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On the Multiaxial High Cycle Fatigue Damage Parameters

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Abstract

It is known that the vast majority of machine components are subjected to cyclic multiaxial stresses. On this basis, in recent years numerous theoretical and experimental studies have been carried out in order to analize the multiaxial fatigue damage mechanisms. In this context, this paper presents a critical analysis of the most important damage parameters used in high cycle fatigue life prediction (finite and infinite life). The authors also propose a new damage parameter, which is verified showing good correlation with some experimental data from the literature.

Keywords: *damage parameter, HCF, multiaxiality, proportional/nonproportional, life prediction*

Introduction

A large number of studies have been carried out in recent years regarding the damage mechanisms under variable multiaxial loading, while the modeling of fatigue crack nucleation and propagation has been taken one step closer to reality by developing new damage parameters for different loading conditions.

This is due to the fact that the vast majority of engineering components, such as machine shafts, turbine blades, and generally all components with notches are subjected to multiaxial cyclic stresses. Successful design of such components mostly depends on the correct life prediction under variable multiaxial loading. In many cases the variable loadings are nonproportional, and multiaxiality is characterized by non-zero mean stresses.

Numerous experimental studies, concerning steels and Al alloys, have lead to the development of low cycle fatigue (LCF) models. However, many applications require life prediction for high cycle numbers (10^3 – 10^6) or even for infinite durability. The present paper analizes in this context a number of proposed damage parameters applicable for multiaxial high cycle fatigue (HCF). These parameters allow the determination of the number of cycles to failure in case of limited life, and the prediction of microcrack nucleation in case of infinite life.

Particular aspects of HCF calculation

As it has been mentioned before, a large number of mechanical components, in case of which periodical inspections for detection and control of microcrack growth are not possible, are designed for long term durability. The most important part of the life of such components is strictly related to the nucleation of fatigue cracks, determining the safe operation of equipment and machines.

HCF damage occurs at lower stress levels than the material's yield point, and it is caused by localized cyclic strains in the material grains. This is followed by the formation of slip bands where cracks nucleate, even though at macroscopic level the loading is elastic. Given the above, shear stresses have a decisive role in the primary phase of crack nucleation, while normal stresses, by opening the already formed cracks, lower the fatigue limit.

Multiaxial HCF calculation can be done in order to verify the material under a specific loading, respectively to compare the damage parameter (which can be a stress) and the fatigue limit of the material. In other cases, the goal of a fatigue calculation is the determination of the number of cycles to failure, using an equation of the general form:

$$(D.P) = A \cdot N_f^b + B \cdot N_f^c \tag{1}$$

where *(D.P)* – damage parameter, A, B, c, d material constants, N_f the number of cycles to failure.

HCF models and damage parameters

Over the years a large number of multiaxial HCF models and criteria have been developed. Each criterion is based on a damage parameter calculated as a function of the characteristics of a multiaxial load cycle $[\sigma_{ij}(t)]_T$ defined on a period T, material properties, such as ultimate strength R_m, yield strength σ_c, fatigue limits for symmetrical and pulsating tension and torsion load cycles (σ_{-1}, σ_0, τ_{-1}, τ_0), etc.

Stresses, sum of stresses, elastic strains, sum of elastic strains, product of stresses, product of stresses and strains, or even dimensionless functions can all be considered as *(D.P)* When dimensionless functions are considered, *(D.P)* ≥ 1 indicates the possibility of crack nucleation.

The multiaxial HCF models and criteria can be classified as follows:
- models based on equivalent stresses;
- critical plane criteria;
- models based on mean stresses in elementary volumes (integral approach);
- energy based models;
- models based on stress invariants;
- mesoscopic models.

Models based on equivalent stresses

As their name suggests, these models consider as *(D.P)* a uniaxial equivalent stress, for which the same fatigue limit is obtained as for multiaxial loads. Initially these models were used for the calculation of an equivalent stress in case of in-phase multiaxial loadings [1][2][3][4]. Later developed models also took into account the effect of phase shift between bending and torsion, applicable for machine shafts [5][6][7].

Such models are frequently used in machine design because of their relative simplicity and because they give conservative results regarding the fatigue life of transmission shafts. The *(D.P)* is usually a stress or the combination of stresses, while the critical value is the tension, torsion or bending fatigue limit of the material (see Table 1.).

