BULETINUL Universităţii Petrol – Gaze din Ploieşti	Vol. LXVII No. 1/2015	93 – 98	Seria Tehnică

On the Calculus of Some Cinematic Parameters of the Sucker Rod Pumping Units

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Abstract

In the paper some results concerning the calculus of the displacement, the speed and the acceleration of the end of the polished rod are presented. First, some results concerning the determination of the exact variation on a cinematic cycle of these parameters using the method of the projection of the independent contours are presented. There are also presented the results obtained by applying the theory of the approximate kinematics and the theory of the elementary kinematics. Then the variation of the parameters mentioned above on a cinematic cycle is approximated with polynomial functions whose coefficients are calculated with the method of the least squares. These coefficients depend only on the dimensions of the component links of the rod pumping unit. Finally, some interesting simulation results in the case of the pumping unit C-640D-365-144 are presented.

Key words: *pumping unit mechanism, exact kinematics, the method of the least squares*

Introduction

Generally the design of the mechanisms requires a rigorous analysis of the variation on a cinematic cycle of different cinematic and dynamic parameters that characterize their functioning [1, 2, 3, 4]. In the case of the sucker rod pumping units the most representative cinematic parameters whose variations on a cinematic cycle have to be determined are the displacement, the speed and the acceleration of the end of the polished rod.

Because the analytical expressions of these parameters are quite complicated [2, 5] sometimes, for an easy assessment, some simplified theories, like: the theory of the approximate kinematics and the theory of the elementary kinematics are utilized [1].

In this paper some results concerning the calculus of the displacement, the speed and the acceleration of the end of the polished rod are presented. First, some results concerning the determination of the exact variation on a cinematic cycle of the parameters mentioned above using the method of the projection of the independent contours are presented. There are also presented the results obtained by applying the theory of the approximate kinematics and the theory of the elementary kinematics.

Then the variation of the displacement, the speed and the acceleration of the end of the polished rod on a cinematic cycle is approximated with polynomial functions whose coefficients are calculated with the method of the least squares.

Finally, some simulation results that permit to compare the variation of these parameters on a cinematic cycle obtained by applying the exact cinematic method, the approximate and elementary kinematics theory and the polynomial functions calculated with the method of the least squares in the case of the pumping unit C-640D-365-144 are presented.

Theoretical Considerations and Simulation Results

In figure 1 the cinematic scheme of the mechanism of the sucker rod pumping unit with conventional type geometry is presented.

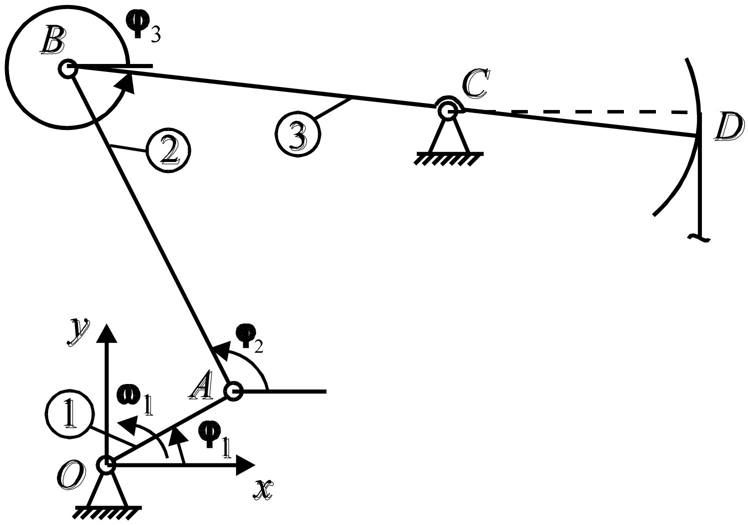

Fig. 1. Pumping unit mechanism with conventional type geometry

The value of the displacement of the end of the polished rod denoted by s_D, can be calculated with the following relation [2]:

$$s_D = (2\pi - \varphi_{3d} + \varphi_{3a}) \cdot l_{3p} \tag{1}$$

where: φ_{3d} and φ_{3a} are the values of the angle φ_3 for the extreme positions of the rocker of the mechanism and $l_{3p} = CD$.

