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Research Concerning the Calculus of the Equilibrium Moment in the Case of a Plane Mechanism Using the Dynamic Model

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Abstract

In this paper some results concerning the establishing of the variation on a cinematic cycle of the equilibrium moment for a plane mechanism using the dynamic model are presented. It is shown that the variation of the equilibrium moment in the analyzed case can be very well approximated with the variation on a cinematic cycle of the reduced moment corresponding to the dynamic model of the mechanism. Some interesting simulation results that highlight the aforesaid for different functioning regimes of the mechanism are presented.

Key words: *mechanism, equilibrium moment, dynamic model*

Introduction

The equilibrium moment is one of the most important dynamic parameter used for an optimum design of the plane mechanisms [1, 2, 3, 4]. Its variation on a cinematic cycle provides useful information regarding the motor moment and the influence of different categories of forces and moments (weight forces, technological forces and moments, inertial forces and moments) that work on the component links of the analyzed mechanism [3, 4, 5].

In this paper some results concerning the establishing of the variation on a cinematic cycle of the equilibrium moment for a plane mechanism using the variation on a cinematic cycle of the reduced moment corresponding to the dynamic model of the mechanism are presented. It is analyzed the possibility of approximation of the variation of the equilibrium moment in the analyzed case with the variation on a cinematic cycle of the reduced moment. The simulations carried out highlight the aforesaid for different functioning regimes of the mechanism.

Theoretical Considerations and Simulation Results

In figure 1 the cinematic scheme of the analyzed plane mechanism is presented. The following elements are considered to be known:

- the dimensions of the component links: *OA*=0.06 m; *AB*=0.3 m; *AK*=0.1 m; *BK*=0.24 m; *KD*=0.3 m; *DE*=0,15 m; $x_E = 0.38\,\text{m}$; $y_E = 0.3\,\text{m}$. The mass centers: C_1, C_4, C_5 are on the middle of the corresponding links and C_2 is on the mass center of the triangle *ABK*.

- the mass of the component links: $m_1 = 1.5$ kg; $m_2 = 7$ kg; $m_3 = 2$ kg; $m_4 = 5.5$ kg; $m_5 = 3$ kg;
- the mass moments of inertia of the links: $I_{C_1} = 0.003\,\text{kgm}^2$; $I_{C_2} = 0.35\,\text{kgm}^2$; $I_{C_4} = 0.12\,\text{kgm}^2$; $I_{C_5} = 0.03\,\text{kgm}^2$. The value of I_{C_3} is neglected.
- the technological forces $\overline{F}_{ru}^{dr}$ and $\overline{F}_{ru}^{st}$ (fig. 1) work to the contrary of the speed $\overline{v}_{C_3}$ of the plunger *3* of the mechanism. Their action is cumulated in the following expression of the technological force F_r:

$$F_r = -\sin(\varphi_1 - \varphi_{13}) \cdot \left(F_r^{st} \cdot \left(1 - \frac{v_{C_3}}{|v_{C_3}|} \right) / 2 + F_r^{dr} \cdot \left(1 + \frac{v_{C_3}}{|v_{C_3}|} \right) / 2 \right) \tag{1}$$

where: φ_{13} is the value of the crank angle φ_1 when $v_{C_3} = 0$; $F_r^{dr} = 1500\,\text{N}$; $F_r^{st} = 700\,\text{N}$;

- the technological moment M_{ru} works to the contrary of the angular speed ω_5 of the rocker *5* (fig. 1). Its variation on a cinematic cycle is given by the following expression:

$$M_{ru} = M_r \cdot \sin\frac{\varphi_1 - \varphi_{1d}}{2} \cdot \sin\frac{\varphi_1 - \varphi_{1a}}{2} \tag{2}$$

where: $M_r = 120\,\text{N}\cdot\text{m}$; φ_{1d} and φ_{1a} are the values of the crank angle φ_1 corresponding to the two extreme positions of the rocker *5*, when the angle φ_5 (fig.1) achieves its two extreme values: φ_{5d} and φ_{5a}, respectively.