Critical plane criteria

These criteria are based on the idea of reducing a multiaxial stress state to an equivalent uniaxial one. Stanfield (1935) [8] was the first to introduce this concept. According to Stanfield, both the normal and shear stresses acting on a plane, considered the critical plane, have their effect in the

Table 1. Multiaxial high cycle fatigue models and corresponding damage parameters

	Model	Damage parameter, *(D.P)*	Observations
Equivalent stress theory	Hohenemser-Prager (1933) [1]	$\sqrt{\tau_a^2+\tau_{-1}^2\frac{\sigma_m}{R_m}}$	Critical equivalent stress, $(D.P)_{cr}=\tau_{-1}$
	Gough-Pollard (1935) [2]	$\sqrt{\tau_a^2+\tau_{-1}^2\frac{f_a^2}{f_{-1}^2}}$	Critical equivalent stress, τ_{-1}; applicable for ductile materials
		$\sqrt{\tau_a^2+\left(f_{-1}-\tau_{-1}\right)\left(\frac{f_a}{f_{-1}}\right)^2\tau_{-1}+\left(2\tau_{-1}-f_{-1}\right)\frac{f_a}{f_{-1}}\tau_{-1}}$	Critical equivalent stress, τ_{-1}; applicable for fragile materials
	Davies (1935) [3]	$\sqrt{f_a+\frac{\tau_m}{\tau_r}f_{-1}^2}$	Critical equivalent stress, f_{-1}
	Nishihara-Kawamoto (1941) [4]	$\sqrt{\tau_a^2+\tau_{-1}^2\frac{f_a^2}{f_{-1}^2}}$	Critical equivalent stress, τ_{-1}; applicable for $f_{-1}/\tau_{-1}\geq\sqrt{3}$
		$\sqrt{\tau_a^2+\frac{1}{2}\left(3\tau_{-1}^2-f_{-1}^2\right)\frac{f_a}{f_{-1}}+\frac{1}{2}\left(f_{-1}^2-\tau_{-1}^2\right)\left(\frac{f_a}{f_{-1}}\right)^2}$	Critical equivalent stress, τ_{-1}; applicable for $f_{-1}/\tau_{-1}<\sqrt{3}$
	Langer (1979) [5]	$\frac{f_a}{\sqrt{2}}\left[1+K^2+\left(1+2K^2\cos(2\Phi)+K^4\right)^{1/2}\right]^{1/2}$	Applicable for out-of-phase symmetrical sinusoidal torsion-bending cycles; $K=2\tau_a/f_a$; Φ – phase-shift
	ASME (1978)	$\frac{f_a}{\sqrt{2}}\left[1+\frac{3}{4}K^2+\left(1+\frac{3}{2}K^2\cos(2\Phi)+\frac{9}{16}K^4\right)^{1/2}\right]^{1/2}$	Applicable for out-of-phase symmetrical sinusoidal torsion-bending cycles; $K=2\tau_a/f_a$; Φ – phase-shift
	Lee (1980) [6]	$f_a\left[1+\left(\frac{\tau_a f_{-1}}{f_a\tau_{-1}}\right)^{\alpha}\right]^{1/\alpha}$	Critical equivalent stress, τ_{-1}; $\alpha=2(1+\beta\sin\Phi)$; β – material constant
	Lee (1989) [7]	$f_a\left[1+\left(\frac{\tau_a f_{-1}}{f_a\tau_{-1}}\right)^{\alpha}\right]^{1/\alpha}\Big/\left[1-\left(\frac{\sigma_m}{R_m}\right)^{n}\right]$	n – material constant
Critical plane theory	Stulen-Cummings (1954) [10]	$\tau_{na}+\alpha\sigma_{n\max}$	The critical plane with normal n, is the plane where σ_n is max.; τ_{na} – shear stress ampl. in the crit. plane; $(D.P)_{cr}=\tau_{-1}$; $\alpha=2(\tau_{-1}/\sigma_{-1})-1$;
	Findley (1957) [9]	$\tau_{na}+\alpha\sigma_{n\max}$	The critical plane with normal n, is the plane where *(D.P)* is critical; $(D.P)_{cr}=\tau_{-1}$; $\alpha=2(\tau_{-1}/\sigma_{-1})-1$;
	Yokobori (1966) [11]	$\max_t\left(\tau_n(t)\right)+\alpha\sigma_{n\max}$	The critical plane with normal n, is the plane where τ_n is max.; σ_{nmax} – normal stress in the same plane; $(D.P)_{cr}=\tau_{-1}$
	Matake (1977) [12]	$\tau_{na}+\alpha\sigma_{n\max}$	The critical plane with normal n, is the plane where τ_{na} is max.; $(D.P)_{cr}=\tau_{-1}$; $\alpha=2(\tau_{-1}/\sigma_{-1})-1$;
	Dang Van (1974) [13]	$\max_t\left(\tau_{na}(t)+\alpha p_H(t)\right)$	$p_H(t)=1/3\cdot\left(\sigma_1(t)+\sigma_2(t)+\sigma_3(t)\right)$; $\alpha=3\left(\tau_{-1}/\sigma_{-1}-1/2\right)$; $(D.P)_{cr}=\tau_{-1}$
	McDiarmid (1995) [14]	$\tau_{na}+\alpha\sigma_{n\max}$	The critical plane with normal n, is the plane where τ_{na} is max.; $\alpha=\tau_{-1}/2\sigma_r$; $(D.P)_{cr}=\tau_{-1}$
	Kandil-Brown-Miller (1982) [15]	$\varepsilon_{na}+\gamma_{na}$	The critical plane with normal n, is the plane where shear strain ampl. γ is max.; ε_{na} – tensile strain ampl. in the same plane; *(D.P)* is a strain
	Fatemi-Socie-Kurath (1988) [16]	$\gamma_{na}\left(1+K\frac{\sigma_{\max}}{\sigma_c}\right)$	The critical plane with normal *n,* is the plane where γ_{na} is max.; $\sigma_{\max}$ – max. normal stress in critical plane
	Robert (1992) [17]	$\max_t\left[\tau_{na}(t)+\alpha\sigma_{na}(t)+\beta\sigma_{nm}\right]$	Critical *(D.P)* value: $(D.P)_{cr}=\tau_{-1}\sqrt{\alpha^2+1}$; α, β – mat. const.

