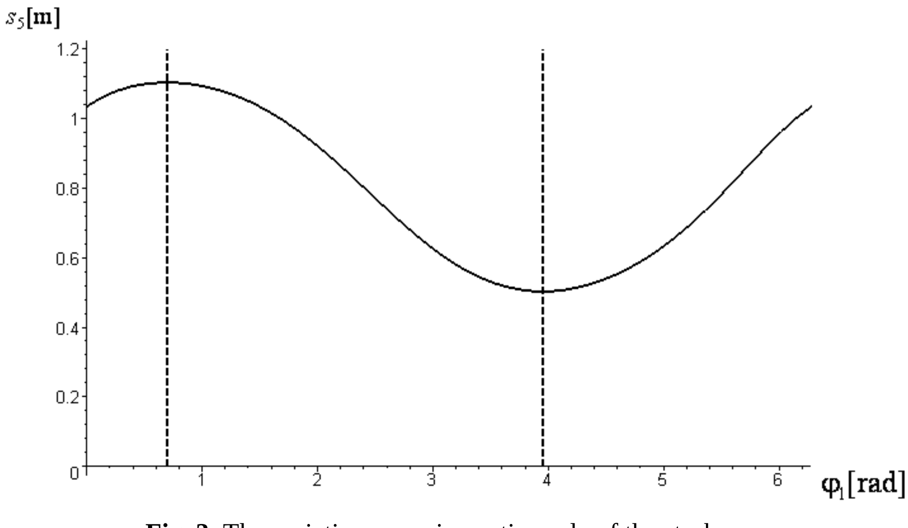

Fig. 3. The variation on a cinematic cycle of the stroke s_5

In Figure 3, the dashed lines correspond to the angles φ_{1d} and φ_{1a} that in this case have the following values: $\varphi_{1d} = 0.691\mathrm{rad}$ (39.571°) and $\varphi_{1a} = 3.954\mathrm{rad}$ (226.567°). For these two values of the crank angle φ_1 the stroke s_5 has the following values: 1.104 m and 0.503 m, respectively.

Further, the values of the stroke s_5 for different values of the crank angle φ_1 and the values of the crank angle φ_{1d} and φ_{1a} will be imposed. So, for $\varphi_1 = 0$, $s_5 = 1\text{m}$, for $\varphi_1 = \varphi_{1d}$, $s_5 = 1.17\text{m}$ and for $\varphi_1 = \varphi_{1a}$, $s_5 = 0.52\text{m}$. The imposed values for the crank angles φ_{1d} and φ_{1a} are: $\varphi_{1d} = 40^\circ$ and $\varphi_{1a} = 220^\circ$.

The dimensions of the component links of the mechanism have been determined using the optimization function *LSSolve* from Maple program [6]. The optimization function *LSSolve* solves a least squares problem, which involves computing the minimum of an objective function having the form:

$$\frac{1}{2}\cdot\left(\left(s_{5,\varphi_1=0} - s_{5T,\varphi_1=0}\right)^2 + \left(s_{5,\varphi_1=\varphi_{1d}} - s_{5T,\varphi_1=\varphi_{1d}}\right)^2 + \left(s_{5,\varphi_1=\varphi_{1a}} - s_{5T,\varphi_1=\varphi_{1a}}\right)^2\right) \tag{13}$$

where: $s_{5,\varphi_1=0}, s_{5,\varphi_1=\varphi_{1d}}$ and $s_{5,\varphi_1=\varphi_{1a}}$ are the values of the stroke s_5 when $\varphi_1 = 0$, $\varphi_1 = \varphi_{1d}$ and $\varphi_1 = \varphi_{1a}$, respectively.

In relation (13), $s_{5T,\varphi_1=0}, s_{5T,\varphi_1=\varphi_{1d}}$ and $s_{5T,\varphi_1=\varphi_{1a}}$ are the target values of the stroke s_5 when $\varphi_1 = 0$ ($s_5 = 1\text{m}$), $\varphi_1 = \varphi_{1d}$ ($s_5 = 1.17\text{m}$) and $\varphi_1 = \varphi_{1a}$ ($s_5 = 0.52\text{m}$), respectively.

