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# On the Calculus of the Equilibrium Moment for a Mechanism with Three Independent Contours

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## Abstract

*In this paper some results concerning the variation on a cinematic cycle of the equilibrium moment for a mechanism with three independent contours are presented. The calculus of the equilibrium moment is developed using the dynamic equilibrium in instantaneous powers of all the forces and moments that work on the component links of the mechanism. Special attention was given to the influence of the inertial forces and moments. Finally, some interesting simulation results that highlight this influence for different functioning regimes of the mechanism are presented.*

**Key words:** *mechanism, dynamics, equilibrium moment*

## Introduction

One of the most important problems for an optimum design of the mechanisms is to analyze the variation on a cinematic cycle of the equilibrium moment. For a safe operation, many times, the calculus of the equilibrium moment requires as a mandatory step a rigorous analysis of the influence of different categories of forces and moments (weight forces, technological forces and moments, inertial forces and moments) on its variation on a cinematic cycle [1].

In this paper some results concerning the variation on a cinematic cycle of the equilibrium moment for a mechanism with three independent contours (fig. 1) are presented. The calculus of the equilibrium moment is developed using the dynamic equilibrium in instantaneous powers of all the forces and moments that work on the component links of the mechanism. Special attention was given to the influence of the inertial forces and moments. For analyzing the influence of all the categories of forces and moments that work on the component links of the mechanism a simulation program using Maple programming language [5] was developed. Finally, some interesting simulation results are presented.

## Theoretical Considerations and Simulation Results

In figure 1 the cinematic scheme of a mechanism with three independent contours is presented.

The following elements are considered to be known:

- the dimensions of the component links: $OA = 0.14\,\text{m}$; $AC = 1.05\,\text{m}$; $AB = 0.525\text{m}$; $BD$ = 0.25 m; $DE$ = $EF$ = 0.225 m; $FG = 1\,\text{m}$; $x_E = 0.5\,\text{m}$; $y_E = -0.3\,\text{m}$; the mass centers:

$C_1, C_2, C_4, C_5$ and $C_6$ are on the middle of the corresponding links;

- the mass of the component links: $m_1 = 5.5$kg; $m_2 = 16$kg; $m_3 = 5$kg, $m_4 = 10$kg, $m_5 = 10$kg, $m_6 = 15.5$kg, $m_7 = 5$kg;

- the moments of inertia of the links: $I_{C_1} = 0.009\,\text{kgm}^2$; $I_{C_2} = 1.47\,\text{kgm}^2$; $I_{C_4} = 0.05\,\text{kgm}^2$, $I_{C_5} = 0.17\,\text{kgm}^2$, $I_{C_6} = 1.3\,\text{kgm}^2$; $I_{C_3}$ and $I_{C_7}$ have negligible values;

- the technological forces: $F_3^{dr} = 1500\,\text{N}$, $F_3^{st} = 150\,\text{N}$, $F_7^{dr} = 1200\,\text{N}$, $F_7^{st} = 120\,\text{N}$;

- the nominal angular speed of the leader link of the mechanism: $\omega_1 = 15$ rad/s .

