Modeling and Calculation of Cracks in Solids with Fractal Interpolation Method

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Abstract

The object of this study is the modeling and the computational calculations for various geometries of cracks in different solid materials, that are were generated within the rules of calculation of fractal geometry. The premises of the paper are using forms similar to those from reality, i.e. fractal forms that, unlike the continuous forms from classical models, manage to better replicate the shapes in the real world. Cracks as found in solid materials, and fractals as computer generated abstract forms have the same common properties of self-similarity. Research tools are discrete mathematical functions obtained by fractal interpolation; for their implementation and delivery in an application, a mathematical programming method involving matrix computation and recursive was used. In order to model the crack geometry, one begins with various arrangements of points in the plane, to obtain in the end result through fractal interpolation to similar forms present in reality. The subject of this paper is the modelling and calculation of cracks found in various materials with particular aspects of interest.

Key words: *modeling cracks, fractal interpolation, crack geometry, solid materials, computer generated.*

Introduction

This paper seeks to generate cracks by numerical methods using the computer. There are considered two types of cracks: those which propagate them selves in one line, and the second type, cracks which have a radial arrangement, the second type is often met in the frontal impact of a body with the materials in question.

Crack generating method uses fractal interpolation applied on random data series which will be named generated data series. These data series are built through integer values accumulation and random values on two directions, horizontal and vertical.

In order to generate and calculate cracks I used algorithms which were implemented in MathCAD.

Next to the cracks generated on PC, I will give several images with cracks from technical and construction fields. Also, in the next pages, there are indicated the values of the used parameters.

Toolbox

In order to model cracks a MathCAD software program is used, see paper [2]. As a study method there are used dynamic systems of IFS type (iterated function system). These systems are built helped by interpolation fractal function. Making algorithm involves working with matrices, recursive functions, functions for generating random numbers and probability functions.

Below are given the calculation relationships for building the fractal interpolation function with their parameters, see the paper [1]. This function interpolates the generated data series.

$$I(r) = \begin{pmatrix} a_n & 0 \\ c_n & d_n \end{pmatrix} r + \begin{pmatrix} e_n \\ f_n \end{pmatrix}, \tag{1}$$

where $r = \begin{pmatrix} x \\ y \end{pmatrix}$

$$a_n = \frac{x_n - x_{n-1}}{x_N - x_0} \tag{2}$$

$$e_n = \frac{x_N x_{n-1} - x_0 x_n}{x_N - x_0} \tag{3}$$

$$c_n = \frac{y_n - y_{n-1} - s(y_N - y_0)}{x_N - x_0} \tag{4}$$

$$f_n = \frac{x_N y_{n-1} - x_0 y_n - s(x_N y_0 - x_0 y_N)}{x_N - x_0} \tag{5}$$

$$d_n = s = \frac{N^{\dim-1}}{N-1} \tag{6}$$

The involved parameters are: s - vertical scaling factor, *dim* - fractal dimension, N - number of dots of the data series to be interpolated, this parameter will be called precision, N_t - the number of points the new data series, this parameter will be called density, and n is the index of the interpolated data series.

Recurring relations that lead to create the rank of generated data series, see the paper [3], they are:

$$K^{<n+1>} = \begin{cases} K^{<n>} + \begin{pmatrix} cS_{0,n} - 1 \\ 0 \end{pmatrix} & if(S_{1,n} = 0) \\ K^{<n>} + \begin{pmatrix} 0 \\ cS_{0,n} - 1 \end{pmatrix} & if(S_{1,n} = 1) \end{cases} \tag{7}$$

and initial condition, $K^{<0>} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

where: $S_{0,n} = floor(rnd(2))$ and $S_{1,n} = floor(rnd(2))$ are random functions that determine the meaning and direction of development of the graph, and c is the coefficient for the step. Terms of values develop after the 4 directions. The generated data series is, therefore, the strange attractor of the dynamical system from above.

Modeling Cracks and Their Identification into the Materials. Specific Cases

Cracks generated by the application can be founded in various materials such as steel, asphalt, concrete, rubber, etc. The appearance is given by the form and by the texture of the crack.

