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Optimum Flow in a Transportation Petri Net

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Abstract

*The problem of the optimum flow at minimum costs applied on a transportation Petri network, capacity restricted, is solved in this paper by means of an algorithm based on the network expressed by the incidence matrix, W and described by position set P, transition set T, capacity vector C and cost vector H. The algorithm – maximum flow at minimum cost (**MFLC**), establishes in the first stage the capacity matrix, the flow vectors corresponding to execution sequences (also determined by the algorithm), the matrices associated to cost vector, H_i,, cost matrix Q and the associated matrix [Φ,Q]. On the last one are selected the flows corresponding to the minimum costs q_i Residual matrices of capacities give the opportunity to determine new components of the flow. The components of the flow vector Φ expected at the end of the algorithm process are the maximum values of the flows along the minimum cost execution sequences. An application in the field of maritime transport checks the algorithm efficiency.*

Key words: *Petri nets, discrete events system, optimum flow, algorithms.*

Introduction

The transportation Petri nets (TPN), defined, made and analyzed in [1] represents an important means that can be used in the quality (logical) description of the dynamics of the discrete events systems, particularly transportation systems.

Problems of practical value, generated by the evolution of a transport system (optimum flow, shortest path, longest path), modeled by a graph of transportation-type network can be associated to transportation Petri network, an aspect highlighted by this paper whose aim is to determine maximum flow, cost and capacity restricted, in a transportation system modeled by TPN. The analysis of the problem computational wise, with a view to determine the "toughness" of the solving process, leads to a process of algorithm development that, in-between the problem and algorithm stages, will go over the described steps in [2] and be subject to selection by the complexity criterion.

The topology of the transportation Petri network, its proprieties and the model of matrix analysis of transportation networks [3] have been the basis and the facilities of the construction of an algorithm in seven steps which allows the determination of the optimum flow component, the value of the minimum cost flow and the cost flow.

Problem Description

A problem which regards the flow that runs through a transportation flow is that of taking into consideration both the capacities of the transportation network (the model of the maritime transportation being a transportation Petri network) and the unity cost of flow transportation through transitions. A Petri network of flow transportation, cost restricted, is a network with $P = \{P_1,\dots P_n\}$, position set, and $T= \{T_1,\dots,T_m\}$ transition set, capacity restricted on transitions, to which is attached a vector H, which gives each transition a non-negative whole value known as unity cost. To such a PNt is therefore attached capacity vectors $C= (c_1,c_2,\dots c_m)$ and cost, $H = (h_1,h_2,\dots,h_m)$. Thus, in a Petri network of flow transportation, capacity and cost restricted, the issue that arises is to find a flow $\phi = (\varphi_1,\dots\varphi_m)$, of maximum value that checks capacities restrictions, $0 \le \varphi_k \le c_k$ and minimizes the cost total of the flow:

$$Z = \sum_{Tj\in T} h_j \varphi_j \tag{1}$$

The problem can be solved by means o an algorithm, applicable on a transportation Petri network, which constitutes, step by step, a flow with maximum components at lowest costs.

Algorithm - Maximum Flow at Lowest Costs (MFLC)

STEP 1. Determining the incidence matrix [W]; →**step 1.1** Determining the matrix of capacities $[C]=[W].[C]_m$. (where $[C]_m$ is the diagonal matrix of capacities); →**step 1.2** Determining the characteristic vectors Si (by algorithm [1]); →**step 1.3.** Determining vectors: $C_i = [S_i]\cdot[C]_m$; →**step 1.4.** Determining minimum $\mathbf{c_i}$ for each vector C_i ; →**step 1.5**. Determining $\varphi_i = c_{i\,min}$; →**step 1.6**. Making vectors $\phi_i = \varphi_i(S_i)$; →**step 1.7.** Determining matrices associated to vectors Hi : $[H_i]=[S_i]\cdot[H]_m$, where $[S_i]$ is the line matrix of the characteristic vector **i** and $[H]_m$, the diagonal matrix of costs; →**step 1.8.** Computing values $q_i = \sum_{hj\in Hi} h_j$; →**step 1.9**. Determining the column matrix $[Q]=[q_i]$;