In [2] the values of the angles φ_{3d} and φ_{3a} and of the angles φ_{1d} and φ_{1a} corresponding to the crank angle φ_1 for the extreme positions of the rocker have been calculated from the following systems of equations obtained by projecting the contour $O-A-B-C-O$ on the *x* and *y* axes (fig. 1) for these two extreme positions of the rocker:

$$\begin{cases} (l_1 + l_2) \cdot \cos\varphi_{1d} + l_3 \cdot \cos\varphi_{3d} - x_C = 0 \\ (l_1 + l_2) \cdot \sin\varphi_{1d} + l_3 \cdot \sin\varphi_{3d} - y_C = 0 \end{cases} \tag{2}$$

$$\begin{cases} (l_1 - l_2) \cdot \cos\varphi_{1a} + l_3 \cdot \cos\varphi_{3a} - x_C = 0 \\ (l_1 - l_2) \cdot \sin\varphi_{1a} + l_3 \cdot \sin\varphi_{3a} - y_C = 0 \end{cases} \tag{3}$$

where: $l_1 = OA$; $l_2 = AB$; $l_3 = BC$.

Some results concerning the determination of the exact variation on a cinematic cycle of the speed and of the acceleration of the end of the polished rod using the method of the projection of the independent contours are presented in [3].

By projecting the contour $O-A-B-C-O$ (fig. 1) on the *x* and *y* axes the unknown angles φ_2 and φ_3 were calculated from the following system of equations:

$$\begin{cases} l_1 \cdot \cos\varphi_1 + l_2 \cdot \cos\varphi_2 + l_3 \cdot \cos\varphi_3 - x_C = 0 \\ l_1 \cdot \sin\varphi_1 + l_2 \cdot \sin\varphi_2 + l_3 \cdot \sin\varphi_3 - y_C = 0 \end{cases} \tag{4}$$

The relations above have been transposed into a computer program using Maple programming language that has integrated powerful functions for symbolical calculus [6].

The angular speeds and accelerations: $\omega_i, \varepsilon_i, i = 2{,}3$, of the links *2* and *3* have been calculated by deriving with time the variation functions of the corresponding angles φ_2 and φ_3, using the derivation function *diff* in Maple programming language [6].

Then, the speed v_D and the acceleration a_D (fig. 1) of the end of the polished rod have been calculated with the following relations: $v_D = \omega_3 \cdot l_{3p}$; $a_D = \varepsilon_3 \cdot l_{3p}$.

The relations for the calculus of the displacement, the speed and the acceleration of the end of the polished rod using the theory of the approximate kinematics [1] are the following:

$$s_D = \frac{l_{3p}}{l_3} \cdot l_1 \cdot (1 - \cos\varphi_1 + \frac{l_1}{2 \cdot l_2} \cdot \sin^2\varphi_1) \tag{5}$$

$$v_D = \frac{l_{3p}}{l_3} \cdot l_1 \cdot \omega_1 \cdot (\sin\varphi_1 + \frac{l_1}{2 \cdot l_2} \cdot \sin 2\varphi_1) \tag{6}$$

$$a_D = \frac{l_{3p}}{l_3} \cdot l_1 \cdot \omega_1^2 \cdot (\cos\varphi_1 + \frac{l_1}{l_2} \cdot \cos 2\varphi_1) \tag{7}$$

and those coresponding to the theory of the elementary kinematics [1] are:

$$s_D = \frac{l_{3p}}{l_3} \cdot l_1 \cdot (1 - \cos\varphi_1) \tag{8}$$

$$v_D = \frac{l_{3p}}{l_3} \cdot l_1 \cdot \omega_1 \cdot \sin\varphi_1 \tag{9}$$

$$a_D = \frac{l_{3p}}{l_3} \cdot l_1 \cdot \omega_1^2 \cdot \cos\varphi_1 \tag{10}$$