In the first case, the application generates cracks that which propagate them selves in one line.
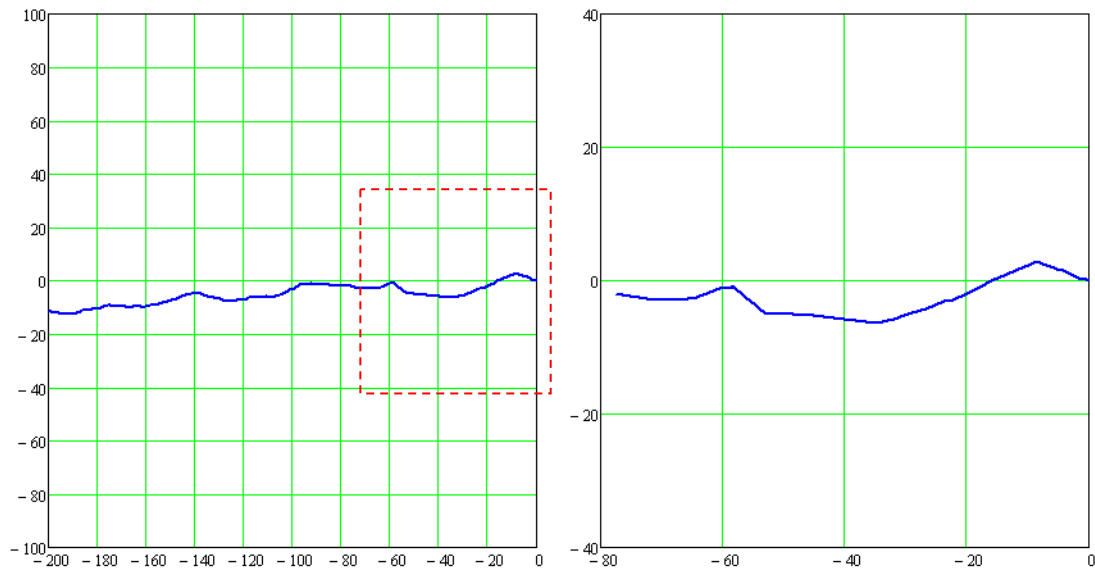

Fig. 1. A crack is generated with: fractal dimension = 1.1, precision = 50, density = 150, viewed at different scales, and the length=293 of the crack is determined on the interval [-200.0] (the crack can be seen at different scales).
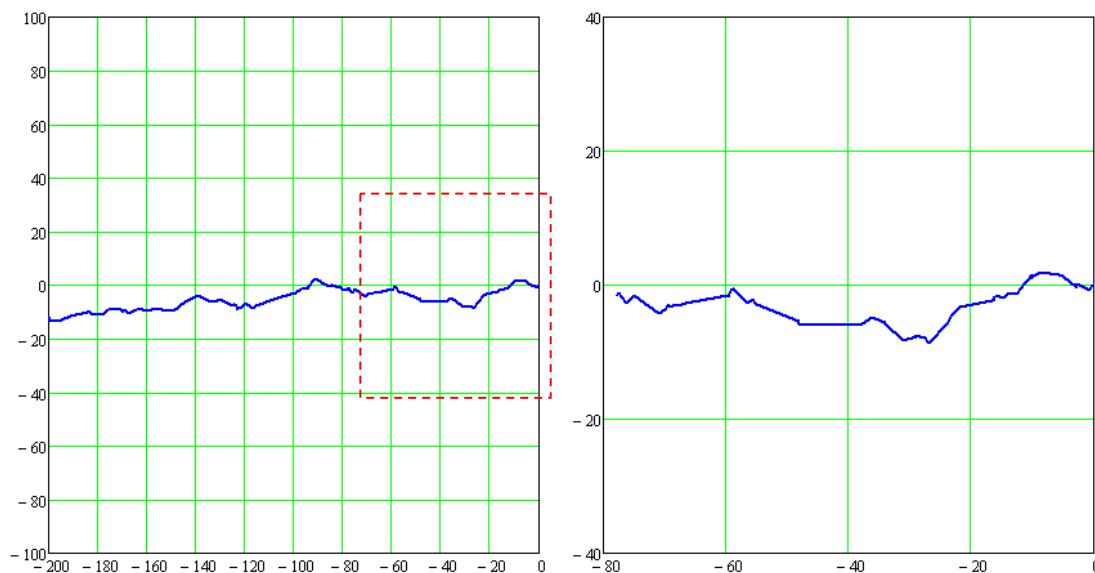

Fig. 2. A crack is generated with: fractal dimension = 1.6, precision = 330, density = 150, viewed at different scales, and the length=310 of the crack is determined on the interval [-200.0] (the crack can be seen at different scales).

