BULETINUL Universităţii Petrol – Gaze din Ploieşti	Vol. LXVI No. 3/2014	95 - 100	Seria Tehnică

On the Kinematics of Some Sucker Rod Pumping Units

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Abstract

In the paper some results concerning the exact kinematics analysis of the mechanism of some sucker rod pumping units are presented. The analysis is realized using the method of the projection of the independent contours. The positional and cinematic parameters are determined in an analytical form depending on the angle of the driving cranks, using the powerful functions for symbolical calculus integrated in Maple program. Special attention was given to the variation of the acceleration of the end of the polished rod. Finally, some interesting simulation results in the case of the Lufkin pumping units: C-1280D-427-192, RM-1280D-427-192, M-1280D-427-192 and A-1280D-427-192 are presented.

Key words: *pumping unit mechanism, exact kinematics*

Introduction

The cinematic study of the mechanism of the pumping units has as the main goal the determination of the variation on a cinematic cycle of the displacement, speed and acceleration of the end of the polished rod. Because the analytical expressions of these parameters are quite complicated sometimes some simplified theories, like: the theory of the approximate kinematics and the theory of the elementary kinematics are utilized [1]. But, generally the design of the mechanism of a sucker rod pumping unit requires a rigorous analysis of the influence of the length of the component links on the variation of different positional, cinematic and dynamic parameters that characterize the installation functioning [2, 3]. Also, in many design situations of these mechanisms it is necessary to solve synthesis problems for establishing the values of the length of the component links that ensure some imposed values of the positional and cinematic parameters [2, 4].

In this paper some results concerning the exact kinematics analysis of the mechanism of the sucker rod pumping units with conventional type geometry and modified geometry are presented. The positional and cinematic parameters are determined in an analytical form depending on the angle of the driving crank, using the powerful functions for symbolical calculus integrated in Maple program [5]. Special attention was given to the variation of the acceleration of the end of the polished rod that has a great influence on the dynamics of the string of sucker rods. Finally, some interesting simulation results in the case of the Lufkin pumping units with conventional type geometry: C-1280D-427-192 and RM-1280D-427-192 and of the Lufkin pumping units with modified geometry: M-1280D-427-192 and A-1280D-427-192 are presented.

Theoretical Considerations and Simulation Results

In figures 1 and 2 the cinematic schemes of the mechanism of the sucker rod pumping units with conventional type geometry and modified geometry, respectively, are presented.

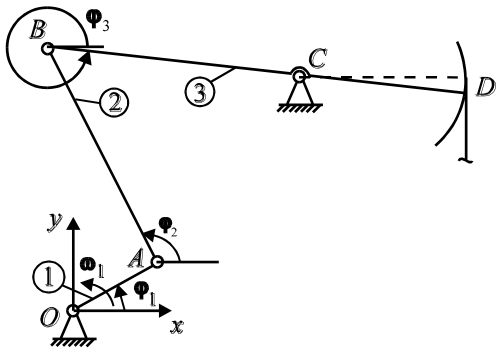

Fig. 1. Pumping unit mechanism with conventional type geometry

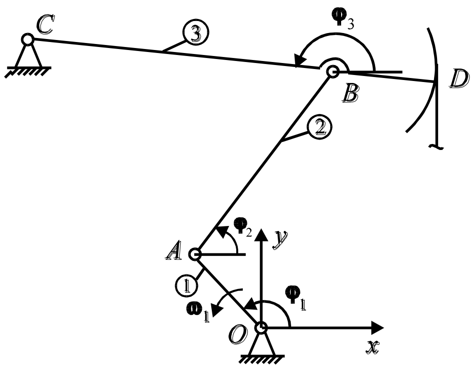

Fig. 2. Pumping unit mechanism with modified geometry

By projecting the contour $O-A-B-C-O$ on the x and y axes for these two mechanisms, the following systems of equations were obtained:

$$\begin{cases} l_1 \cdot \cos\varphi_1 + l_2 \cdot \cos\varphi_2 + l_3 \cdot \cos\varphi_3 - x_C = 0 \\ l_1 \cdot \sin\varphi_1 + l_2 \cdot \sin\varphi_2 + l_3 \cdot \sin\varphi_3 - y_C = 0 \end{cases} \quad (1)$$

where: $l_1 = OA$; $l_2 = AB$; $l_3 = BC$.