Now, the variation on a cinematic cycle of the acceleration of the end of the polished rod is approximated with a polynomial function whose coefficients are calculated with the method of the least squares [7]. In this case the acceleration a_D of the end of the polished rod can be calculated with the following relation:

$$a_D = \omega_1^2 \cdot P_m(\varphi_1) \tag{11}$$

where:

$$P_m(\varphi_1) = c_0 + c_1 \cdot \varphi_1 + c_2 \cdot \varphi_1^2 + \ldots + c_m \cdot \varphi_1^m \tag{12}$$

The values of the coefficients $c_i, i=\overline{0,m}$, that gives the best approximation of a_D/ω_1^2 can be determined by solving the following system of equations [7]:

$$\begin{cases} c_0\cdot(n+1)+c_1\cdot\sum_{i=0}^{n}\varphi_{1i}+c_2\cdot\sum_{i=0}^{n}\varphi_{1i}^2+\ldots+c_m\cdot\sum_{i=0}^{n}\varphi_{1i}^m=\frac{1}{\omega_1^2}\sum_{i=0}^{n}a_D(\varphi_{1i}) \\ c_0\cdot\sum_{i=0}^{n}\varphi_{1i}+c_1\cdot\sum_{i=0}^{n}\varphi_{1i}^2+c_2\cdot\sum_{i=0}^{n}\varphi_{1i}^3+\ldots+c_m\cdot\sum_{i=0}^{n}\varphi_{1i}^{m+1}=\frac{1}{\omega_1^2}\sum_{i=0}^{n}\varphi_{1i}\cdot a_D(\varphi_{1i}) \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ c_0\cdot\sum_{i=0}^{n}\varphi_{1i}^m+c_1\cdot\sum_{i=0}^{n}\varphi_{1i}^{m+1}+c_2\cdot\sum_{i=0}^{n}\varphi_{1i}^{m+2}+\ldots+c_m\cdot\sum_{i=0}^{n}\varphi_{1i}^{2m}=\frac{1}{\omega_1^2}\sum_{i=0}^{n}\varphi_{1i}^m\cdot a_D(\varphi_{1i}) \end{cases} \quad (13)$$

where: $\varphi_{1i}=\varphi_{1d}+2\cdot\pi\cdot i/n$.

The speed and the displacement of the end of the polished rod can be determined by starting from the expression (11) of the acceleration a_D with the following relations:

$$v_D=\omega_1\cdot(Q_{m+1}(\varphi_1)+C_v) \quad (14)$$

$$s_D=R_{m+2}(\varphi_1)+C_s \quad (15)$$

where:

$$Q_{m+1}(\varphi_1)=\int_0^{\varphi_1}P_m(\varphi_1)\,\mathrm{d}\varphi_1 \quad (16)$$

$$C_v=-Q_{m+1}(\varphi_{1d}) \quad (17)$$

$$R_{m+2}(\varphi_1)=\int_0^{\varphi_1}(Q_{m+1}(\varphi_1)+C_v)\,\mathrm{d}\varphi_1 \quad (18)$$

$$C_s=-R_{m+2}(\varphi_{1d}) \quad (19)$$

By transposing the relations above into a computer program using Maple programming language the polynomial function $P_m(\varphi_1)$ that gives the best approximation of a_D/ω_1^2 in the case of C-640D-365-144 pumping unit, whose dimensions of the component links and the value of the crank angle φ_{1d} are presented in table 1, has the following expression:

$$\begin{aligned} P_m(\varphi_1) &= 20.601-33.013\cdot\varphi_1+23.789\cdot\varphi_1^2-8.222\cdot\varphi_1^3+1.146\cdot\varphi_1^4+0.013\cdot\varphi_1^5- \\ &-0.019\cdot\varphi_1^6+0.003\cdot\varphi_1^7-611\cdot10^{-6}\cdot\varphi_1^8+79.2\cdot10^{-6}\cdot\varphi_1^9-0.681\cdot10^{-5}\cdot\varphi_1^{10}+ \\ &+0.7398\cdot10^{-6}\cdot\varphi_1^{11}+0.216\cdot10^{-7}\cdot\varphi_1^{12}-0.156\cdot10^{-7}\cdot\varphi_1^{13}+0.938\cdot10^{-9}\cdot\varphi_1^{14} \end{aligned} \quad (20)$$