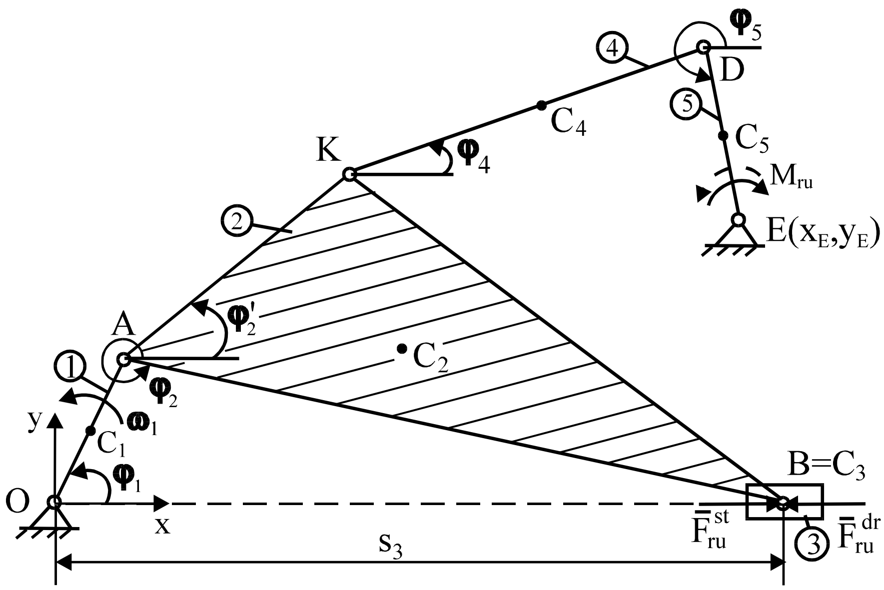

Fig. 1. Plane mechanism

The variation on a cinematic cycle of the equilibrium moment M_e can be obtained by expressing the dynamic equilibrium in instantaneous powers of all the forces and moments that work on the component links of the mechanism [1], with the following relation:

$$\overline{M}_e \cdot \overline{\omega}_1 + \sum_{j=1}^{5} \overline{G}_j \cdot \overline{v}_{C_j} + \sum_{j=1}^{5} \left(\overline{F}_{ij} \cdot \overline{v}_{C_j} + \overline{M}_{ij} \cdot \overline{\omega}_j \right) + \overline{F}_r \cdot \overline{v}_B + \overline{M}_{ru} \cdot \overline{\omega}_5 = 0 \qquad (3)$$

where:

$G_j = m_j \cdot g,\ j = \overline{1,5}$, are the weight forces corresponding to the component links ($g = 9.81\text{m/s}^2$ is the gravitational acceleration);

$\overline{F}_{ij} = -m_j \cdot \overline{a}_{C_j},\ j = \overline{1,5}$, are the inertial forces, where $\overline{a}_{C_j}$ is the acceleration of the mass center of the *j* link;

$\overline{M}_{ij} = -I_{C_j} \cdot \overline{\varepsilon}_j,\ j \in \{1,2,4,5\}$, are the inertial moments, where $\overline{\varepsilon}_j$ is the angular acceleration of the *j* link;

$\overline{v}_{C_j}$ is the speed of the mass center of the *j* link;

$\overline{\omega}_j$ is the angular speed of the *j* link;

$\overline{v}_B$ is the speed of the point *B*, where: $\overline{v}_B = \overline{v}_{C_3}$ (fig. 1).

The components of the speeds and of the accelerations of the mass centers $C_i, i = \overline{1,5}$, on the axes of the coordinates system (*Oxy*) can be determined by deriving with time the expressions of the coordinates $x_{C_i} = x_{C_i}(\varphi_1)$; $y_{C_i} = y_{C_i}(\varphi_1), i = \overline{1,5}$, with the following relations [6,7,10]:

$$\begin{cases} (v_{C_i})_x = \dfrac{\mathrm{d}x_{C_i}}{\mathrm{d}\varphi_1}\dfrac{\mathrm{d}\varphi_1}{\mathrm{d}t} = \omega_1 \cdot \dfrac{\mathrm{d}x_{C_i}}{\mathrm{d}\varphi_1} \\ (v_{C_i})_y = \dfrac{\mathrm{d}y_{C_i}}{\mathrm{d}\varphi_1}\dfrac{\mathrm{d}\varphi_1}{\mathrm{d}t} = \omega_1 \cdot \dfrac{\mathrm{d}y_{C_i}}{\mathrm{d}\varphi_1} \end{cases} \qquad (4)$$

$$\begin{cases} (a_{C_i})_x = (\dot{v}_{C_i})_x = \varepsilon_1 \cdot \dfrac{\mathrm{d}x_{C_i}}{\mathrm{d}\varphi_1} + \omega_1^2 \cdot \dfrac{\mathrm{d}^2 x_{C_i}}{\mathrm{d}\varphi_1^2} \\ (a_{C_i})_y = (\dot{v}_{C_i})_y = \varepsilon_1 \cdot \dfrac{\mathrm{d}y_{C_i}}{\mathrm{d}\varphi_1} + \omega_1^2 \cdot \dfrac{\mathrm{d}^2 y_{C_i}}{\mathrm{d}\varphi_1^2} \end{cases} \qquad (5)$$

