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Dynamic Investigation, Identification and Decoupling of a Multivariable 2X2 Temperature Process

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Abstract

Most of the real systems are multivariable, having in addition to the input-output direct interactions, unwanted input-output cross interactions. The objective of this paper is to find a simple and feasible mathematical model of a heat transfer multivariable 2X2 process using the results of the process dynamic investigation. A simple and accurate model is useful in the design of a 2X2 structure decoupler connected in series with the process, which aims to cancel the multivariable process natural cross interactions in order to obtain a process having only direct interactions, which can easily be controlled.

Key words: *multivariable process, dynamic investigation, process identification, 2X2 decoupler design.*

Introduction

There are processes with one input variable and one output variable, which are referred as Single-Input Single-Output (SISO) and processes with more than two input variables and two output variables, and at least one output depends on at least two input variables, which are named Multi-Input Multi-Output (MIMO) processes. In most of practical industrial applications the case of SISO systems in not often found, the majority of the processes being multivariable. Such examples are chemical reactors, distillation columns, heat exchangers etc. A common case of MIMO systems is the particular case of systems with two input variables and two output variables, referred as 2X2, which are characterized by input-output (I-O) direct and cross interactions [7].

Because of the I-O cross interactions, considering a control structure with good performance is difficult to do; a decoupler connected in front of the process has to be considered. The decoupler is designed based on the process models on each input-output channel and aims to cancel the unwanted process cross interactions.

The serial connection between the decoupler and process is named pseudo-process (decoupled process) and it is characterized only by direct interactions/process channels [4].

Obviously, the modeling errors will affect the decoupling performance and consequently the performance of the entire control system.

The objective of this paper is to obtain a simple and accurate model of a 2X2 mltivariable temperature process which will be further used in order to design a decoupler with the same structure with the process, which will be connected in front of the process, that tends to reduce or to cancel the unwanted natural process cross interactions. This is done in order to offer the possibility to easily control the process.

Dynamic Investigation of the Multivariable Temperature Process

The process used for this study is a multivariable 2X2, having 2 inputs and two outputs (fig.1). The process consists of two chambers, with one bulb each, located close one to the other. The process inputs are the two voltages which control the degree of the light and the two outputs are the two chamber temperatures. Because the chambers are close one to each other when we want to increase the temperature in one chamber, we increase the voltage, the degree of light increases and the temperature will increase but, unwanted will increase also the temperature in the other chamber.

In Figure 1, is presented the multivariable process block diagram, having the two voltages as inputs (U1 and U2) and the two chamber temperatures as outputs (Y1 and Y2). Controlling this multivariable process cannot be done because of the cross interactions between U1 and Y2 and between U2 and Y1.

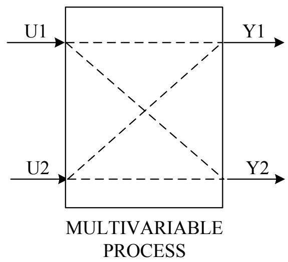

Fig. 1. Multivariable 2X2 process block diagram: U1 and U2 – process inputs (voltages), Y1 and Y2 – process outputs (temperatures).

The process dynamics was investigated considering step changes in the process inputs U1 and U2 and recording the process output (Y1 and Y2) trends.

In Figure 2, are presented the multivariable process outputs (Y1 and Y2) when the process input U1 increases with 10%. We can see that the process output Y1 increases from 20 to 52°C and the process output Y2 increases from 20 to 25°C. The process dynamics on these two process channels (U1-Y1 and U1-Y2) is similar to the behavior of a second order system without dead time [1].

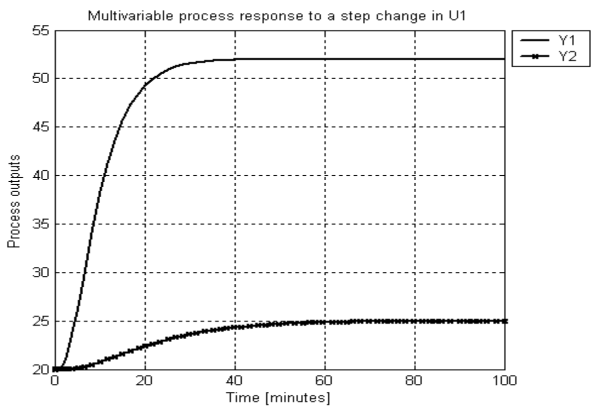

Fig. 2. Multivariable process response (Y1 and Y2 [°C]) to a 10% step change in U1

Similarly, in Figure 3, are presented the multivariable process output (Y1 and Y2) trends when the process input U2 increases with 10%. We can see that the process output Y1 increases from

20 to 27°C and the process output Y2 increases from 20 to 47°C. Like in the previous case, the process dynamics on these two process channels (U2-Y1 and U2-Y2) is similar to the behavior of a second order system without dead time.