STEP 2. Making the matrix $[\phi,Q]$, matrix Φ whose rows are the vectors ϕ_i, and Q the column matrix;

STEP 3. In matrix $[\phi,Q]$, in column Q, selects $q_{i\,min\,im}$; →**step 3.1**. In matrix $[\phi,Q]$, selects row ϕ_i, corresponding to q_i, if there are more flows at lowest costs, selects maximumϕ_i ; →**step 3.2**. Comparing value φ_i with element r_{ij} in matrix R (**j** is the index of entrance transition into sequence S_i);

- **If ..** $\varphi_i \le |r_{ij}|$ →**step 3.3**
- **If** $\varphi_i > |r_{ij}|$ →sets $\varphi_i = r_{ij}$; →**step 3.3** Determining diagonal matrix $[\phi_i]_m$; **step 3.4**. Determining matrix $[\phi_i]=[W]\cdot[\phi_i]_m$;

STEP 4. Determining the matrix of residual capacity: $[R]=[C]\cdot[\phi_i]$; →**step 4.1**. Computing flow cost: $Z_k = \varphi_i \cdot q_i$; →**step 4.2**. Computing total cost: $Z_t = \sum Z_k$;

STEP 5. In matrix $[\phi, Q]$, eliminates line ϕ_i; →**step 5.1**. If in matrix R, in row P_1 are non-zero elements, goes through step 3 again; If in matrix R, there is no path from P_1 to P_8, it proceeds to the next step.

STEP 6. Determining the flow vector $\phi = \sum(\phi_i)$. Computing $\varphi_{max} = \sum \varphi_i$; $Z_t = \sum Z_k$;

STEP 7. Stop.

Application

A problem of mixed transportation (maritime and on railway).A metallurgical plant , noted as P_8, gets its supplies of iron ore from three loading ports P_2, P_3, P_4 and the ore is carried by sea to three discharging ports P_5,P_6,P_7, from where it is transported by rail to the plant. The ore is brought to ports P_2,P_3,P_4 from the location P_1. The transport capacities between ports are given, and so is the unit cost of transport.The requirement is to make up a transportation plan towards the metallurgical plant, P_8, so that the total amount of carried iron ore is maximum and at the lowest costs.