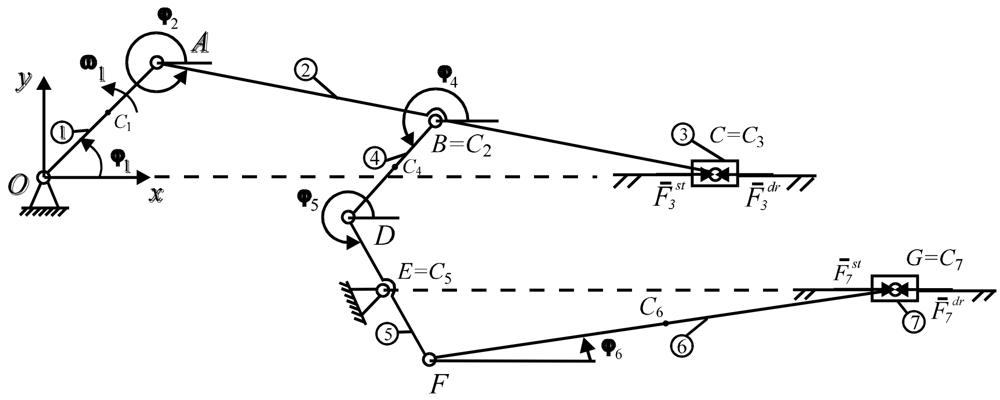


**Fig. 1.** Mechanism with three independent contours

The variation on a cinematic cycle of the equilibrium moment $M_e$ can be obtained by expressing the dynamic equilibrium in instantaneous powers of all the forces and moments that work on the component links of the mechanism [1], with the following relation:

$$\overline{M}_e \cdot \overline{\omega}_1 + \sum_{j=1}^{7} \overline{G}_j \cdot \overline{v}_{C_j} + \sum_{j=1}^{7} \left( \overline{F}_{ij} \cdot \overline{v}_{C_j} + \overline{M}_{ij} \cdot \overline{\omega}_j \right) + \overline{F}_3 \cdot \overline{v}_C + \overline{F}_7 \cdot \overline{v}_G = 0 \qquad (1)$$

where:

$G_j = m_j \cdot g,\ j = \overline{1,7}$, are the weight forces corresponding to the component links ( $g = 9.81\text{m/s}^2$ is the gravitational acceleration);

$\overline{F}_{ij} = -m_j \cdot \overline{a}_{C_j},\ j = \overline{1,7}$, are the inertial forces, where $\overline{a}_{C_j}$ is the acceleration of the mass center of the *j* link;

$\overline{M}_{ij} = -I_{C_j} \cdot \overline{\varepsilon}_j,\ j \in \{1,2,4,5,6\}$, are the inertial moments, where $\overline{\varepsilon}_j$ is the angular acceleration of the *j* link;

$\overline{v}_{C_j}$ is the speed of the mass center of the *j* link;

$\overline{\omega}_j$ is the angular speed of the *j* link;

$\overline{v}_C$ and $\overline{v}_G$ are the speeds of the points *C* and *G* respectively, where: $\overline{v}_C = \overline{v}_{C_3}$ and $\overline{v}_G = \overline{v}_{C_7}$ ;

$\overline{F}_j, j = \overline{3,7}$, are the technological forces equal to $\overline{F}_j^{dr}, j = \overline{3,7}$, when the *j* element moves to the right and equal to $\overline{F}_j^{st}, j = \overline{3,7}$, when the movement is to the left.

Some results concerning the calculus of the positional and cinematic parameters by applying the method of the projection of the independent contours are presented in [2].

The mechanism has three independent contours: *0-1-2-3-0* (*O-A-C-O*); *0-1-2-4-5-0* (*O-A-B-D-E-O*) and *0-5-6-7-0* (*E-F-G-E*). By projecting the three independent contours mentioned above on the *x* and *y* axes (fig. 1) the following systems of equations were obtained:

$$\begin{cases} l_1 \cdot \cos\varphi_1 + l_2 \cdot \cos\varphi_2 - s_3 = 0 \\ l_1 \cdot \sin\varphi_1 + l_2 \cdot \sin\varphi_2 = 0 \end{cases} \tag{2}$$

$$\begin{cases} l_1 \cdot \cos\varphi_1 + l_{21} \cdot \cos\varphi_2 + l_4 \cdot \cos\varphi_4 + l_{51} \cdot \cos\varphi_5 - x_E = 0 \\ l_1 \cdot \sin\varphi_1 + l_{21} \cdot \sin\varphi_2 + l_4 \cdot \sin\varphi_4 + l_{51} \cdot \sin\varphi_5 + |y_E| = 0 \end{cases} \tag{3}$$

$$\begin{cases} l_{52} \cdot \cos\varphi_5 + l_6 \cdot \cos\varphi_6 - s_7 = 0 \\ l_{52} \cdot \sin\varphi_5 + l_6 \cdot \sin\varphi_6 = 0 \end{cases} \tag{4}$$

where: $l_1 = OA$; $l_2 = AC$; $l_{21} = AB$; $l_4 = BD$; $l_{51} = DE$; $l_{52} = EF$; $l_6 = FG$; $s_3 = OC$; $s_7 = EG$.