	Skibicki (2007) [18]	$(\tau_{na}+C_1\sigma_{na}+C_2\sigma_{nm})\left(1-\frac{\tau_{-1}}{\sigma_{-1}}H^3\right)$	The critical plane with normal n, is the plane where τ_{na} is max.; σ_{na} and σ_{nm} are crit. plane normal stress ampl. and mean value C_1, C_2 – material constants; H – weight function; $(D.P)_{cr}=\tau_{-1}$
Integral theory	Papadopoulos (1994) [19]	$\sqrt{\langle T_a^2\rangle}+\alpha p_{H\max}$ $\sqrt{\langle T_a^2\rangle}=\sqrt{5}\sqrt{\frac{1}{8\pi^2}\int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{\chi=0}^{2\pi}T_a(\varphi,\theta,\chi)d\chi\sin\theta d\theta d\varphi}$	$\langle T_a\rangle$ – mean value of shear stress ampl. in all slip directions belonging to a plane (integration with resp. to χ), respectively to all planes through a considered point in an elementary volume (integration with resp. to φ and θ) $(D.P)_{cr}=\tau_{-1}$; $\alpha=3\tau_{-1}/\sigma_{-1}-\sqrt{3}$
	Papuga (2008) [20]	$\sqrt{\frac{1}{4\pi}\int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\pi}\left[\frac{5}{2}C_a^2+\alpha\left(N_a+\frac{\sigma_{-1}}{\tau_{-1}}N_m\right)\right]\sin\theta d\theta d\varphi}$	C_a, N_a, N_m – shear stress ampl., normal stress ampl. and mean value in a given plane; $(D.P)_{cr}=\tau_{-1}$; $\alpha=\sigma_{-1}\left(3\sigma_{-1}^2/\tau_{-1}^2-1\right)$
Energy theory	Macha (1999) [22]	$4\sqrt{GU_f}$	$(D.P)_{cr}=\tau_{-1}$
	Shariyat (2009) [23]	$a\sqrt{U_f}+\bar{\beta}p_H$	a, $\bar{\beta}$ – mat. const.; $(D.P)_{cr}=\tau_{-1}$
	Chu-Conle-Bonnen (1993) [25]	$\max_t\left[\gamma_{na}(t)\tau(t)+\varepsilon_{na}(t)\sigma(t)\right]$	*(D.P)* is a shear energy, determined in the plane where it reaches max. value
Stress invariant theory	Sines (1955) [26]	$\sqrt{J_{2a}}+K(I_1)_m$	$\sqrt{J_{2a}}$ – equiv. shear stress ampl.; $(I_1)_m=p_{Hm}$ – mean hydrostatic stress; $K=3\tau_{-1}/\sigma_0-\sqrt{3}$; $(D.P)_{cr}=\tau_{-1}$
	Crossland (1986) [27]	$\sqrt{J_{2a}}+K\left[(I_1)_m+(I_1)_a\right]$	$(I_1)_m+(I_1)_a=(I_1)_{max}=p_{Hmax}$; $K=3\tau_{-1}/\sigma_{-1}-\sqrt{3}$; $(D.P)_{cr}=\tau_{-1}$
	Kakuno-Kawada (1979) [28]	$\sqrt{J_{2a}}+K(I_1)_m+\lambda(I_1)_a$	$(I_1)_m=p_{Hm}$; $(I_1)_a=p_{Ha}$; $K=3\tau_{-1}/\sigma_{-1}-\sqrt{3}$; $\lambda=3\tau_{-1}/\sigma_0-\sqrt{3}$; $(D.P)_{cr}=\tau_{-1}$
Mesoscopic theory	Dang Van (1989) [38]	$\max_t\left\{\max_{\vec{n}}\left[\left\|\tau_a(\vec{n},t)+\alpha p_H(t)\right\|\right]\right\}$	$\alpha=\frac{\tau_{-1}-\sigma_1/2}{\sigma_{-1}/3}$; $(D.P)_{cr}=\tau_{-1}$
	Morel (1998) [39]	$\frac{1}{N_i}\left(p\ln\frac{C_a}{C_a-\tau_{\lim}}+q\frac{\tau_{\lim}}{C_a-\tau_{\lim}}-\frac{r}{C_a}\right)$	N_i – no. of cycles to crack nucleation; C_a – macroscopic shear stress ampl.; p, q, r – mat. const.; $\tau_{lim}=\tau_c$; $(D.P)_{cr}=1$