The figure below presents several simulations of fractal interpolation function. In this case the function is represented by dots.

The generated cracks have changed form and texture.

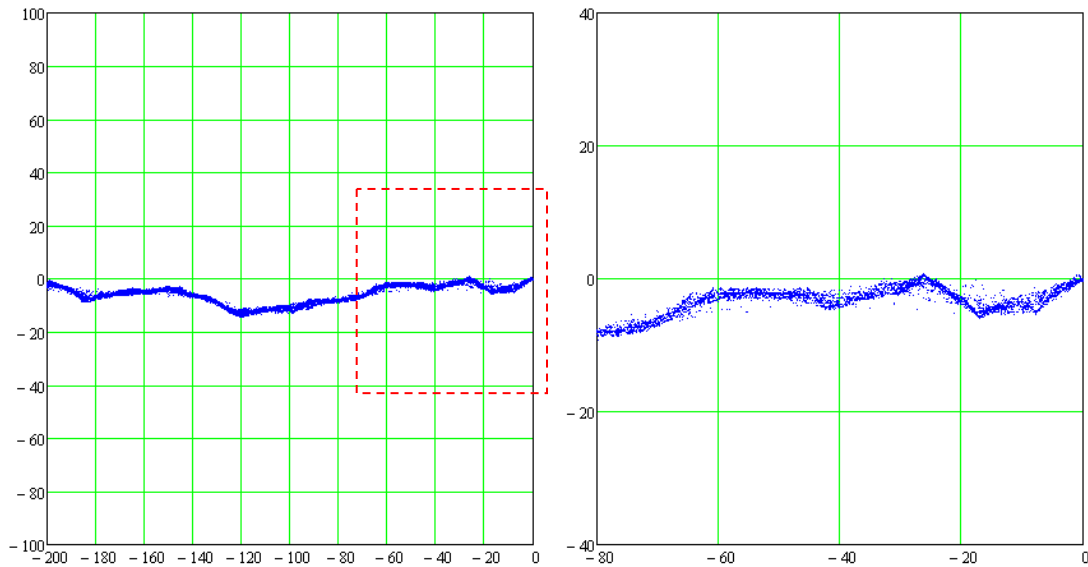These changes are determinate by considered parameters.

Fig. 3. A generated crack with: fractal dimension = 1.1, precision = 50, density = 150, viewed at different scales and at different number of simulations = 111.
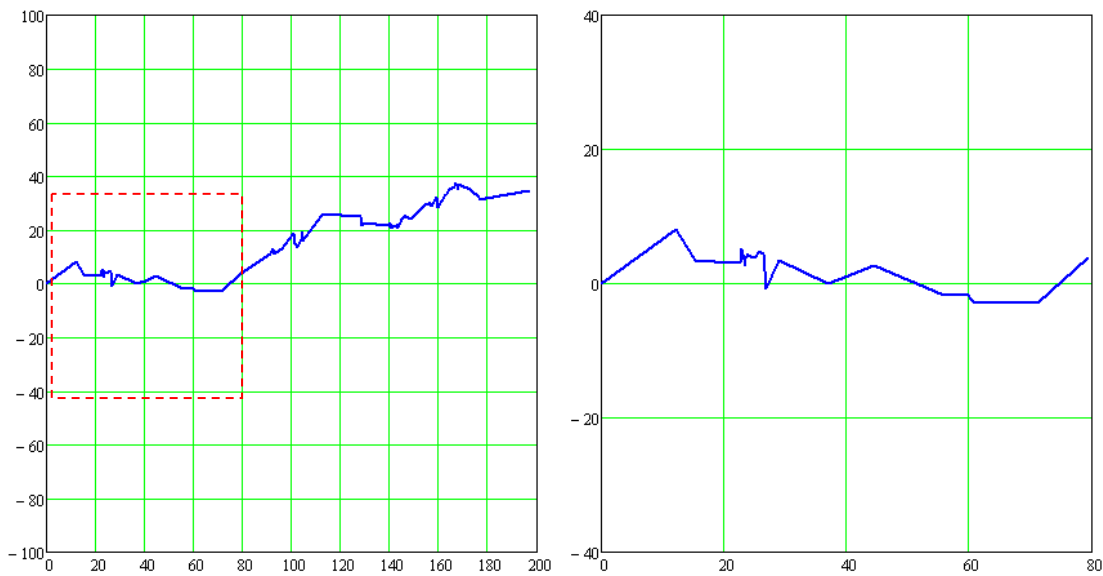

Fig. 4. A generated crack with: fractal dimension = 1.5, precision = 100, density = 100, viewed at different scales and with range coefficient = 3.