The resolving of the system of equations (2), (3) and (4) for the determination of the unknown parameters: $\varphi_2, \varphi_4, \varphi_5, \varphi_6, s_3$ and $s_7$ is presented in [2].

The coordinates of the mass centers $C_i, i = \overline{1,7}$, can be calculated with the following relations:

$$\begin{cases} x_{C_1} = OC_1 \cdot \cos\varphi_1 \\ y_{C_1} = OC_1 \cdot \sin\varphi_1 \end{cases} \tag{5}$$

$$\begin{cases} x_{C_2} = OA \cdot \cos\varphi_1 + AC_2 \cdot \cos\varphi_2 \\ y_{C_2} = OA \cdot \sin\varphi_1 + AC_2 \cdot \sin\varphi_2 \end{cases} \tag{6}$$

$$\begin{cases} x_{C_3} = s_3 \\ y_{C_3} = 0 \end{cases} \tag{7}$$

$$\begin{cases} x_{C_4} = OA \cdot \cos\varphi_1 + AB \cdot \cos\varphi_2 + BC_4 \cdot \cos\varphi_4 \\ y_{C_4} = OA \cdot \sin\varphi_1 + AB \cdot \sin\varphi_2 + BC_4 \cdot \sin\varphi_4 \end{cases} \tag{8}$$

$$\begin{cases} x_{C_5} = x_E \\ y_{C_5} = y_E \end{cases} \tag{9}$$

$$\begin{cases} x_{C_6} = OA \cdot \cos\varphi_1 + AB \cdot \cos\varphi_2 + BD \cdot \cos\varphi_4 + DF \cdot \cos\varphi_5 + FC_6 \cdot \cos\varphi_6 \\ y_{C_6} = OA \cdot \sin\varphi_1 + AB \cdot \sin\varphi_2 + BD \cdot \sin\varphi_4 + DF \cdot \sin\varphi_5 + FC_6 \cdot \sin\varphi_6 \end{cases} \tag{10}$$

$$\begin{cases} x_{C_7} = x_E + s_7 \\ y_{C_7} = y_E \end{cases} \tag{11}$$

The components of the speeds and of the accelerations of the mass centers $C_i, i=\overline{1,7}$, on the axes of the coordinates system ($Oxy$) can be determined by deriving with time the expressions of the coordinates $x_{C_i} = x_{C_i}(\varphi_1), i=\overline{1,7}$ and $y_{C_i} = y_{C_i}(\varphi_1), i=\overline{1,7}$, established before with the following relations [3,4]:

$$\begin{cases} (v_{C_i})_x = \dfrac{\mathrm{d}x_{C_i}}{\mathrm{d}\varphi_1}\dfrac{\mathrm{d}\varphi_1}{\mathrm{d}t} = \omega_1 \cdot \dfrac{\mathrm{d}x_{C_i}}{\mathrm{d}\varphi_1} \\ (v_{C_i})_y = \dfrac{\mathrm{d}y_{C_i}}{\mathrm{d}\varphi_1}\dfrac{\mathrm{d}\varphi_1}{\mathrm{d}t} = \omega_1 \cdot \dfrac{\mathrm{d}y_{C_i}}{\mathrm{d}\varphi_1} \end{cases} \tag{12}$$

$$\begin{cases} (a_{C_i})_x = (\dot{v}_{C_i})_x = \varepsilon_1 \cdot \dfrac{\mathrm{d}x_{C_i}}{\mathrm{d}\varphi_1} + \omega_1^2 \cdot \dfrac{\mathrm{d}^2 x_{C_i}}{\mathrm{d}\varphi_1^2} \\ (a_{C_i})_y = (\dot{v}_{C_i})_y = \varepsilon_1 \cdot \dfrac{\mathrm{d}y_{C_i}}{\mathrm{d}\varphi_1} + \omega_1^2 \cdot \dfrac{\mathrm{d}^2 y_{C_i}}{\mathrm{d}\varphi_1^2} \end{cases} \tag{13}$$