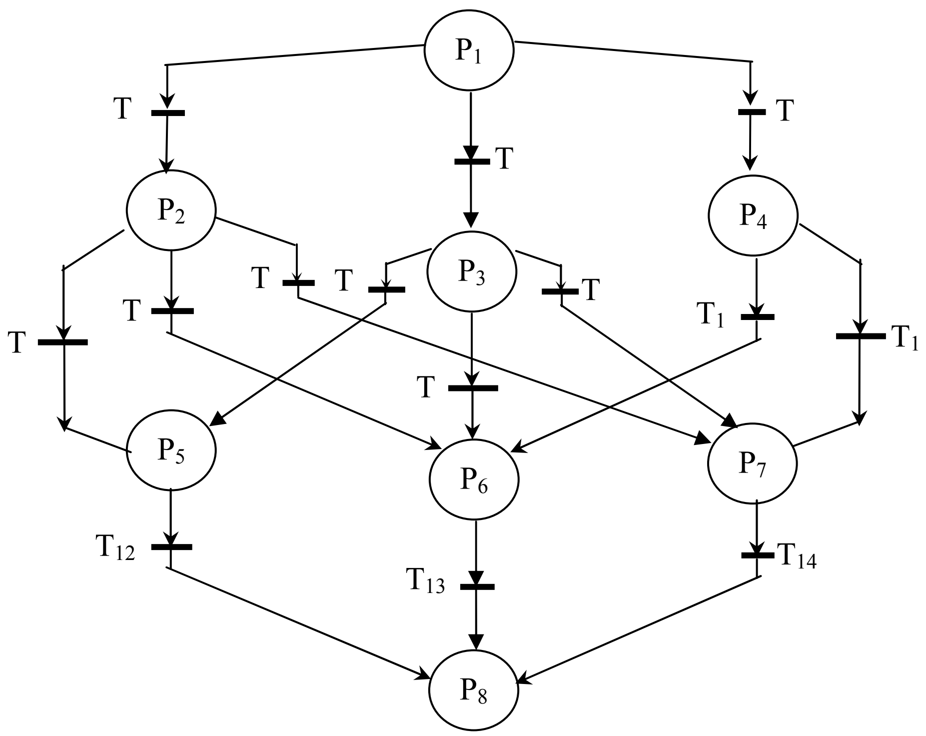

Fig. 1.

Solution:
The problem of mixed transportation (on railway and at sea) is modeled by a transportation Petri network whose transitions are attached apart from unit capacities and costs.In fig. 1 is presented the model of the problem stated before. The positions set is $P = (P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8)$ while the transitions set is $T = (T_1, T_2, T_3 ... T_{14})$. The transport capacities between positions are the components of the capacity vector $C = (10,10,10, 3,4, 1, 5, 4,1, 4, 5, 8,12,10)$. The unit

costs of the transitions make the cost vector $H = (5, 4, 1, 4, 3, 1, 4, 3, 1, 4, 5, 3, 5, 4)$. These being known, it is applied the algorithm – maximum flow at minimum cost.

STEP 1. Determining the incidence matrix W:

$W = W^{+} - W^{-} =$

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}
P_1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0
P_2	1	0	0	-1	-1	-1	0	0	0	0	0	0	0	0
P_3	0	1	0	0	0	0	-1	-1	-1	0	0	0	0	0
P_4	0	0	1	0	0	0	0	0	0	-1	-1	0	0	0
P_5	0	0	0	1	0	0	1	0	0	0	0	-1	0	0
P_6	0	0	0	0	1	0	0	1	0	1	0	0	-1	0
P_7	0	0	0	0	0	1	0	0	1	0	0	0	0	-1
P_8	0	0	0	0	0	0	0	0	0	0	0	1	1	1

Step 1.1. Determining the capacity matrix $[C] = [W] \cdot [C]_m$:

C =

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}
P_1	-10	-10	-10	0	0	0	0	0	0	0	0	0	0	0
P_2	10	0	0	-3	-4	-1	0	0	0	0	0	0	0	0
P_3	0	10	0	0	0	0	-5	-4	-1	0	0	0	0	0
P_4	0	0	10	0	0	0	0	0	0	-4	-5	0	0	0
P_5	0	0	0	3	0	0	5	0	0	0	0	-8	0	0
P_6	0	0	0	0	4	0	0	4	0	4	0	0	-12	0
P_7	0	0	0	0	0	1	0	0	1	0	5	0	0	-10
P_8	0	0	0	0	0	0	0	0	0	0	0	8	12	10

Step 1.2. Determining the characteristic vectors Si(by algorithm)

S_1=(1,0,0,1,0,0,0,0,0,0,0,1,0,0) S_2=(1,0,0,0,1,0,0,0,0,0,0,0,1,0)
S_3=(1,0,0,0,0,1,0,0,0,0,0,0,0,1) S_4=(0,1,0,0,0,0,1,0,0,0,0,1,0,0)
S_5=(0,1,0,0,0,0,0,1,0,0,0,0,1,0) S_6=(0,1,0,0,0,0,0,0,1,0,0,0,0,1)
S_7=(0,0,1,0,0,0,0,0,0,1,0,0,1,0) S_8=(0,0,1,0,0,0,0,0,0,0,1,0,0,1)

Step 1.3. Determining vectors $C_i = [S_i] \cdot [C]_m$:

C_1=(10,0,0,3,0,0,0,0,0,0,0,8,0,0) ; C_2=(10,0,0,0,4,0,0,0,0,0,0,0,12,0);
C_3=(10,0,0,0,0,1,0,0,0,0,0,0,0,10) ; C_4=(0,10,0,0,0,0,5,0,0,0,0,8,0,0);
C_5=(0,10,0,0,0,0,0,4,0,0,0,0,12,0) ; C_6=(0,10,0,0,0,0,0,0,1,0,0,0,0,10);
C_7=(0,0,10,0,0,0,0,0,0,4,0,0,12,0); C_8=(0,0,10,0,0,0,0,0,0,0,5,0,0,10);

Step 1.5. Determining $\varphi_i = c_{i\,min}$:

$\varphi_1 = c_{1\,min} = 3$ $\varphi_2 = c_{2\,min} = 4$
$\varphi_3 = c_{3\,min} = 1$ $\varphi_4 = c_{4\,min} = 5$
$\varphi_5 = c_{5\,min} = 4$ $\varphi_6 = c_{6\,min} = 1$
$\varphi_7 = c_{7\,min} = 4$ $\varphi_8 = c_{8\,min} = 5$

Step 1.6. Making up vectors $\phi_i = \varphi_i \cdot S_i$

$\phi_1 = (3,0,0,3,0,0,0,0,0,0,0,3,0,0)$ $\phi_2 = (4,0,0,0,4,0,0,0,0,0,0,0,4,0)$
$\phi_3 = (1,0,0,0,0,1,0,0,0,0,0,0,0,1)$ $\phi_4 = (0,5,0,0,0,0,5,0,0,0,0,5,0,0)$
$\phi_5 = (0,4,0,0,0,0,0,4,0,0,0,0,4,0)$ $\phi_6 = (0,1,0,0,0,0,0,0,1,0,0,0,0,1)$
$\phi_7 = (0,0,4,0,0,0,0,0,0,4,0,0,4,0)$ $\phi_8 = (0,0,5,0,0,0,0,0,0,0,5,0,0,5)$

Step 1.7. Determining the matrices associated to vectors Hi $[H_i]=[S_i]\cdot[H]_m$, where $[S_i]$ is the row matrix and $[H_i]_m$ is the diagonal cost matrix:

$$[H_1] = [S_1]\cdot[H]_{14} = [5,0,0,4,0,0,0,0,0,0,0,3,0,0]$$
$$[H_2] = [S_2]\cdot[H]_{14} = [5,0,0,0,3,0,0,0,0,0,0,0,5,0]$$
$$[H_3] = [S_3]\cdot[H]_{14} = [5,0,0,0,0,1,0,0,0,0,0,0,0,4]$$
$$[H_4] = [S_4]\cdot[H]_{14} = [0,4,0,0,0,0,4,0,0,0,0,3,0,0]$$
$$[H_5] = [S_5]\cdot[H]_{14} = [0,4,0,0,0,0,0,3,0,0,0,0,5,0]$$
$$[H_6] = [S_6]\cdot[H]_{14} = [0,4,0,0,0,0,0,0,1,0,0,0,0,4]$$
$$[H_7] = [S_7]\cdot[H]_{14} = [0,0,1,0,0,0,0,0,0,4,0,0,5,0]$$
$$[H_8] = [S_8]\cdot[H]_{14} = [0,0,1,0,0,0,0,0,0,0,5,0,0,4]$$

Step 1.8. Computing values $q_i = \sum_{hj\in Hi} h_j$

$$q_1 = \sum_{hj\in H_1} h_j = 12 \qquad q_4 = 11$$
$$q_2 = \sum_{hj\in H_2} h_j = 13 \qquad q_5 = 12$$
$$q_3 = \sum_{hj\in H_3} h_j = 10 \qquad q_6 = 9$$
$$q_7 = 10$$
$$q_8 = 10$$

Step 1.9. Determining the column matrix $[Q]=[q_i]$: $[Q]=[12, 13, 10, 11, 12, 9, 10, 10]^t$

STEP 2. Making up the matrix $[\phi, Q]$.. having j+1columns and i rows.