By solving the equations system (1), the unknown angles φ_2 and φ_3 were calculated from the following relations:

$$\begin{cases} A_2 \cdot \sin\varphi_2 + B_2 \cdot \cos\varphi_2 = C_2 \\ A_3 \cdot \sin\varphi_3 + B_3 \cdot \cos\varphi_3 = C_3 \end{cases} \quad (2)$$

where:

$$\begin{cases} A_2 = 2 \cdot l_1 \cdot l_2 \cdot \sin\varphi_1 - 2 \cdot l_2 \cdot y_C \\ B_2 = 2 \cdot l_1 \cdot l_2 \cdot \cos\varphi_1 - 2 \cdot l_2 \cdot x_C \\ C_2 = l_3^2 - l_1^2 - l_2^2 - x_C^2 - y_C^2 + 2 \cdot x_C \cdot l_1 \cdot \cos\varphi_1 + 2 \cdot y_C \cdot l_1 \cdot \sin\varphi_1 \end{cases} \quad (3)$$

$$\begin{cases} A_3 = 2\cdot l_1\cdot l_3\cdot \sin\varphi_1 - 2\cdot l_3\cdot y_C \\ B_3 = 2\cdot l_1\cdot l_3\cdot \cos\varphi_1 - 2\cdot l_3\cdot x_C \\ C_3 = l_2^2 - l_1^2 - l_3^2 - x_C^2 - y_C^2 + 2\cdot x_C\cdot l_1\cdot\cos\varphi_1 + 2\cdot y_C\cdot l_1\cdot \sin\varphi_1 \end{cases} \tag{4}$$

The value of the crank angle φ_{1d}, corresponding to the beginning of the upward movement of the polished rod, in the case of the mechanism of the sucker rod pumping units with conventional type geometry (fig. 1), can be calculated from the following equations system, obtained by projecting the contour $O-A-B-C-O$ on the *x* and *y* axes when the rod *2* is in the prolongation of the crank *1*:

$$\begin{cases} (l_1+l_2)\cdot\cos\varphi_{1d} + l_3\cdot\cos\varphi_{3d} - x_C = 0 \\ (l_1+l_2)\cdot\sin\varphi_{1d} + l_3\cdot\sin\varphi_{3d} - y_C = 0 \end{cases} \tag{5}$$

where: φ_{3d} is the value of the angle φ_3 for this extreme position of the rocker of the mechanism.

By solving the system of equations (5), the unknown angle φ_{1d} was calculated from the following relation:

$$A_{1d}\cdot\cos\varphi_{1d} + B_{1d}\cdot\sin\varphi_{1d} = C_{1d} \tag{6}$$

where:

$$\begin{cases} A_{1d} = -2\cdot x_C\cdot(l_1+l_2) \\ B_{1d} = -2\cdot y_C\cdot(l_1+l_2) \\ C_{1d} = l_3^2 - (l_1+l_2)^2 - x_C^2 - y_C^2 \end{cases} \tag{7}$$

In the case of the mechanism of the sucker rod pumping units with modified geometry (fig. 2), the crank angle φ_{1d}, corresponding to the beginning of the upward movement of the polished rod, can be calculated from the following equations system, obtained by projecting the contour $O-A-B-C-O$ on the *x* and *y* axes when the rod *2* overlaps the crank *1*:

$$\begin{cases} (l_1-l_2)\cdot\cos\varphi_{1d} + l_3\cdot\cos\varphi_{3d} + |x_C| = 0 \\ (l_1-l_2)\cdot\sin\varphi_{1d} + l_3\cdot\sin\varphi_{3d} - y_C = 0 \end{cases} \tag{8}$$