The angular speeds and accelerations $\omega_j, \varepsilon_j, j = 2,4,5,6$, can be determined by deriving with time the expressions of the angles $\varphi_j(\varphi_1), j = 2,4,5,6$, with the following relations [3,4]:

$$\begin{cases} \omega_j = \dot{\varphi}_j = \dfrac{\mathrm{d}\varphi_j}{\mathrm{d}\varphi_1} \cdot \dfrac{\mathrm{d}\varphi_1}{\mathrm{d}t} = \omega_1 \cdot \dfrac{\mathrm{d}\varphi_j}{\mathrm{d}\varphi_1} \\ \varepsilon_j = \ddot{\varphi}_j = \varepsilon_1 \cdot \dfrac{\mathrm{d}\varphi_j}{\mathrm{d}\varphi_1} + \omega_1^2 \cdot \dfrac{\mathrm{d}^2\varphi_j}{\mathrm{d}\varphi_1^2} \end{cases} \tag{14}$$

The relations above have been transposed into a computer program using Maple programming language [5] that has powerful symbolic computation functions. For obtaining the analytical expressions of the speeds and the accelerations mentioned above, the derivatives with respect to the crank angle $\varphi_1$ of the position parameters have been calculated using the derivation function *diff* in Maple programming language [5].

In Figures 2, 3, 4 and 5 the variation on a cinematic cycle of the equilibrium moment for $\omega_1 = 15$ rad/s, $\omega_1 = 10$ rad/s, $\omega_1 = 7$ rad/s and $\omega_1 = 5$ rad/s respectively, when the effect of the inertial forces and moments is taken into consideration and when this effect is neglected is presented.

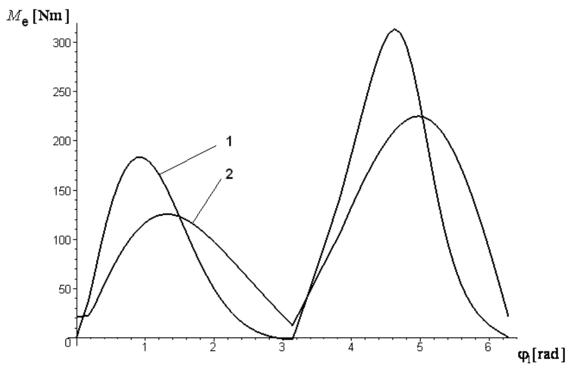


**Fig. 2.** The variation on a cinematic cycle of the equilibrium moment for $\omega_1 = 15$ rad/s, when the effect of the inertial forces and moments is taken into consideration (curve *1*) and when this effect is neglected (curve *2*)

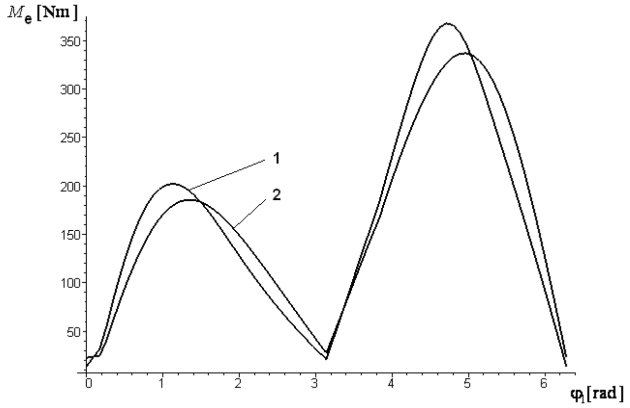


**Fig. 3.** The variation on a cinematic cycle of the equilibrium moment for $\omega_1 = 10$ rad/s, when the effect of the inertial forces and moments is taken into consideration (curve *1*) and when this effect is neglected (curve *2*)