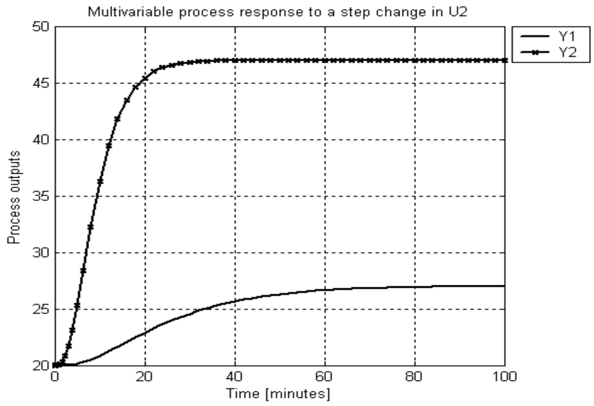

Fig. 3. Multivariable process response (Y1 and Y2 [°C]) to a 10% step change in U2

Using the data from Figures 2 and 3, the process was identified. The process model block diagram is presented in Figure 4. The process model has the same structure as the real process.

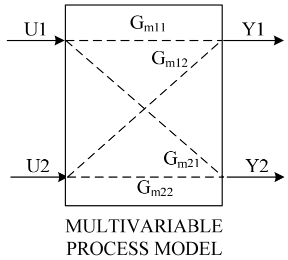

Fig. 4. Multivariable 2X2 process model block diagram: U1 and U2 – process inputs, Y1 and Y2 – process outputs, G_{m11} - the transfer function for U1-Y1 process channel, G_{m21} - the transfer function for U1-Y2 process channel, G_{m12} - the transfer function for U2-Y1 process channel and G_{m22} - the transfer function for U2-Y2 process channel.

Multivariable Process Identification

From the process dynamic investigations was observed that it behaves on each of the four channels as a second order without dead time system. So the process channels will be represented as a reunion of four models (transfer functions): G_{m11} is the transfer function for U1-Y1 process channel, G_{m21} is the transfer function for U1-Y2 process channel, G_{m12} is the transfer function for U2-Y1 process channel and G_{m22} is the transfer function for U2-Y2 process channel, having the following expressions:

$$G_{m11}(s) = \frac{k_{11}}{(T_{11} \cdot s + 1)^2} = \frac{3.2}{(4.4 \cdot s + 1)^2}, \quad (1)$$

$$G_{m21}(s) = \frac{k_{21}}{(T_{21} \cdot s + 1)^2} = \frac{0.5}{(10.9 \cdot s + 1)^2}, \quad (2)$$

$$G_{m12}(s) = \frac{k_{12}}{(T_{12} \cdot s + 1)^2} = \frac{0.7}{(12.9 \cdot s + 1)^2}, \quad (3)$$

$$G_{m22}(s) = \frac{k_{22}}{(T_{22} \cdot s + 1)^2} = \frac{2.7}{(4.1 \cdot s + 1)^2}, \qquad (4)$$

The above transfer function gains (k_{11}, k_{21}, k_{11} and k_{21}) are computed as the process output percent variation divided by the process input percent variation:

$$k_{11} = \frac{52\% - 20\%}{10\%} = 3.2, \qquad (5)$$

$$k_{21} = \frac{25\% - 20\%}{10\%} = 0.5, \qquad (6)$$

$$k_{12} = \frac{27\% - 20\%}{10\%} = 0.7, \qquad (7)$$

$$k_{22} = \frac{47\% - 20\%}{10\%} = 2.7. \qquad (8)$$

The transfer function time constants (T_{11}, T_{21}, T_{11} and T_{21}) are computed as the process channel settling time divided by 6. The process settling time is the time in which the process output reaches 98% from its steady-state value:

$$T_{11} = \frac{26.5\,\text{min}}{6} = 4.4\,\text{min}, \qquad (9)$$

$$T_{21} = \frac{61\,\text{min}}{6} = 10.9\,\text{min}, \qquad (10)$$

$$T_{12} = \frac{77.5\,\text{min}}{6} = 12.9\,\text{min}, \qquad (11)$$

$$T_{11} = \frac{24.4\,\text{min}}{6} = 4.1\,\text{min}. \qquad (12)$$

Because the transducer range is 0-100°C, the process outputs values in °C are the same in percent.