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}	Q
ϕ_1	3	0	0	3	.	.	.	.	.	.	.	3	0	0	12
ϕ_2	4	.	.	.	4	.	.	.	.	.	.	.	4	0	13
ϕ_3	1	.	.	.	.	1	.	.	.	.	.	.	.	1	10
ϕ_4	.	5	.	.	.	.	5	.	.	.	.	5	.	.	11
ϕ_5	.	4	.	.	.	.	.	4	.	.	.	.	4	.	12
ϕ_6	.	1	.	.	.	.	.	.	1	.	.	.	.	1	9
ϕ_7	.	.	4	.	.	.	.	.	.	4	.	.	4	.	10
ϕ_8	.	.	5	.	.	.	.	.	.	.	5	.	.	5	10

STEP 3. In matrix $[\phi, Q]$, in column Q, it is selected q_i=minim, $-q_{i\,min} = q_6 = 1$

Step 3.1. Selecting, in matrix $[\phi, Q]$, flow $\phi_6, \ldots \phi_6 = (0,1,0,0,0,0,0,0,1,0,0,0,0,1)$

Step 3.2. Determining the diagonal matrix $[\phi_6]_{14}$

Step 3.3. Determining the matrix : $[\phi_6]=[W]\cdot[\phi_6]_{14}$.

STEP 4 Determining the matrix $R_1 = [C]-[\phi_6]$:

$[R_1] =$

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}
P_1	-10	-9	-10	.	.	.	.	.	.	.	.	.	.	.
P_2	10	.	.	-3	-4	-1	.	.	.	.	.	.	.	.
P_3	.	9	.	.	.	.	-5	-4	.	.	.	.	.	.
P_4	.	.	10	.	.	.	.	.	.	-4	-5	.	.	.
P_5	.	.	.	3	.	.	5	.	.	.	.	-8	.	.
P_6	.	.	.	.	4	.	.	4	.	4	.	.	-12	.
P_7	.	.	.	.	.	1	.	.	.	.	5	.	.	-9
P_8	.	.	.	.	.	.	.	.	.	.	.	8	12	9

Step 4.1. Computation of flow cost:

$$Z_1 = \varphi_6 \cdot q_6 = 1 \cdot 9 = 9$$

$$Z_t = 9$$

STEP 5. In matrix $[\phi, Q]\ldots$ by eliminating row $\phi_6 \Rightarrow [\phi, Q]_{(i-1)15} \Rightarrow$ matrix

$[\phi, Q]^r =$

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}	Q
ϕ_1	3	.	.	3	.	.	.	.	.	.	.	3	.	.	12
ϕ_2	4	.	.	.	4	.	.	.	.	.	.	.	4	.	13
ϕ_3	1	.	.	.	.	1	.	.	.	.	.	.	.	1	10
ϕ_4	.	5	.	.	.	.	5	.	.	.	.	5	.	.	11
ϕ_5	.	4	.	.	.	.	.	4	.	.	.	.	.	.	12
ϕ_7	.	.	4	.	.	.	0	0	0	4	.	.	.	.	10
ϕ_8	.	.	5	.	.	.	.	.	.	.	5	.	.	5	10

Step 5.1. If in matrix R_1, in row P_1, are non-null elements, step 3 is performed again.

STEP 3. In matrix $[\phi, Q]^r$, in column Q is selected $q_{i\,min} \Rightarrow q_{i\,min} = q_3 = q_7 = q_8 = 10 \ldots$

Step 3.1. Flows corresponding to costs . q_3, q_7, q_8 are $\phi_3 = 1, \phi_7 = 4$ and $\phi_8 = 5$.

Selecting $\phi\,\max im \Rightarrow \phi_{i\,max} = \phi_8 = 5 \ldots.$

Step 3.2. Comparing ϕ_8 with r_{13} in matrix R_1.