damage process, being possible to combine them in a mathematical relation. However, these planes do not necessarily coincide with the principal planes. Twenty years passed until this concept was further developed, when Findley (1959) respectively Stulen and Cummings (1954) introduced the term "critical", while describing their experimental procedures based on the critical plane concept [9][10]. The *(D.P)* of the first critical plane criteria were stresses, then strains and strain energy (combination of the first two) were introduced. In Table 1 some critical plane criteria with the corresponding *(D.P)* are presented.

Models based on mean stresses in elementary volumes

These models, also known as the integral approach criteria, are based on mean stress values acting in an elementary volume V. Generally these mean values are described by double integrals with respect to the spherical coordinates φ and θ of the plane Δ's normal direction vector $\vec{n}$. With respect to φ the integrals are calculated on the interval $[0; 2\pi]$, while with respect to θ on $[0; \pi]$.

Widely accepted integral approach criteria are: Papadopoulos [19] and Papuga [20]. Another criterion based on mean stresses has been proposed by Grubisic and Simburger [21].

It has to be mentioned that the Papadopoulos criterion is in good correlation with experimental data in case of in-phase loading. The Papuga criterion brings the necessary corrections regarding out-of-phase loading, achieving satisfactory correlations with the corresponding experimental data.

Energy based models

Energy models can be divided into three groups, depending on the type of specific strain energy dissipated in one load cycle:

- models based on the elastic strain energy;
- models based on the plastic strain energy;
- models based on both elastic and plastic strain energy.

Criteria based on the elastic strain energy are used for HCF calculation, $N > 5\cdot 10^4$ or 10^5 cycles. Plastic energy based criteria are more apropriate for LCF calculation, $N < 10^4$ or $5\cdot 10^4$ cycles. Criteria based on both elastic and plastic energy can be applied for either LCF or HCF calculation.

Table 1 presents the *(D.P)* from a number of models based on the elastic strain energy, applicable for HCF calculation. These *(D.P)* have the dimension of stress, and can be determined based on the distorsion (shear) strain energy (Macha [22] and Shariyat [23]).