For the first case, I will identify cracks without ramifications in some materials that are technically used.

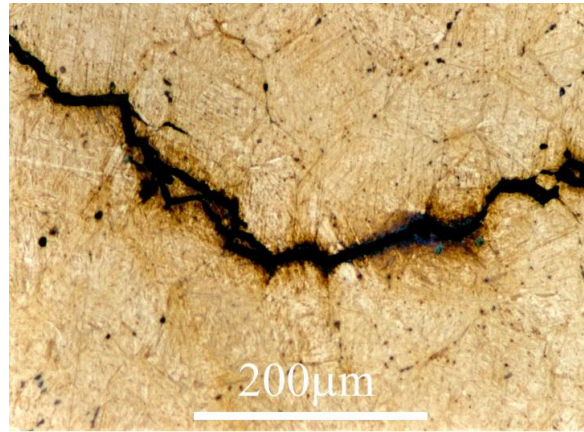**Fig. 5.** Cracks in a piece of composite material [4]

Fig. 6. Crack in steel [5]

Fig. 7. Crack in a tire [6]

Fig. 8. Crack in concrete foundation [7]

I will identify in the following situations cracks in some materials that are commonly used in the construction industry.
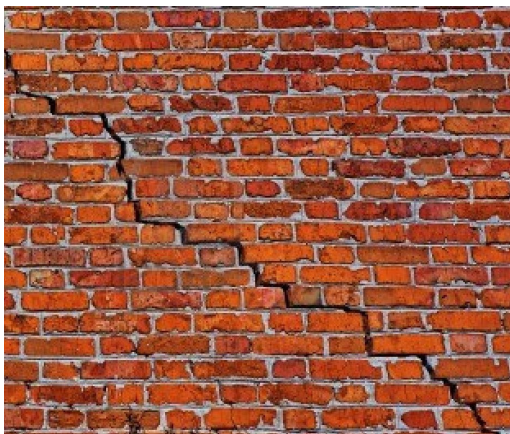

Fig. 9. Crack in brick wall [8]

Fig. 10. Crack in asphalt [9]