By solving the system of equations (8), the unknown angle φ_{1d} can be calculated from a relation similar to the relation (6), where:

$$\begin{cases} A_{1d} = 2\cdot|x_C|\cdot(l_1-l_2) \\ B_{1d} = -2\cdot y_C\cdot(l_1-l_2) \\ C_{1d} = l_3^2 - (l_1-l_2)^2 - x_C^2 - y_C^2 \end{cases} \tag{9}$$

The relations above have been transposed into a computer program using Maple programming language that has integrated powerful functions for symbolical calculus [5]. The angular speeds and accelerations of the links *2* and *3* have been calculated with the following relations, by deriving with time the variation functions of the corresponding angles φ_2 and φ_3, determined from the equations (2):

$$\omega_j = \dot{\varphi}_j = \frac{d\varphi_j}{d\varphi_1}\cdot\frac{d\varphi_1}{dt} = \omega_1\cdot\frac{d\varphi_j}{d\varphi_1};\quad j = 2,3 \tag{10}$$

$$\varepsilon_j = \ddot{\varphi}_j = \varepsilon_1 \cdot \frac{\mathrm{d}\varphi_j}{\mathrm{d}\varphi_1} + \omega_1^2 \cdot \frac{\mathrm{d}^2\varphi_j}{\mathrm{d}\varphi_1^2};\ \ j = 2,3 \tag{11}$$

For obtaining the analytical expressions of the speeds and the accelerations mentioned above, the derivatives with respect to the crank angle φ_1 of the angles φ_2 and φ_3 have been calculated using the derivation function *diff* in Maple programming language [5].

The speed v_D and the acceleration a_D (figs. 1 and 2) of the end of the polished rod can be calculated with the following relations: $v_D = \omega_3 \cdot l_{3p}$; $a_D = \varepsilon_3 \cdot l_{3p}$, where: ω_3 and ε_3 are the angular speed and the angular acceleration of the rocker and $l_{3p} = CD$.

In figures 3÷6 the variation curves of the acceleration a_D for a cinematic cycle, beginning with the value of the crank angle equal to φ_{1d}, in the case of the Lufkin pumping units with conventional type geometry: C-1280D-427-192 and RM-1280D-427-192 and of the Lufkin pumping units with modified geometry: M-1280D-427-192 and A-1280D-427-192 are presented. The dimensions of the component links of these pumping units and the value of the crank angle φ_{1d} are presented in table 1. We considered that the angular speed of the cranks is: $n_1 = 10 \mathrm{rot/min}$.

Table 1. The dimensions of the component links and the value of the crank angle φ_{1d} for some pumping units

Pumping unit	l_1 [m]	l_2 [m]	l_3 [m]	l_{3p} [m]	x_C [m]	y_C [m]	φ_{1d} [°]
C-1280D-427-192	1.346	4.382	3.048	5.334	3.048	4.458	87.264
RM-1280D-427-192	1.143	4.343	2.997	5.804	4.140	4.153	75.477
M-1280D-427-192	1.803	5.793	7.772	9.754	- 5.793	5.436	243.871
A-1280D-427-192	1.060	5.486	3.086	7.010	- 2.908	5.486	270.131

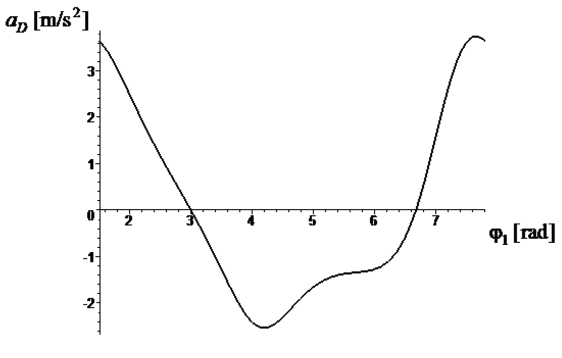

Fig. 3. The variation curve of the acceleration a_D for C-1280D-427-192 pumping unit