$|r_{13}| \gtrless \varphi_8 \Rightarrow |-10| > 5$ – on to step 3.3.

Step 3.3. Determining the diagonal matrix $[\varphi_8]_{14} \ldots$

Step 3.4. Determining matrix $[\varphi_8] = [W] \cdot [\phi_8]_{14}$

STEP 4. Determining matrix $[R_2] = [R_1] - [\phi_8]$:

Step 4.1. Computing $Z_2 = \varphi_8 \cdot q_8 = 5 \cdot 10 = 50 \ldots$

Step 4.2. $Z_t = \sum_i Z_i = Z_1 + Z_2 = 9 + 50 = 59 \ldots$

STEP 5. In matrix $[\phi, Q]^r$, row ϕ_8 is eliminated.

The resulting matrix is:

$[\phi, Q]^{r_1} =$

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}	Q
ϕ_1	3	.	.	3	.	.	.	.	.	.	.	3	.	.	12
ϕ_2	4	.	.	.	4	.	.	.	.	.	.	.	4	.	13
ϕ_4	1	5	.	.	.	1	.	.	.	.	.	.	.	1	10
ϕ_5	.	4	.	.	.	.	5	.	.	.	.	5	.	.	11
ϕ_6	.	.	.	.	.	.	.	4	.	.	.	.	4	.	12
ϕ_7	.	.	4	.	.	.	.	.	.	4	.	.	4	.	10

Step 5.1. If in matrix $[R_2]$ in row P_1 are non-null elements – take step 3 again.

STEP 3. In matrix $[R_2]$ in column Q is selected $q_{i\,min\,im} \Rightarrow q_{i\,min\,im} = q_3 = q_7 = 10$

Step 3.1. Flows corresponding to costs q_3 and q_7 are $\phi_3=1$ and $\phi_7=4$. By selecting the maximum flow $\Rightarrow \varphi_{max} = \varphi_7 = 4$.

Step 3.2. $[\phi_7]$ has the first non-null element in column T_3, then compare $|r_{13}|$ in matrix R_2 with φ_7 $\left(|r_{13}| \gtrless \varphi_{7,3}\right)$ $|-5| > 4$

Step 3.3. Determining the diagonal matrix $[\phi_7]_{14}$:

Step 3.4. Determining matrix $[\phi_8] = [W] \cdot [\phi_8]_{11}$.

STEP 4. Determining the matrix of residual capacities $R_3 = [R_2] - [\phi_8]$.

Step 4.1. Computation of $Z_3 = \varphi_7 \cdot q_7 = 4 \cdot 10 = 40 \ldots$

Step 4.2. Computation of $Z_t = \sum_i Z_i = Z_1 + Z_2 + Z_3 = 59 + 40 = 99$

STEP 5. In matrix $[\phi, Q]^r$ row ϕ_7. is eliminated. The resulting matrix is:

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}	Q
ϕ_1	3	.	.	3	.	.	.	.	.	.	.	3	.	.	12
ϕ_2	4	.	.	.	4	.	.	.	.	.	.	.	.	.	13
ϕ_3	1	.	.	.	.	1	.	.	.	.	.	.	.	1	10
ϕ_4	.	5	.	.	.	.	5	.	.	0	0	5	0	0	11
ϕ_5	.	4	.	.	.	.	.	4	.	.	.	.	4	.	12

Step 5.1. If in matrix R_3, in row P_1 are non-null elements, take step 3 again.

STEP 3. In matrix $[\phi, Q]$, in column Q, selects $q_{i\,min\,im} \Rightarrow q_{i\,min} = q_3 = 10$.

Step 3.1. The flow corresponding to cost q_3 is ϕ_3.. Select maximum flow $\phi_3 = 4$.