The second case: to generate a radial arrangement of the crack. This situation is frequently encountered at the impact of the materials with other bodies.
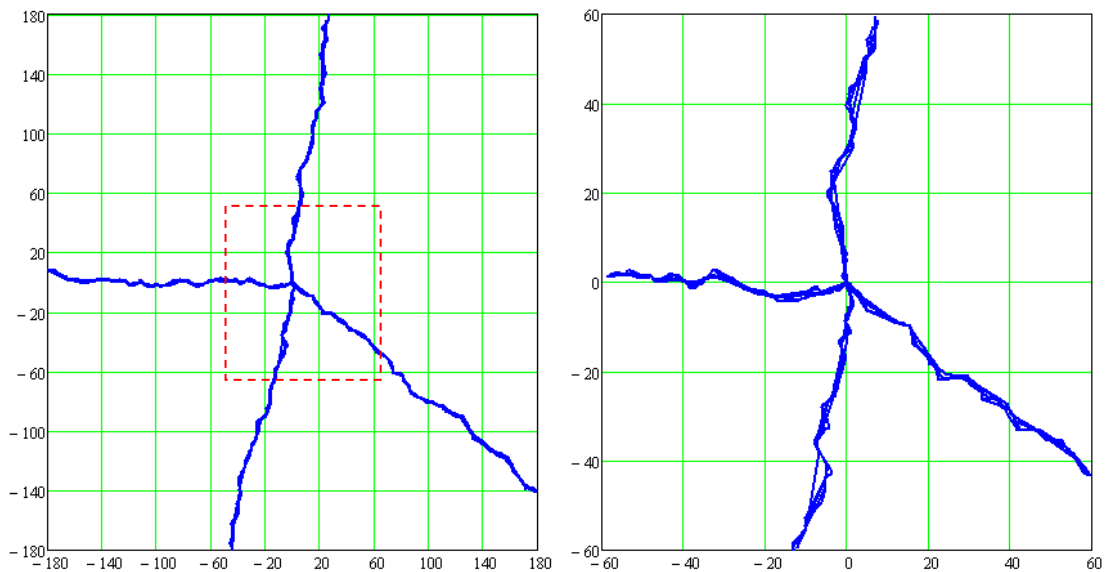
Fig. 11. Generated cracks with: fractal dimension = 1.1, precision = 150, density = 150, viewed at different scales and number of simulation = 4.
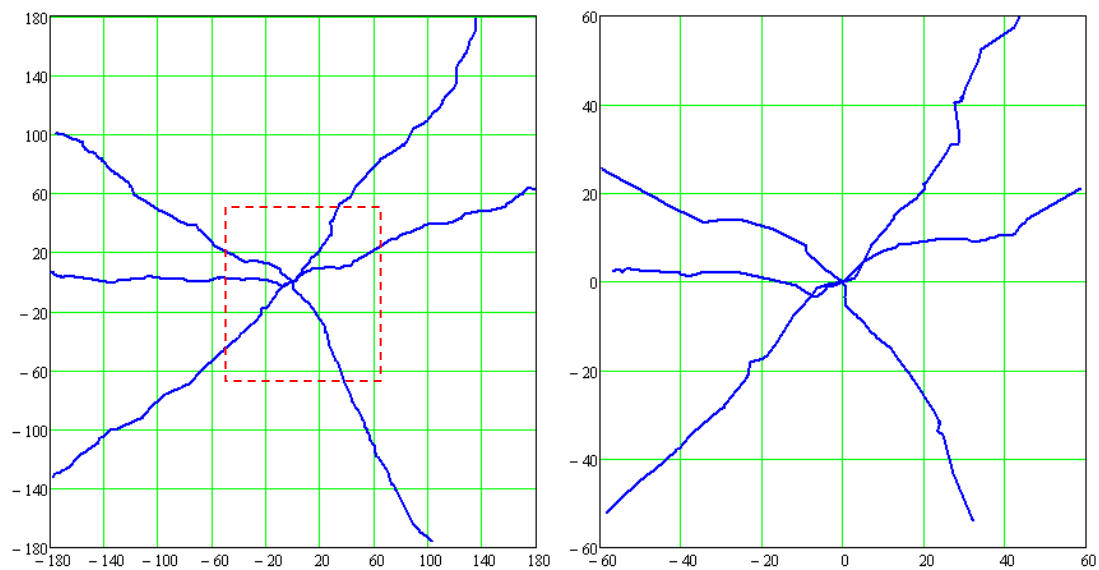

Fig. 12. Generated cracks with: fractal dimension = 1.1, precision = 50, density = 150, viewed at different scales and number of simulation = 6.

For the second case, I will identify some radial cracks encountered in different materials. The cracks are similar to those generated above.

The appearance of cracks presented before, is different from one material to another, for instance the crack in wood is different from that in the composite material. The one in wood has smoother features and straighter shapes (fig.14), while the one in composite materials shows higher deviations from direction (fig. 5). The micro-crack in steel has sudden deviations from direction (fig. 6). On the other hand, it can be said that there is a similarity of traits of the cracks that are from completely different materials, for example the crack from composite materials with the crack from concrete foundation (fig. 5 and fig. 8). For this, the application has parameters that can adjust the local deviation from the direction. The most important parameter for interpolation function is *fractal dimension* between 1 and 2, this gives fractured features,

depending on the set value (fig. 1 and fig. 2). Besides the parameters interpolation function, the characteristics of the crack can be changed by changing the value of the coefficient of generating function (fig. 4).

Fig. 13. Radial cracks propagated as a result of the impact of a bullet with a ceramic plate [10]

Fig. 14. Radial cracks found in a trunk [11]

Fig. 15. Radial cracks, and not only, in a pavement [12]

At some cracks can be seen that not only the propagation line presents irregularities but the crack thickness also (fig. 6). Through repeated dialing of interpolation function it can be obtained a set of graphs that are locally getting closer one of another or locally getting further, thus providing the variable thickness of the generated crack (fig. 11). Only by changing the mode of the representation, is obtained a field points which approximates the generated data series (fig. 3).