In figures 2, 3 and 4 the variation curves of a_D/ω_1^2, v_D/ω_1 and s_D on a cinematic cycle, beginning with the value of the crank angle equal to φ_{1d}, in the case of C-640D-365-144 pumping unit by considering the exact cinematic theory, the theory of the approximate kinematics, the theory of the elementary kinematics and the approximated formulation with polynomial functions are presented.

Table 1. The dimensions of the component links and the value of the crank angle φ_{1d} for C-640D-365-144 pumping unit

l_1 [m]	l_2 [m]	l_3 [m]	l_{3p} [m]	x_C [m]	y_C [m]	φ_{1d} [˚]
1.1938	3.7719	3.0488	4.572	3.048	3.8354	87.522

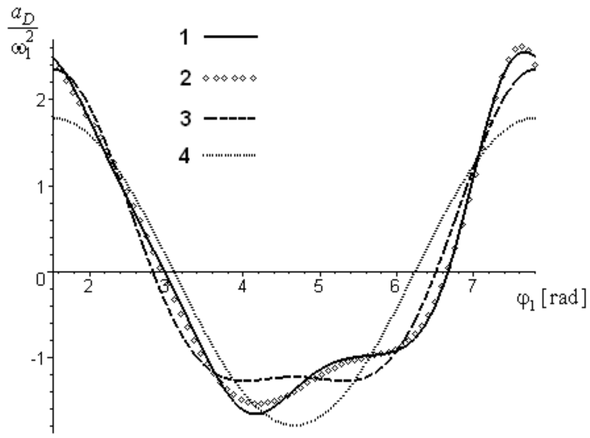

Fig. 2. The variation on a cinematic cycle of a_D / ω_1^2 for C-640D-365-144 pumping unit in the case of the exact cinematic theory (curve 1), the approximated formulation with polynomial functions (curve 2), the theory of the approximate kinematics (curve 3) and the theory of the elementary kinematics (curve 4)

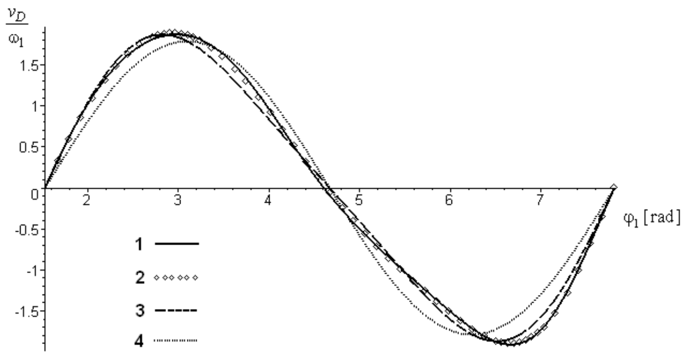

Fig. 3. The variation on a cinematic cycle of v_D / ω_1 for C-640D-365-144 pumping unit in the case of the exact cinematic theory (curve 1), the approximated formulation with polynomial functions (curve 2), the theory of the approximate kinematics (curve 3) and the theory of the elementary kinematics (curve 4)