Step 3.2. (ϕ_3) has the first non-null in column T_1, then compare $|r_{11}|$ in matrix R_3 with φ_3 $|r_{11}| \gtrless \varphi_3 \Rightarrow |-10| > 1$

Step 3.3. Determining the diagonal matrix $[\phi_3]_{14} \ldots$:

Step 3.4. Determining matrix $[\phi_3] = [W] \cdot [\phi_3]_{14}$.

SIEP 4. Determining the matrix of residual capacities $[R_4] = [R_3] - [\phi_3]$. The resulting matrix is: R_4

Step 4.1. Computation of $Z_4 = \varphi_3 \cdot q_3 = 1 \cdot 10 = 10 \ldots$

Step 4.2. Computation of $Z_t = \sum_{i}^{3} Z_i = Z_1 + Z_2 + Z_3 + Z_4 = 99 + 10 = 109$

STEP 5. In matrix $[\phi, Q]$, row ϕ_3. is eliminated. The resulting matrix is:

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}	Q
ϕ_1	3	.	.	3	.	.	.	.	.	.	.	3	.	.	12
ϕ_2	4	.	.	.	4	.	.	.	.	.	.	.	4	.	13
ϕ_4	.	5	.	.	.	.	5	.	.	.	.	5	.	.	11
ϕ_5	.	4	.	.	.	.	.	4	.	.	.	.	4	.	12

Step 5.1. If in matrix R_4, in row P_1 are non-null elements, take step 3 again.

STEP 3. In matrix $[\phi, Q]$, in column Q, select q_i $\min im \Rightarrow q_{i\min} = q_4 = 11$ →**Step 3.1**. The flow corresponding to cost q_4 is φ_4 whose value is 5. Select flow $[\varphi_4]$./

Step 3.2. The first element of flow vector ϕ_4 is non-null, corresponding to T_2, then compare, in matrix R_4, $|r_{12}|$ with value φ_4., $|r_{12}| {>}\!\!/\!\!{<} \varphi_4 \Rightarrow |-9| > 5$

Step 3.3. Determining the diagonal matrix $[\phi_4]_{14}$:

Step 3.4. Determining matrix $[\phi_4] = [W] \cdot [\phi_4]_{14}$.

STEP 4. Determining the matrix of residual capacities $[R_5] = [R_4] - [\phi_4]$ →**Step 4.1**. Computation of $Z_5 = \varphi_4 \cdot q_4 = 5 \cdot 11 = 55$ →**Step 4.2**. Computation of Zt $Z_t = \sum_{i=1}^{5} Z_i = 109 + 55 = 164$.

STEP 5. In matrix $[\phi, Q]$. row ϕ_4. is eliminated. The resulting matrix is:

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}	Q
ϕ_1	3	.	.	3	.	.	.	.	.	.	.	3	.	.	12
ϕ_2	4	.	.	4	.	.	.	.	.	.	.	.	4	.	13
ϕ_5	.	4	.	.	.	.	.	4	.	.	.	.	4	.	12

Step 5.1. If in matrix R_5, in row P_1 are non-null elements, take step 3 again.

STEP 3. In matrix $[\phi, Q]$, in column Q, select q_i $\min im \Rightarrow q_{i\min} = q_1 = q_5 = 12$.→

Step 3.1. The flow corresponding to cost 12 are $\phi_1 = 3$ and $\phi_5 \Rightarrow$ $\phi_5 = 4$.

Step 3.2. The first element of vector ϕ_5 is non-null, corresponding to T_2, then compare, in matrix R_5, $|r_{12}|$ with value . ϕ_5 $|r_{12}| \gtrless \phi_5 \Rightarrow |-4| = \varphi_5$.

Step 3.3. Determining the diagonal matrix $[\phi_5]_{14}$:

Step 3.4. Determining matrix $[\phi_5] = [W] \cdot [\phi_5]_{14}$.