The angular speeds and accelerations $\omega_j, \varepsilon_j, j = 2,4,5$, can be determined by deriving with time the expressions of the angles $\varphi_j(\varphi_1), j = 2,4,5$ (fig. 1), with the following relations [10,11]:

$$\begin{cases} \omega_j = \dot{\varphi}_j = \dfrac{\mathrm{d}\varphi_j}{\mathrm{d}\varphi_1} \cdot \dfrac{\mathrm{d}\varphi_1}{\mathrm{d}t} = \omega_1 \cdot \dfrac{\mathrm{d}\varphi_j}{\mathrm{d}\varphi_1} \\ \varepsilon_j = \ddot{\varphi}_j = \varepsilon_1 \cdot \dfrac{\mathrm{d}\varphi_j}{\mathrm{d}\varphi_1} + \omega_1^2 \cdot \dfrac{\mathrm{d}^2\varphi_j}{\mathrm{d}\varphi_1^2} \end{cases} \qquad (6)$$

The coordinates of the mass centers $C_i, i = \overline{1,5}$, can be calculated with the following relations:

$$\begin{cases} x_{C_1} = OC_1 \cdot \cos\varphi_1 \\ y_{C_1} = OC_1 \cdot \sin\varphi_1 \end{cases} \qquad (7)$$

$$\begin{cases} x_{C_2} = (x_A + x_B + x_K)/3 \\ y_{C_2} = (y_A + y_B + y_K)/3 \end{cases} \qquad (8)$$

where:

$$\begin{cases} x_A = OA \cdot \cos\varphi_1; \quad y_A = OA \cdot \sin\varphi_1 \\ x_B = s_3; \quad y_B = 0 \\ x_K = x_A + AK \cdot \cos\varphi_2^{'}; \quad y_K = y_A + AK \cdot \sin\varphi_2^{'} \end{cases} \tag{9}$$

where: $\varphi_2^{'} = \varphi_2 - 2\pi + \angle KAB$ (fig. 1);

$$\begin{cases} x_{C_3} = s_3 \\ y_{C_3} = 0 \end{cases} \tag{10}$$

$$\begin{cases} x_{C_4} = x_K + KC_4 \cdot \cos\varphi_4 \\ y_{C_4} = y_K + KC_4 \cdot \sin\varphi_4 \end{cases} \tag{11}$$

$$\begin{cases} x_{C_5} = x_K + KD \cdot \cos\varphi_4 + DC_5 \cdot \cos\varphi_5 \\ y_{C_5} = y_K + KD \cdot \sin\varphi_4 + DC_5 \cdot \sin\varphi_5 \end{cases} \tag{12}$$

By projecting the vector contour $\overline{OA} + \overline{AB} + \overline{BO} = 0$ on the *x* and *y* axes (fig. 1), the following system of equations is obtained:

$$\begin{cases} OA \cdot \cos\varphi_1 + AB \cdot \cos\varphi_2 - s_3 = 0 \\ OA \cdot \sin\varphi_1 + AB \cdot \sin\varphi_2 = 0 \end{cases} \tag{13}$$

By solving the system of equations (13) the unknown parameters φ_2 and s_3 can be calculated from the following relations:

$$\begin{cases} \sin\varphi_2 = -\dfrac{OA}{AB} \cdot \sin\varphi_1 \\ s_3 = OA \cdot \cos\varphi_1 + \sqrt{AB^2 - OA^2 \cdot \sin^2\varphi_1} \end{cases} \tag{14}$$

The coordinates of the point *D*, x_D and y_D, can be determined by solving the following system of equations:

$$\begin{cases} (x_D - x_K)^2 + (y_D - y_K)^2 = KD^2 \\ (x_D - x_E)^2 + (y_D - y_E)^2 = DE^2 \end{cases} \tag{15}$$

where the coordinates of the point *K* have been calculated using the relation (9).

Then, the angles φ_4 and φ_5 can be calculated from the following relations:

$$\begin{cases} \sin\varphi_4 = \dfrac{y_D - y_K}{KD} \\ \sin\varphi_5 = \dfrac{y_E - y_D}{DE} \end{cases} \tag{16}$$

The relations above have been transposed into a computer program using Maple programming language [12] that has powerful symbolic computation functions. For obtaining the analytical expressions of the speeds and of the accelerations mentioned above, the derivatives with respect to the crank angle φ_1 of the position parameters in relations (4), (5) and (6) have been calculated using the derivation function *diff* in Maple programming language [12].

