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Considerations Regarding the Behaviour of Cylindrical Structures

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Abstract

This paper contains the main formulas used for calculating the internal, uniform pressure on cylindrical one piece and multilayer structures. By analysing the distribution of the tension fields and the behaviour of the structures, several observations have been made, especially as a result of the tests carried out in special lab conditions

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Key words: *cylinder, stress, strain, pressure, multilayer, one piece, ultimate state, fissure*.

Introduction

The recipients can be subjected to the action of thermal fields (besides even or uneven pressure, internal or external, or any other force applied on the interior or exterior surface), causing uneven dilatation or constriction. The mechanical or thermal stresses are constant or vary with time. When thinking about mechanical, thermal or volume based actions, the strength of the recipient's structure reacts, creating a state of stress. This stress is associated with a strain.

Based on the concept of ultimate state, the evaluation of the recipient's strength under pressure depends on the natural phenomenon associated with breaking- the type of ultimate state- and on the level of load limit. This aspect can be solved by applying an analytical method of calculation or by using experimental methods. The functionality of the under pressure recipients (from a technical point of view) must be guaranteed by firm calculations and conception, without over sizing them, because their construction should be economical.

The State of Stress in Cylindrical Shapes with Thick Walls

The body of the "high pressure recipients" should be built from one piece walls or multilayer. The safety of an under pressure recipient can be obtained by assuring its strength, rigidity and tightness. By carefully analysing the constructive solutions, the fundamental theory, the practical evaluation should lead to the necessary degree of safety under economical terms.

The recipients can be covers or revolution shapes, defined by the fact that the middle surface is generated by rotating a curve around a fixed axis, situated in the same plane and which does not intersect with the curve. Take a cylinder with a thick wall that is long enough, loaded with the internal and external uniform pressures p and p_1, interior radius r_o and exterior radius r_1 (Figure. 1.).

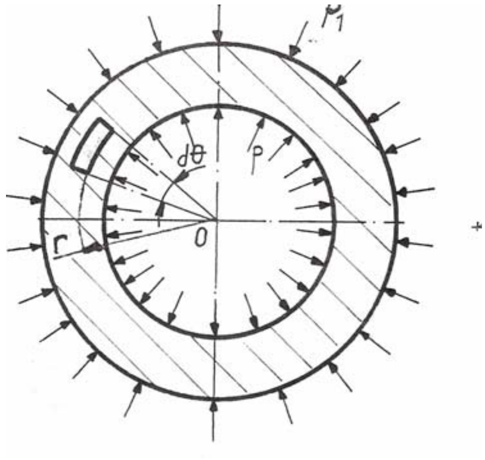

Fig. 1. Cylindrical shape- under internal and external pressure

The axial stress σ_z is zero, when there are no axial constraints applied to the cylinder. Because of the symmetry of the axial strains, the cutting stresses are zero. A state of plane stress will be born, with the main stresses: σ_r–radial stress and σ_θ-circumferential stress. A fibre at the distance r from the cylinder's axis (Fig. 1.) will move radially with the quantity: u. The variation laws of stresses and radial movements, depending on the radius r, are,

$$\sigma_\theta, \sigma_r = \frac{p r_o^2 - p_1 r_1^2}{r_1^2 - r_o^2} \pm \frac{(p - p_1)}{r_1^2 - r_o^2} \frac{r_o^2 r_1^2}{r^2}; \tag{1}$$

$$u = \frac{(1-\nu)(p r_o^2 - p_1 r_1^2)}{E(r_1^2 - r_o^2)} r + \frac{(1+\nu)}{E} \frac{p - p_1}{r_1^2 - r_o^2} \frac{r_o^2 r_1^2}{r}. \tag{2}$$

To make the above expressions easier, we will mark: $ß=r/r_o$, $ß_1=r_1/r_o$, which represent the geometrical coefficients of the cylinder and the load is produced by the internal pressure p and external pressure p_1. The general equations (1) and (2) can become specific, for the different loads of the thick walls cylinders.

By analysing the variation of the radial and circumferential stresses, when the cylinder load is produced be an internal pressure p, we can observe that the maximum stress load appears on the interior surface of the cylinder, where $ß_1=1$. As the difference between the exterior diameter and the interior diameter becomes bigger, the values for σ_r/p and σ_θ/p become smaller, and the degree of stress variation on the thick wall becomes bigger.