STEP 4. Determining the matrix of residual capacities $[R_6] = [R_5] - [\phi_5]$ The resulting matrix is:

$[R_6] =$

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}
P_1	-9	.	-1	.	.	.	.	.	.	.	.	.	.	.
P_2	9	.	.	-3	-4	.	.	.	.	.	.	.	.	.
P_3	.	.	.	.	.	.	.	.	.	.	.	.	.	.
P_4	.	.	1	.	.	.	.	.	.	.	.	.	.	.
P_5	.	.	.	3	.	.	.	.	.	.	.	-3	.	.
P_6	.	.	.	.	4	.	.	.	.	.	.	.	-4	.
P_7	.	.	.	.	.	.	.	.	.	.	.	.	.	-3
	.	.	.	.	.	.	.	.	.	.	.	3	4	3

Step 4.1. Computation of $Z_6 = \varphi_5 \cdot q_5 = 4 \cdot 12 = 48$

Step 4.2. Computation of $Z_t = \sum_{i=1}^{6} Z_i = 164 + 48 = 212$

STEP 5. In matrix $[\phi, Q]$ row ϕ_5. is eliminated. The resulting matrix is:

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}	Q
ϕ_1	3	.	.	3	.	.	.	.	.	.	.	3	.	.	12
ϕ_2	4	.	.	.	4	.	.	.	.	.	.	..	4	.	13

Step 5.1. If in matrix R_6, in row P_1 are non-null elements, take step 3 again.

Step 3. $q_{i\,min} = q_1 = 12$

STEP 4. Determining the matrix of residual capacities $[R_7] = [R_6] - [\phi_1]$

Step 4.1. Computation of $Z_7 = 3 \cdot 12 = 36$

Step 4.2. Computation of Zt=164+36=200…

STEP 5. In matrix $[\phi, Q]$. row Φ1 is eliminated. ϕ_1

STEP 3. $q_{i\,min} = q_2 = 13$

STEP 4. Determining the capacities matrix $[R_8] = [R_6] - [\phi_1]$

Step 4.1. Computation of $Z_8 = 4 \cdot 13 = 52$

Step 4.2. Computation of Z_t=200+52=252

STEP 5. $[\phi, Q] = 0$

Step 5.1. In matrix R_8 there is no path from P_1 to P_8.

STEP 6. Computation of $\phi_{max} = \sum \phi_i = 27$ →Print Zt=252

STEP 7. Stop.

Conclusions

The matrix-type description of TPN allows the making of an algorithm which determines the maximum flow of minimum cost in a capacity restricted network and during its application, after finding each flow components, it computes the cost of the component and the total cost at that stage.

The advantage of this algorithm lies in the fact that it calculates directly the maximum components of the flow at lowest costs, unlike other algorithms which generate an initial flow that is successively upgraded by creating several negative circuits regarding.

The intermediary values that are obtained can be useful in the management of sea transport as modeled by a transportation Petri network.

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Fluxul optim pe o rețea Petri de transport

Rezumat

Problema fluxului optim cu cost minim aplicatã pe o retea Petri de transport cu restrictie de capacitate este rezolvatã în lucrare printr-un algoritm construit pe reteaua exprimatã prin matricea de incidentã, W si descrisã prin multimea P, a pozitiilor, multimea T, a tranzitiilor, vectorul capacitate C si vectorul cost, H. Algoritmul – flux optim cu cost minim(FOCM), determinã în prima sa parte, matricea capacitate, vectorii flux corespunzãtori secventelor de executie (determinate si ele de algoritm), matricile asociate vectorilor cost, H_i, matricea cost Q si matricea asociatã [Φ,Q]. Pe aceasta din urmã se selecteazã fluxurile corespunzãtoare costurilor q_i, minime.Matrici reziduale ale capacitãtilor dau posibilitatea determinãrii de noi componente ale fluxului.Componentele vectorului flux, Φ, la finalul derulãrii algoritmului sunt valorile maxime ale fluxurilor de-a lungul secventelor de executie cu cost minimO aplicatie din domeniul transportului maritim verificã competentele algoritmului.