For the angles $\varphi_{13}, \varphi_{1d}$ and φ_{1a} in relations (1) and (2) have been obtained the following values: $\varphi_{13} = 180^\circ$, $\varphi_{1d} = 223.035^\circ$ and $\varphi_{1a} = 58.075^\circ$.

The variation on a cinematic cycle of the reduced moment corresponding to the dynamic model of the mechanism from figure 1 can be determined with the following relation [1,8,9]:

$$M_{red} = \frac{1}{\omega_1} \cdot \left(\sum_{j=1}^{5} \overline{G}_j \cdot \overline{v}_{C_j} + \overline{F}_r \cdot \overline{v}_B + \overline{M}_{ru} \cdot \overline{\omega}_5 \right) \tag{17}$$

It can be noticed by comparing the relations (3) and (17) that if the inertial forces and the inertial moments are neglected then the following relation is true: $M_e = -M_{red}$.

In figures 2 and 3 the variation on a cinematic cycle of the equilibrium moment M_e and of $-M_{red}$ for $\omega_1 = 10$ rad/s and $\omega_1 = 15$ rad/s is presented.

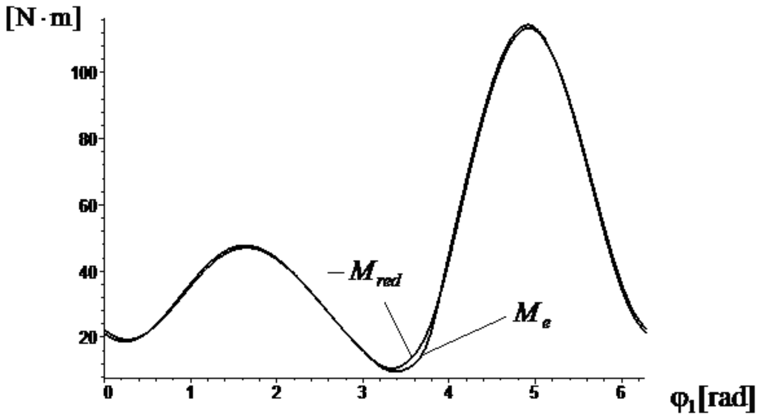

Fig. 2. The variation on a cinematic cycle of M_e and of $-M_{red}$ for $\omega_1 = 10$ rad/s

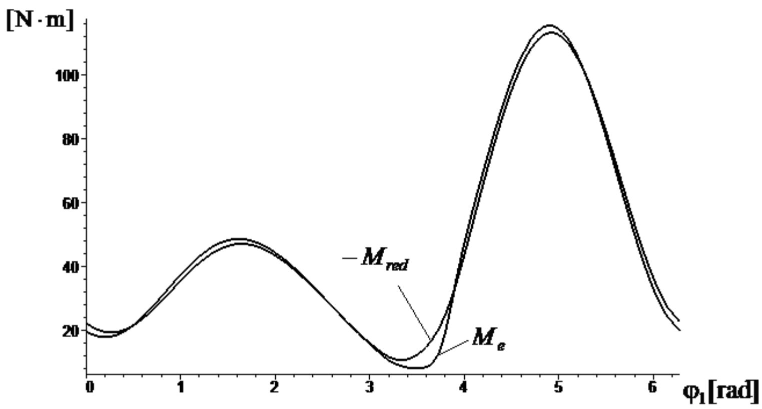

Fig. 3. The variation on a cinematic cycle of M_e and of $-M_{red}$ for $\omega_1 = 15$ rad/s

Conclusions

In this paper some results concerning the establishing of the variation on a cinematic cycle of the equilibrium moment for a plane mechanism using the variation on a cinematic cycle of the reduced moment corresponding to the dynamic model of the mechanism have been presented. The simulation results presented in figures 2 and 3 highlight the possibility of approximation of the variation of the equilibrium moment in the analyzed case with the variation on a cinematic cycle of $-M_{red}$, where M_{red} is the reduced moment.

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Cercetări privind calculul momentului de echilibrare în cazul unui mecanism plan folosind modelul dinamic

Rezumat

In articol sunt prezentate o serie de rezultate privind stabilirea variaţiei pe un ciclu cinematic a momentului de echilibrare pentru un mecanism plan folosind modelul dinamic. Se arată că variaţia momentului de echilibrare în cazul analizat poate fi foarte bine aproximată cu variaţia pe un ciclu cinematic a momentului redus corespunzător modelului dinamic al mecanismului. Sunt prezentate câteva rezultate interesante ale simulărilor care evidenţiază cele menţionate mai înainte pentru diferite regimuri de funcţionare ale mecanismului.