As a result, the exterior surface is less strained, thus leading to the ability of building it from materials with less strength. Observations so far refer to a long cylinder with thick walls, under radial pressure, with zero axial load, thus with open ends. It will be called a tube. The cylindrical tanks break due to incredibly high strains, in the most stressed points, which are the interior surface. As we can see, it is also the interior fibres that are the most stressed, so:

$$|\sigma_\theta|_{r=r_o} > |\sigma_r|_{r=r_o}. \tag{3}$$

For strains in the elastic area, the theories and models that help determine the thickness of the "under-pressure recipient's" wall, refer to the resistance flow σ_c or to the ultimate strength σ_R of the material, determined by the "ultimate breaking experiment" on a standard test piece.

By using these numbers as a starting point for finding the ultimate breaking load of a "under-pressure tank", it can be stated that two materials (for which strain is not considered) are as resistant if σ_R or σ_c are the same.

However, in reality, the phenomenon has proved to be different. Taking all these aspects into consideration, several methods for determining the maximum pressure load for a cylindrical, one piece wall or multilayer recipient, have been developed.

According to the work carried out so far, the thickness of the recipient walls is calculated so that when reaching the allowed pressure, there should still be a "safety space" until the beginning of the flow of the interior fibres.

In the field of engineering, there are certain empirical formulas used to determine the dimensions of the "under-pressure" recipients. However, these formulas cannot be generalised. They only stand in a very narrow field, experimentally tested.

As seen by analysing the stresses in the thick wall cylinders, the exterior area is less strained. Thus, it is rational to build the cylindrical recipients strained from huge internal pressures, out of materials with high mechanical strength (combined with corrosive strength if necessary), and the exterior part out of ordinary materials. A more rational allocation of stresses can be obtained in a multilayer construction, by introducing some initial compression stresses, by joining some layers of the material through pressing.

The Hoping of Clindrical Shapes

For cylinders subjected to internal pressure p, when the total stress is two times the internal pressure, the wall thickness becomes infinite. This would mean that the internal pressure should be half as small as the allowable load of the material of the cylinder.

However, this is rarely the care in reality, because the pressure is very big. Even the best materials known to man would not handle this situation. This is why a series of constructive solutions have been found. One of them is introducing an initial state of stress, by using the hooped cylinders.

The most common technological solution is to introduce (usually at a relatively high temperature) an exterior cylinder over the one subjected to the internal pressure. The result is fit shrink, the fit resulting because of the fact that the interior diameter of the exterior piece is smaller than the exterior diameter of the interior piece. The difference between the two radiuses is called fastening and is symbolised by Δ.

After the fitting, a contact pressure (called hoping pressure) appears on the contact surface of the two pieces. This pressure brings a friction force which stops the relative movements of the pieces.By hoping a certain evenness of the allocation of tensions is obtained, due to the internal pressure.

The pressing is chosen so that the stress is identical in the most strained points of the two tubes. By analysing the stress state, we can observe the fact that the highest value is reached in the interior surface of the exterior tube.

The pressing can be calculated so that in the most strained points of the cylinders, the state of stress should have the same safety coefficient. By applying the maximum circumferential stress theory, it is recommended that the circumferential stresses be equal: $\tau_{max1}=\tau_{max2}$, thus the hooping pressure will be,

$$p_f = p\frac{r_2^2}{r_2^2 - r_o^2}\frac{1-\left(\frac{r_o}{r_1}\right)^2}{\frac{r_2^2}{r_2^2 - r_1^2}+\frac{r_1^2}{r_1^2 - r_o^2}}. \tag{4}$$

From the condition that $\tau_{max1,2}<\sigma_c/2$, the value of the internal pressure (which determines the reaching of the limit flow in area r_1 for the two shapes) can be determined:

$$p_{ic} = \tau_c[2 - (\frac{1}{\beta_1^2} + \frac{1}{\beta_2^2})]. \tag{5}$$

The optimum hoping pressure is determined with the formula,

$$p_{fopt} = \frac{\tau_c(\beta - 1)^2}{\beta(\beta + 1)}, \quad p_{fopt} = \frac{E\Delta}{2r_o\sqrt{\beta}}\frac{\beta - 1}{\beta + 1}. \tag{6}$$

From the equality of the formulas (6), the value of the optimum fastening (which delivers the optimum pressure hooping $p_{fopt.}$) can be deduced

$$\Delta_{opt} = \frac{2r_o\tau_c}{E}\frac{(\beta - 1)}{\sqrt{\beta}}. \tag{7}$$

The hooping has a positive effect only in the case of thick wall tubes, when a shrinking of almost two times of the circumferential stress is obtained through this construction. For thin wall tubes, hooping is not recommended.