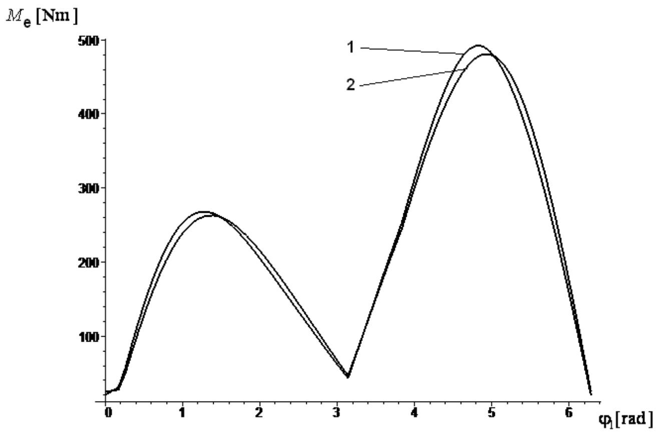


**Fig. 4.** The variation on a cinematic cycle of the equilibrium moment for $\omega_1 = 7$ rad/s, when the effect of the inertial forces and moments is taken into consideration (curve *1*) and when this effect is neglected (curve *2*)

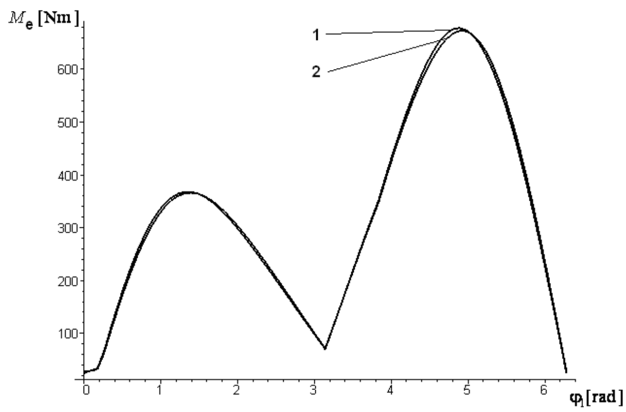


**Fig. 5.** The variation on a cinematic cycle of the equilibrium moment for $\omega_1 = 5$ rad/s, when the effect of the inertial forces and moments is taken into consideration (curve *1*) and when this effect is neglected (curve *2*)

## Conclusions

In this paper some results concerning the variation on a cinematic cycle of the equilibrium moment for a mechanism with three independent contours have been presented. The calculus of the equilibrium moment has been developed using the dynamic equilibrium in instantaneous powers of all the forces and moments that work on the component links of the mechanism. Special attention was given to the influence of the inertial forces and moments.

Figures 2, 3, 4 and 5 highlight this influence for different functioning regimes of the mechanism. Figures 2 and 3 show that when the angular speed $\omega_1$ of the leader link of the mechanism takes high values the effect of the inertial forces and moments is very important regarding the variation on a cinematic cycle of the equilibrium moment. When the mechanism moves slowly (the angular speed $\omega_1$ has small values) the effect of the inertial forces and moments can be neglected as can be seen in figure 4 and especially in figure 5.

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# Asupra calculului momentului de echilibrare pentru un mecanism cu trei contururi independente

## Rezumat

*In articol sunt prezentate o serie de rezultate privind variaţia pe un ciclu cinematic a momentului de echilibrare pentru un mecanism cu trei contururi independente. Calculul momentului de echilibrare s-a realizat folosind echilibrul dinamic în puteri instantanee ale tuturor forţelor şi momentelor care acţionează asupra elementelor componente ale mecanismului. O atenţie deosebită a fost acordată influenţei forţelor şi momentelor de inerţie. In final, sunt prezentate câteva rezultate interesante care evidenţiază această influenţă pentru diferite regimuri de funcţionare ale mecanismului.*