The validation of the process model mathematical models consisted of estimated data validation, computing the standard error using [5]:

$$e\_standard = \frac{\sqrt{\sum_{i=1}^{n} \left(y_{pov}(i) - y_{meov}(i)\right)^2}}{n}, \qquad (13)$$

where i is the current time, y_{pov} the process output value, y_{meov} is the model based estimated output value and n the number of computed values.

According to (13) we have the standard error values from Table 1.

As we can see from the above mentioned values, we can consider that the process model is adequate for its scope, to use it in a further control step, the modeling error values being insignificant.

Also, in order to validate the identified process model, the process model and real process responses to a step change in the two process inputs are represented in the same figures (see figs. 5 and 6).

From Table 1 and Figures 5 and 6, we can see that the standard error values are smaller than 2% and the process model behaves close to the behavior of the real process so it can be further used in a further control stage.

Table 1. Standard error values.

Process channel	e_standard
U1-Y1	0.0123
U1-Y2	0.0016
U2-Y1	0.0018
U2-Y2	0.0110

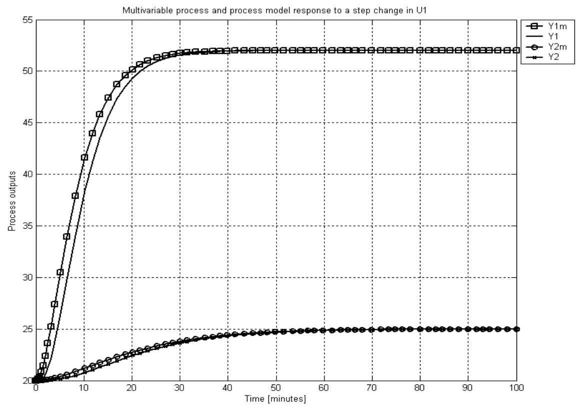

Fig. 5. Multivariable process response (Y1 and Y2 [°C]) and identified process model response (Y1m and Y2m [°C]) to a 10% step change in U1

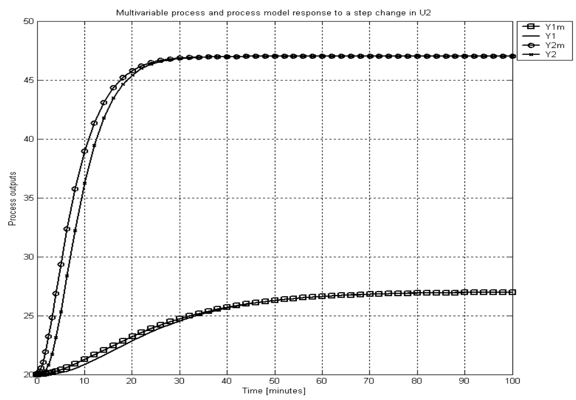

Fig. 6. Multivariable process response (Y1 and Y2 [°C]) and identified process model response (Y1m and Y2m [°C]) to a 10% step change in U2.

2X2 Dedicated Decoupler Design

In order to have a controllable process we have to cancel the process in I-O cross interactions. This is done using a decoupler with the same structure as the process (two inputs and two outputs) connected in series, in front of the process. The multivariable process with decoupler is presented in Figure 7.

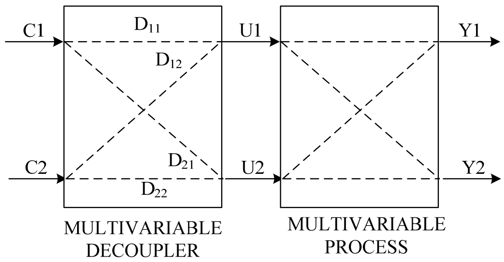

Fig. 7. Multivariable process with decoupler block diagram: C1 and C2 – decoupler inputs, U1 and U2 – decoupler outputs/process inputs, Y1 and Y2 – process outputs

The decoupler models on each of the four input output channels are found so that we obtain the following pseudo-process (decoupled process) structure.