Hooped Objects obtained from more Cylinders

By hooping of various cylindrical objects, a thick wall recipient is obtained. This body may result from concentric cylindrical covers that have an initial clearance δ_i, so it does not use the advantages of the hooping.

When applying the internal pressure p, first the initial clearance δ_i is annulled by straining the cylindrical covers and afterwards applying the hooping pressure. It may come to the point where the interior parts of the cylinder will touch a plastic state of stress.

Another constructive shape of structure is formed out of initially hooped cylindrical covers. On a base cylinder indentified with the number 1, n-1 coaxial cylinders are applied, so that for cylinder i, we have:

$$\beta_i = \frac{r_i}{r_{i-1}}, \beta_{r,i} = \frac{r_i}{r}, r \in [r_{i-1}, r_i]. \tag{8}$$

The pressure between the cylinders "i" and "i+1" is marked p_{fi} according to the figure 2. The cylinder "i" will be under the internal pressure p_{fi-1} and an external pressure p_{fi}.The state of stress in this cylinder is determined with the help of the Lamé formulas.

By applying the total strain energy theory for a point on the interior surface of the cylinder "i", the stress is,

$$(\sigma_{echiv})_{r_{i-1}} = \frac{\sqrt{3}\beta_i^2}{\beta_i^2 - 1}(p_{fi-1} - p_{fi}). \tag{9}$$

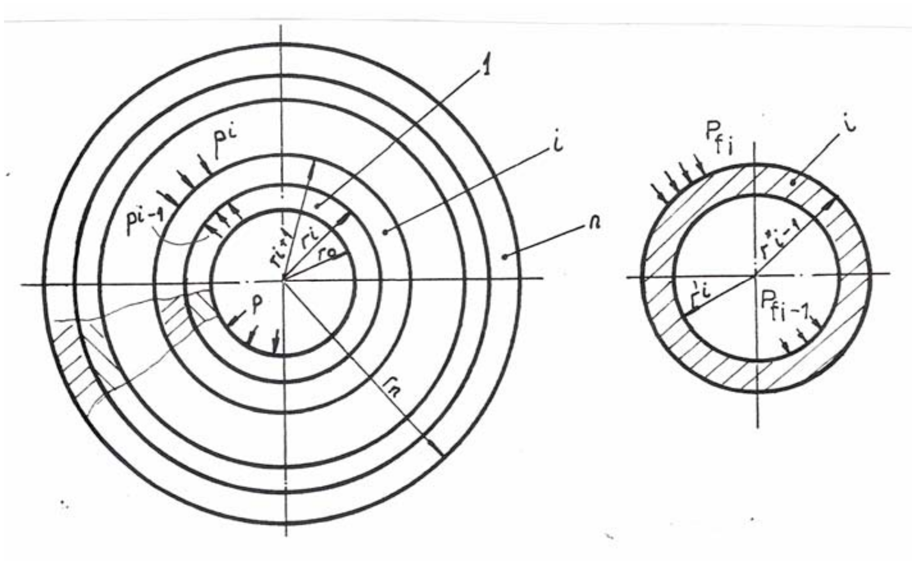

Fig.2. Cylindrical body obtained from more coaxial covers

Under the condition that $(\sigma_{ech})_{r(i-1)}=\sigma_a$, the result is:

$$p_{fi\text{-}1} - p_{fi} = \frac{\sigma_a(\beta_i^2 - 1)}{\sqrt{3}\,\beta_i^2}, \beta_i = \frac{1}{\sqrt{1 - \sqrt{3}\frac{p_{fi\text{-}1} - p_{fi}}{\sigma_a}}}. \tag{10}$$

For the cases most often used in practice, there is a formula between the hooping pressures $p_{fi-1}>p_{fi}$ and the cylinders are made from the same material, the calculation can be made for $ß_i=r_i/r_o$ and then apply it specifically for a shape make from “n” cylinders, so i=n,

$$\beta_{n'} = \frac{r_n}{r_0} = \prod_{j=1}^{n} \frac{1}{\sqrt{1 - \sqrt{3}\frac{p_{fi\text{-}1} - p_{fi}}{\sigma_a}}}. \tag{11}$$