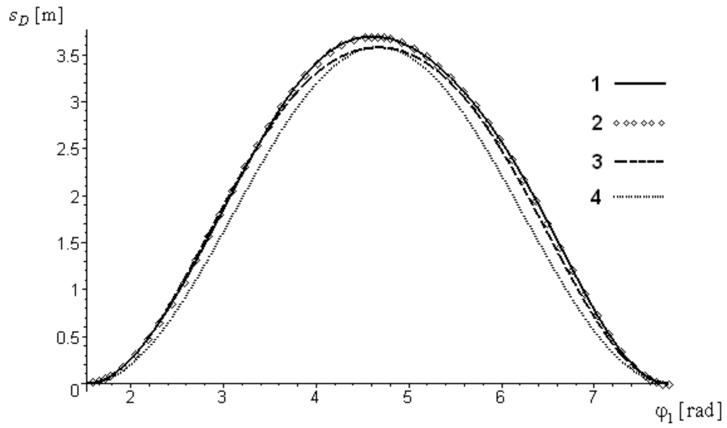

Fig. 4. The variation on a cinematic cycle of the displacement s_D for C-640D-365-144 pumping unit in the case of the exact cinematic theory (curve 1), the approximated formulation with polynomial functions (curve 2), the theory of the approximate kinematics (curve 3) and the theory of the elementary kinematics (curve 4)

Conclusions

In this paper a method that permits the determination of the variation of the displacement, the speed and the acceleration of the end of the polished rod on a cinematic cycle using polynomial functions whose coefficients are calculated with the method of the least squares has been presented. The results obtained in this case have been compared with those obtained by applying the exact cinematic method, the approximate and elementary kinematics theory for the pumping unit C-640D-365-144. Figures 2, 3 and 4 show a good accordance between the results obtained by approximating the variation of the displacement, the speed and the acceleration of the end of the polished rod on a cinematic cycle using polynomial functions with those obtained by applying the exact cinematic method, especially in the case of the variation of the speed and of the displacement (figs. 3 and 4). This accordance is much higher than in the case when there are used the approximate and elementary kinematics theory.

References

1. Popovici, Al. – *Utilaj pentru exploatarea sondelor de petrol,* Editura Tehnică, Bucuresti, 1989
2. Toma, G., Pupăzescu, Al., Bădoiu, D. – On a synthesis problem of the mechanism of a sucker rod pumping unit, *Buletinul Universităţii Petrol-Gaze din Ploieşti, Seria Tehnică*, Vol. LXV, Nr. 4, 2013.
3. Toma, G., Pupăzescu, Al., Bădoiu, D. – On the kinematics of some sucker rod pumping units, *Buletinul Universităţii Petrol-Gaze din Ploieşti, Seria Tehnică*, Vol. LXVI, Nr. 3, 2014.
4. Petre, N., Chiţu-Militaru, P. – *Extracţia ţiţeiului prin pompaj cu prăjini,* Editura Tehnică, Bucureşti, 1986.
5. Toma, G., Bădoiu, D. – Research Concerning the Influence of Some Constructive Errors on the Dynamics of a Pumping Unit, *Buletinul Universităţii Petrol-Gaze din Ploieşti, Seria Tehnică*, Vol. LXIII, Nr. 4, 2011.
6. Monagan, M.B., Geddes, K.O., Heal, K.M., Labahn, G., Vorkoetter, S.M., McCarron, J., DeMarco, P. – *Maple Introductory Programming Guide*, Maplesoft, a division of Waterloo Maple Inc., 2005
7. Dodescu, Gh., Toma, M. – *Metode de calcul numeric,* Editura Didactică şi Pedagogică, Bucureşti, 1976.

Asupra calculului unor parametri cinematici ai unităţilor de pompare cu prăjini

Rezumat

In articol sunt prezentate o serie de rezultate privind calculul deplasării, vitezei şi acceleraţiei la capătul prăjinii lustruite. Mai întâi, sunt prezentate o serie de rezultate privind determinarea variaţiei exacte pe un ciclu cinematic a parametrilor menţionaţi anterior folosind metoda proiecţiei contururilor independente. Sunt de asemenea prezentate rezultatele obţinute prin aplicarea teoriei cinematicii aproximative şi a teoriei cinematicii elementare. Apoi variaţia pe un ciclu cinematic a parametrilor menţionaţi este aproximată cu funcţii polinomiale ale căror coeficienţi sunt calculaţi cu metoda celor mai mici pătrate. Aceşti coeficienţi depind numai de dimensiunile elementelor componente ale unităţii de pompare. In final, sunt prezentate o serie de rezultate ale simulărilor realizate în cazul unităţii de pompare C-640D-365-144.