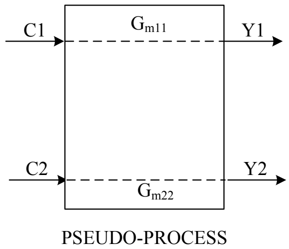

Fig. 8. Decoupled process/pseudo-process block diagram: C1 and C2 – decoupled process inputs, Y1 and Y2 – decoupled process outputs.

Connecting the decoupler in series with the process, in order to obtain a process which has two inputs and two outputs and behaves as two monovariable processes, as in Figure 8, leads to the following decoupling condition [2, 3, 6]:

$$G_m(s)\cdot D(s)=\begin{bmatrix} G_{m11}(s) & G_{m12}(s) \\ G_{m21}(s) & G_{m22}(s) \end{bmatrix}\cdot\begin{bmatrix} D_{11}(s) & D_{12}(s) \\ D_{21}(s) & D_{22}(s) \end{bmatrix}=\begin{bmatrix} G_{m11}(s) & 0 \\ 0 & G_{m22}(s) \end{bmatrix}, \tag{14}$$

which leads to the dedicated decoupler models:

$$D_{11}(s)=\frac{G_{m11}(s)\cdot G_{m22}(s)}{G_{m11}(s)\cdot G_{m22}(s)-G_{m12}(s)\cdot G_{m21}(s)}, \tag{15}$$

$$D_{21}(s)=-\frac{G_{m11}(s)\cdot G_{m21}(s)}{G_{m11}(s)\cdot G_{m22}(s)-G_{m12}(s)\cdot G_{m21}(s)}, \tag{16}$$

$$D_{12}(s)=-\frac{G_{m22}(s)\cdot G_{m12}(s)}{G_{m11}(s)\cdot G_{m22}(s)-G_{m12}(s)\cdot G_{m21}(s)}, \tag{17}$$

$$D_{22}(s)=\frac{G_{m11}(s)\cdot G_{m22}(s)}{G_{m11}(s)\cdot G_{m22}(s)-G_{m12}(s)\cdot G_{m21}(s)}, \tag{18}$$

where G_{m11}, G_{m21}, G_{m12} and G_{m22} are the process transfer functions on the fourth channels having the expressions from (1-4).

Results

The behavior of the multivariable process with decoupling (fig.7) was investigated further in Figures 9-12, to step changes in the two inputs C1 and C2.

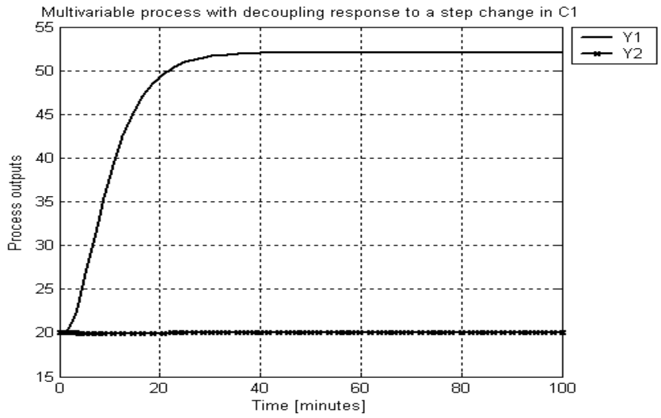

Fig. 9. Multivariable process with decoupling response (Y1 and Y2 [°C]) to a 10% step change in C1

Because the decoupler model depends on the process model, which has modeling errors, the decoupling is not perfect, as we can see in Figure 10, which magnifies the trend of the output Y2 to the change of input C1, from Figure 9.