At cracks from materials with a radial arrangement, even if they have a non deterministic propagation character, the position of the branches has not large deviations from the equidistant position (fig. 13, fig. 14 and fig. 15). When modeling, that was taken into account, the position of each branch was determined by probabilistic calculation.

Looking at different scales, it can be observed the property of self-similarity of the appearance of cracks in different materials. This property is found also in cracks generated by the application, as shown above (fig. 11 and fig. 12).

In various situations it is necessary to know the length of the crack in order to achieve the various calculations of strength. Thus, a crack from a certain material can be approximated by fractal interpolation after that it can be easily determined the crack length (fig. 1 and fig. 2), where the tensions that produced it have occurred.

The Principle of Realization for the Modeling Algorithm

The basic principle from which we started is the fracturated appearance of crack and mathematical apparatus suitable in this situation uses iterated systems that lead to fractals generation.

The characteristic of the shape from which has started to generate cracks is a similar texture with that of the crack in the brick wall from the above example (fig. 11). If we consider the phenomenon in a locally way, the bricks of the wall require the propagation of the crack in only two directions, horizontally and vertically. However, on a larger scale, the crack can propagate itself in any direction. In conclusion, as the building blocks require only two directions of propagation and the quantification of the phenomenon, in the same way the generated data series are quantified in a rectangular grid of integers. For the value of generated data series coefficient that is equal to 1, the ideal direction of development of the graph of random values that are accumulated at an angle of 45 degrees to the horizontal.

Since fractal interpolation function contains the scaling factor *s* vertically, the chart generator tool will be rotated horizontally for optimum interpolation.

Once prepared, the generated data series is interpolated by the function call. After that, the next call of the interpolation function it is obtained another set of values. Thus, at any number of dials, it is obtained a set of graphs which are represented by lines or dots.

In case of crack representation with radial arrangement, the position of each branch is determined by the probability function for the normal distribution, the mean is determined by the number of branches, and the standard deviation is setted by the user.

Conclusions

In this paper I insisted on modeling by generating and not by copying forms. The algorithm presents enough parameters for the geometrical adjustment that lead to achieving desired shapes and features for the generated crack. The spectrum of generated geometries for the two case situations is great. Once the appearance of crack is found, it can be made a determination of the lengths.

Also by adjusting the parameters, the generated crack can be viewed at different scales.

By changing the value of the coefficient of generated data series, the obtained diagrams can be extended to representing the breaks of materials.

The fractal interpolation model implemented here can be extended and used also for other types of data series, such as: data series obtained with continuous and differentiable functions, data series read from place to place with reading instruments and than transferred to the computer, or other data series that are of interest.

This paper aims to respect the non deterministic character of propagation phenomenon and fractal appearance of cracks in the cracks generating and calculating process using computer.

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Modelarea şi calculul fisurilor în materiale solide prin metoda interpolării fractale

Rezumat

Obiectul prezentului studiu constă în modelarea şi calculul computaţional pentru diverse geometrii de fisuri care se găsesc la diferite materiale solide, generarea lor respectând regulile de calcul ale geometriei fractale. Premisele lucrării sunt utilizarea formelor similare cu cele din realitate, adică a formelor fractale care spre deosebire de formele continue din modelele clasice, reuşesc să copieze mai bine formele din lumea reală. Fisurile ca aspect, întâlnit la materialele solide, şi fractalii ca formă abstractă generată de calculator, au aceleaşi proprietăţi comune de auto-asemănare. Instrumentele de cercetare sunt funcţiile matematice discrete obţinute prin interpolare fractală, iar pentru implementarea şi concretizarea lor într-o aplicaţie, s-a folosit metoda programării matematice ce implică calcul matriceal şi recursiv. Pentru modelarea geometriei fisurilor se porneşte de la diverse dispuneri ale punctelor în plan, pentru a se ajunge apoi prin interpolare fractală, la forme similare prezente în realitate. Lucrarea urmăreşte modelarea şi calculul unor fisuri din diverse materiale ce prezintă anumite aspecte particulare şi de interes.