In the optimization function *LSSolve* the following constraints have been introduced:

- *Grashof* condition [3,4] that in the analyzed case has the following form: $l_1 + l_0 \le l_2 + l_3$;
- $(l_3 + a)/l_4 \le 1$ and $(-l_3 + a)/l_4 \ge -1$;

The last two constraints conditions are coming from the condition: $|\cos\varphi_4| \le 1$, where the expression of $\cos\varphi_4$ is given by the first relation in (6).

The dimensions of the component links of the mechanism obtained using the optimization function *LSSolve* are: $OA = 0.272\text{m}$, $AB = 0.337\text{m}$, $BC = 0.393\text{m}$, $OC = 0.441\text{m}$, $BD = 0.816\text{m}$, $a = 0.271\text{m}$.

In Figure 4, it is shown the variation on a cinematic cycle of the stroke s_5 for the new dimensions of the component links.

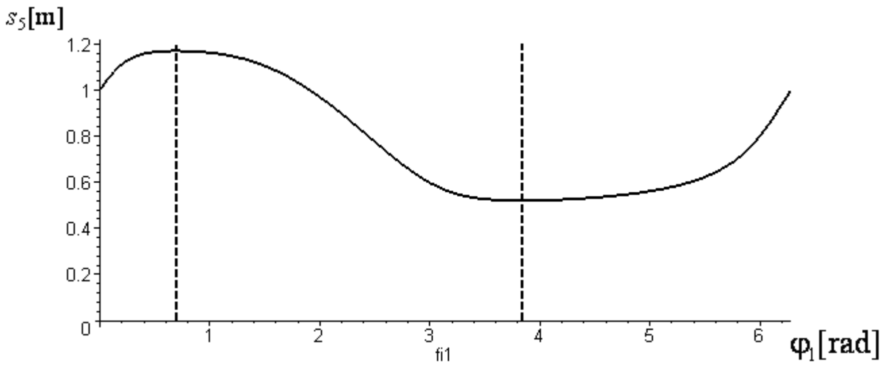

Fig. 4. The variation on a cinematic cycle of the stroke s_5 for the new dimensions of the links

In Figure 4, the dashed lines correspond to the angles φ_{1d} and φ_{1a} that in this case have the following values: $\varphi_{1d} = 0.698\text{rad}$ (40º) and $\varphi_{1a} = 3.840\text{rad}$ (220º).

Conclusions

In this paper, a positional synthesis problem for a plane mechanism with two independent contours has been analyzed. The positional analysis has been transposed into a computer program using the Maple programming language. The optimization function *LSSolve* from Maple program has been used to determine the dimensions of the component links of the mechanism when the values of the stroke of the component plunger for different values of the crank angle and the values of the crank angle for the maximum and minimum values of that stroke have been imposed.

References

1. Bădoiu, D. – *Analiza dinamică a mecanismelor şi maşinilor,* Editura Didactică şi Pedagogică, Bucureşti, 2003.
2. Bădoiu, D. – On the dynamic behavior of a quadrilateral mechanism, *Buletinul Universităţii Petrol-Gaze din Ploieşti, Seria Tehnică*, nr. 2, 2012.
3. Bădoiu, D. – *Analiza structurală şi cinematică a mecanismelor,* Editura Tehnică, 2001.
4. Handra-Luca, V. – *Functiile de transmitere in studiul mecanismelor,* Editura Academiei, Bucuresti, 1983.
5. Dodescu, Gh., Toma, M. – *Metode de calcul numeric,* Editura Didactică şi Pedagogică, Bucureşti, 1976.
6. Monagan, M.B., Geddes, K.O., Heal, K.M., Labahn, G., Vorkoetter, S.M., McCarron, J., DeMarco, P. – *Maple Introductory Programming Guide*, Maplesoft, a division of Waterloo Maple Inc., 2005.

Asupra sintezei poziţionale a unui mecanism cu două contururi independente

Rezumat

In articol sunt prezentate o serie de rezultate privind o problemă de sinteză poziţională a unui mecanism . cu două contururi independente. Analiza poziţională, transpusă într-un program de calculator folosind limbajul de programare Maple, s-a realizat folosind metoda proiecţiei contururilor independente. Prin impunerea valorilor cursei pistonului component pentru diferite valori ale unghiului de manivelă şi a valorilor unghiului de manivelă pentru valoarea minimă şi maximă a acestei curse dimensiunile elementelor componente ale mecanismului sunt determinate folosind funcţia de optimizare LSSolve inclusă în programul Maple.