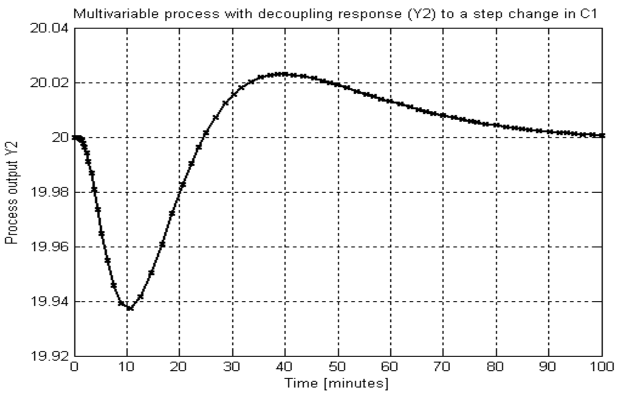

Fig. 10. Multivariable process with decoupling response (Y2 [°C]) to a 10% step change in C1

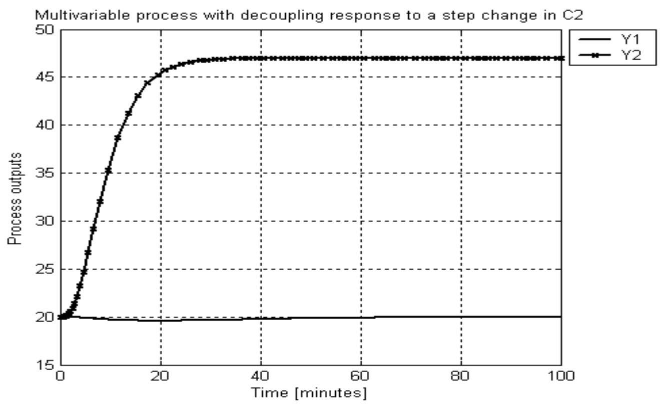

Fig. 11. Multivariable process with decoupling response (Y1 and Y2 [°C]) to a 10% step change in C2

Also in Figure 12 we have the magnified trend of the output Y1 to the change of input C2, from Figure 10. We can also see that the decoupling is not perfect because of the modeling errors.

Using the decoupler connected in series with the multivariable process we can consider, as we see from Figures 9-12, that the obtained process behaves approximately as two monovariable processes, so in order to further control it we can use two monovariable PID controllers, one for each direct I-O process channel.

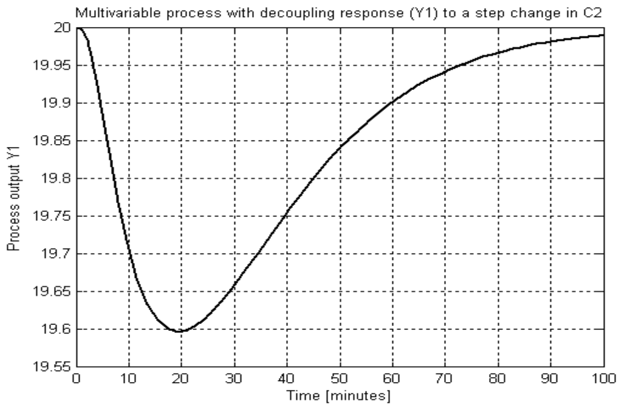

Fig. 12. Multivariable process with decoupling response (Y1 [°C]) to a 10% step change in C2

Conclusions

This paper objective was to study the dynamics of a multivariable 2X2 temperature process and to find a simple, feasible and accurate mathematical model, from a further process control action point of view.

The process models were determined using the process outputs trend to the input variables step changes. The models were validated comparing the trends at input variables step changes with the ones of the real process. Using the identified process, a 2X2 decoupler was designed in order to eliminate the I-O cross interactions and to obtain two monovariable systems (one pseudo-process without cross interactions).

In order to validate the proposed dedicated decoupler, the obtained pseudo-process dynamic behavior was tested for different step changes in the input variables. From the test results we can consider, with enough precision, that the system is decoupled and can easily be controlled further.

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Investigarea Dinamicii, Identificarea şi Decuplarea unui Proces Multivariabil de Temperatură 2X2

Rezumat

Marea majoritate a sistemelor sunt multivariabile, caracterizate prin prezenţa canalelor încrucişate de interacţiune intrare-ieşire, în plus faţă de cele directe. Obiectivul acestei lucrări este găsirea unui model matematic simplu şi fezabil pentru un proces de transfer termic multivariabil cu strucutură 2X2, pe baza rezultatelor obţinute în urma analizei dinamicii procesului. Acest model este necesar în etapa de proiectare a unui decuplor având aceeaşi structură cu procesul, conectat prin înseriere în faţa acestuia, al cărui scop este diminuarea sau chiar anularea interacţiunilor naturale încrucişate din proces şi obţinerea unui sistem ce are numai interacţiuni directe, care să poată fi uşor reglat.