From the condition of the minimum weight, the hooping pressures p_{fi} can be deduced and by starting from the exterior load, that is for k=n-1, $p_{fn}=0$ you get,

$$p_{fn\text{-}1} = \frac{p_{fn\text{-}2}}{2};\ p_{fn\text{-}2} = \frac{p_{fn\text{-}3} + p_{fn\text{-}1}}{2} \tag{12}$$

The following is obtained:

$$\beta_{n'} = \frac{1}{\sqrt{(1 - \frac{\sqrt{3}}{n}\frac{p}{\sigma_a})^n}}. \tag{13}$$

If the thickness of a layer would go towards zero and the number of layers, thus cylinders, would grow towards infinite, then,

$$\beta_{\infty'} = \lim_{n\to\infty} \beta_{n'} = \frac{1}{e^{-\frac{\sqrt{3}}{2}\frac{p}{\sigma_a}}} = e^{\frac{\sqrt{3}}{2}\frac{p}{\sigma_a}}. \tag{14}$$

ß'$_n$ defines the value of the exterior radius of the body with thick multihooped wall, minimum allowed in theory.This formula is also used with objects made from more than one layer of welding placed successively.The minimum number of hooped objects is calculated from the condition that ß'$_n$ is positive and applying the third strength theory, the result is,

$$\beta_n^{'} = \frac{1}{\sqrt{1 - \frac{2}{n}\frac{p}{\sigma_a}}}. \tag{15}$$

The pressing Δ, necessary for joining two hooped cylinders is obtained considering the contact elasticity between two contiguous surfaces.The joining pressing Δ_i between the cylindrical shapes "i" and "i+1" is,

$$\Delta_i = u_i + \frac{(1-\nu^2)(r_{i'}^{\,2} - r_{i''-1}^{\,2})}{Er_{i'}}\sigma_a. \tag{16}$$

The dimensions of the joining are defined by r_i' , which has the exterior radius of the interior object "i" and r''_{i-1} the interior radius of the object "i", both before joining (Fig.2.).

The pressure p_{fi} for the third strength theory is:

$$p_{fi} = p - \frac{\sigma_a}{2}\sum_{k=1}^{i}\frac{\beta_k^2 - 1}{\beta_k^2}, \tag{17}$$

where $ß_k=r_k'/r_{k-1}''$, k=1,2.....n-1.

Once the numbers for the hooping pressure p_{fi} are known, the stress state from each cylinder is determined with the Lamé formulas.In order to analyse the behaviour of the multilayer-wall cylinder subjected to an internal pressure and another one piece one, various experiments have been carried out. Cylinders of successive diameters have been hooped and joined together one inside another, thus resulting a multilayer cylindrical body.

This behaviour of the multilayer cylinder is explained like this: as a result of the fabrication method, while dressing and welding the plates, a state of compression stress is produced because of the contraction of the welding ribs. It is assumed that approximately 70% of the free contraction of the welding produces the compression stresses, the rest contributing to the intimate contact between the plates.

When installing the internal pressure, the interior cylinders are apparently more strained, due to the overlapping of the initial stresses with the stresses produced by the pressure p. Once the internal pressure is bigger, all the layers are in contact and the multilayer wall behaves similar to the one piece wall.By raising the pressure, first the flow comes into the interior fibres and then until it reaches the pure plastic state when the flow begins in the exterior fibres. In this stress state the initial stresses were cancelled and the behaviour of the multilayer recipient is identical to the one of a one piece recipient.

It has to be noticed that the strain of a multilayer recipient is much better than the strain of a one piece recipient with the same size.In order to demonstrate this statement, some 40 mm sheaves have been cut from the multilayer cylinder and compressed on a universal trying machine. None of the sheaves has cracked when bent, proving the fact that this danger does not manifest itself with thicker walls. This does not apply to the cylinder with a one piece wall. In figure 3 two bent samples are represented: one is a cylinder with a one piece wall and the other one with a multilayer wall.