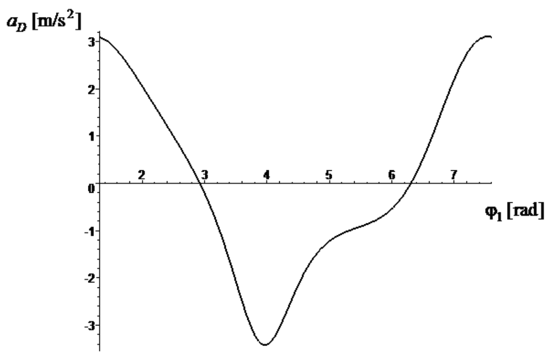

Fig. 4. The variation curve of the acceleration a_D for RM-1280D-427-192 pumping unit

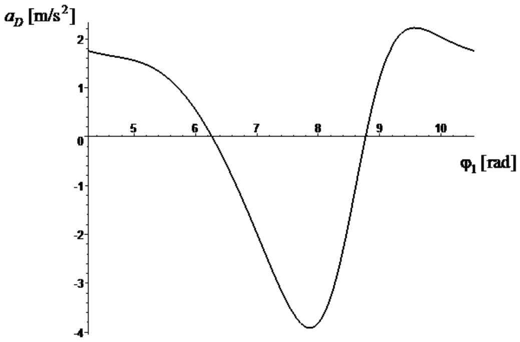

Fig. 5. The variation curve of the acceleration a_D for M-1280D-427-192 pumping unit

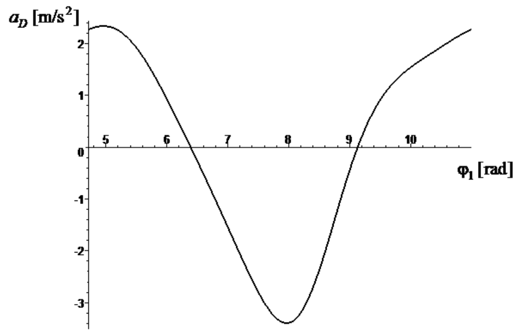

Fig. 6. The variation curve of the acceleration a_D for A-1280D-427-192 pumping unit

Figures 3÷6 show that the maximum values of the acceleration a_D at the beginning of the upward stroke in the case of the pumping units with conventional type geometry are much higher than in the case of the pumping units with modified geometry. On the other hand the values of the acceleration a_D at the downward stroke are higher in module in the case of the pumping units with modified geometry (especially for M-1280D-427-192 pumping unit).

Conclusions

In this paper a method that permits the exact kinematics analysis of the mechanism of the sucker rod pumping units with conventional type geometry and modified geometry is presented. The positional and cinematic parameters have been determined in an analytical form depending on the angle of the driving crank, using the powerful functions for symbolical calculus integrated in Maple program. Special attention was given to the variation of the acceleration of the end of the polished rod which has a great influence on the dynamics of the string of sucker rods. Some interesting simulation results concerning the variation of the acceleration of the end of the polished rod on a cinematic cycle, in the case of the Lufkin pumping units with conventional type geometry: C-1280D-427-192 and RM-1280D-427-192 and of the Lufkin pumping units with modified geometry: M-1280D-427-192 and A-1280D-427-192 have been presented.

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Asupra cinematicii unor unităţi de pompare cu prăjini

Rezumat

In articol sunt prezentate o serie de rezultate privind analiza cinematică a mecanismului unor unităţi de pompare cu prăjini. Analiza este realizată folosind metoda proiecţiei contururilor independente. Parametrii poziţionali şi cinematici sunt determinaţi în formă analitică în funcţie de unghiul manivelelor conducătoare, folosind puternicele funcţii de calcul simbolic integrate în programul Maple. O atenţie deosebită a fost acordată variaţiei acceleraţiei în capătul prăjinii lustruite. In final, se prezintă o serie de rezultate interesante ale simulărilor efectuate în cazul unităţilor de pompare Lufkin: C-1280D-427-192, RM-1280D-427-192, M-1280D-427-192 şi A-1280D-427-192.