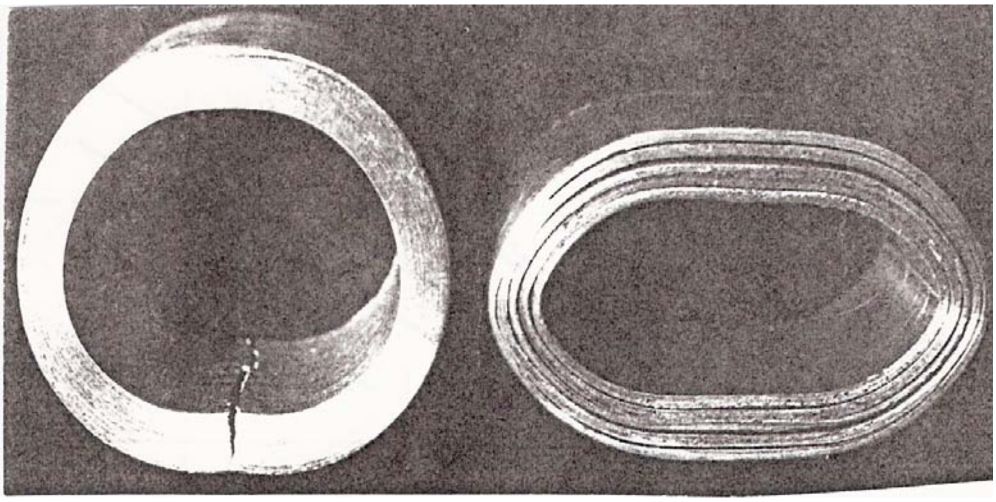

Fig. 3. Bent samples made from a one piece wall cylinder (a) and a multilayer wall one (b)

Conclusions and Comments

From the analysis of the theoretical conclusions presented above, it is deduced that pre-tensioning does not lead to large material savings, especially when the internal pressure is rising. As a result of pre-tensioning, the base cylinder is strained by an external pressure and if the value of this pressure goes beyond the ultimate, cracks may emerge (Fig.4).

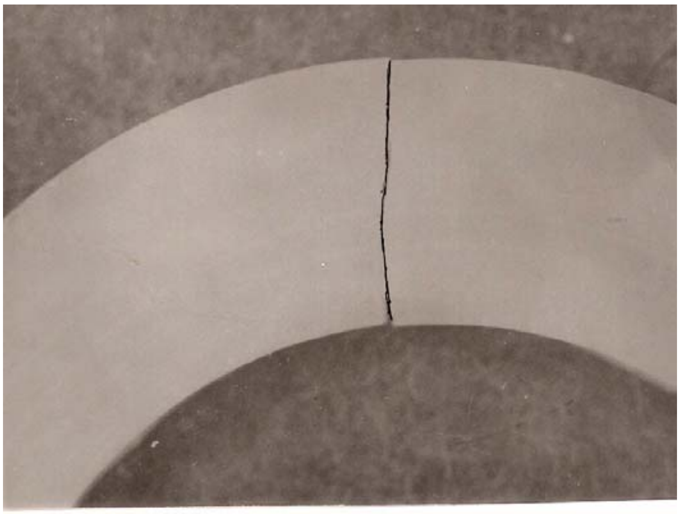

Fig. 4. Detail of a crack in a model cylinder

It is necessary that the state of stresses in the base cylinder, subjected to pre-tensioning, be under permanent control, because inducing a crack may lead to severe consequences when the structure is loaded with the work pressure.

References

1. Dawson, V.C.D., *High pressure containement in cylindrical vssels,* High Pressure Technology, editor I.L. Spain, New York, 1977

2. Pavel, A., *Elastostabilitata recipientelor cilindrice,* Editura Academiei, Bucureşti, 1983

3. Sorohan, M., *Introducere în fizica şi tehnica presiunilor înalte,* Editura Tehnică, Bucureşti, 1977

4. Vasiloaica, C., *Structuri cilindrice pretensionate*, Editura Universității ,,Lucian Blaga", Sibiu, 1996.

Considerații referitoare la comportarea structurilor cilindrice

Rezumat

Lucrarea conține principalele relații de calcul pentru structurile cilindrice monobloc şi multistrat solicitate de o presiune interioară uniformă. Analizând distribuțiile câmpurilor de tensiuni şi comportarea acestor structuri sunt prezentate câteva observații rezulate în urma unor încercări efectuate în condiții de laborator.