Banvillet [24] considers the volumetric strain energy density $U_g(M)$ as *(D.P)* in a material point M, during a load period T, as follows:

$$U_g(M) = \sum_{i=1}^{3}\sum_{j=1}^{3}\int_T \left\langle \sigma_{ij}(M,t)\cdot \varepsilon_{ij}^{e}(M,t)\right\rangle dt \tag{2}$$

Models based on stress invariants

The stress invariant models are based on the correlation between the fatigue limit, the second deviatoric stress invariant J_2 and the first stress invariant I_1. In general form:

$$f\left((I_1)_a,(J_2)_a,(I_1)_m,(J_2)_m\right) = \xi \tag{3}$$

where $I_1 = \sigma_1 + \sigma_2 + \sigma_3$; $J_2 = \frac{1}{6}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right]$; a, m amplitudes and ξ is a material constant calibrated according to the fatigue limit.

The most widely accepted stress invariant models are the following: Sines [26], Crossland [27] and Kakuno-Kawada [28]. Application of these three criteria, especially the most frequently utilized Crossland criterion, runs into some complications when determining the equivalent shear stress amplitude $\sqrt{J_{2a}}$. In case of in-phase multiaxial loading $\sqrt{J_{2a}}$ can be determined directly:

$$\sqrt{J_{2a}} = \sqrt{\frac{1}{6}\left[(\sigma_{xa}-\sigma_{ya})^2 + (\sigma_{ya}-\sigma_{za})^2 + (\sigma_{za}-\sigma_{xa})^2 + 6\left(\tau_{xya}^2 + \tau_{yza}^2 + \tau_{xza}^2\right)\right]} \tag{4}$$

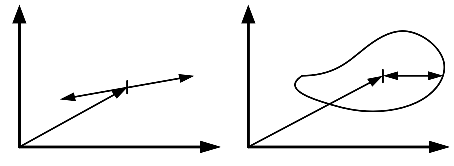

Fig. 1. Shear stress amplitude for proportional and nonproportional loading

In case of out-of-phase loading, determining $\sqrt{J_{2a}}$ requires much more complex mathematics. The vector representing $\sqrt{J_{2a}}$ has variable magnitude and direction during one load cycle. Fig. 1 shows the difference between proportional and nonproportional loadings. Consider a material point on a plane Δ defined by the unitary normal vector $\vec{n}$, having the spherical coordinates φ and θ. The stress vector $\vec{S}_n$ can be resolved into the normal stress vector $\vec{N}$ and the shear stress vector $\vec{C}$ (Fig. 2).

During a load cycle, $\vec{S}_n$ describes a closed curve ψ', and the projection ψ of ψ', on the plane in consideration described by $\vec{C}$, represents the load trajectory (Fig. 3).

The shear stress amplitude depends on the position of plane Δ, $C_a = f(\varphi, \theta)$. In order to find $C_{a\max}$, the maximum value of the function $C_a = f(\varphi, \theta)$ has to be calculated for all possible values of φ and θ.

When the stress invariant method is applied, $\sqrt{J_{2a}}$ remains the same for any position of plane Δ.

To find $\sqrt{J_{2a}}$, there have been proposed a number of methods, by Papadopoulos [29], Deperrois [30], Duprat [31], Bin Li [32], Mamiya-Araujo [33] and Balthazar [34].

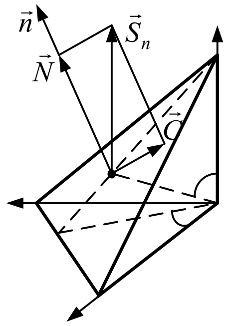

Fig. 2. Total stress vector $\vec{S}_n$ and components $\vec{N}$ and $\vec{C}$ acting in plane Δ

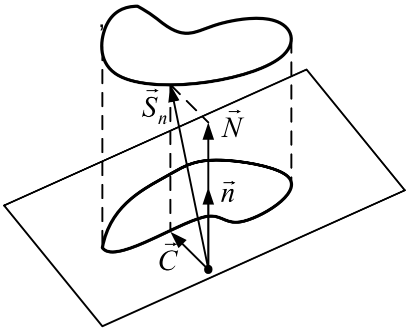

Fig. 3. Trajectory of load ψ' described by stress vector $\vec{S}_n$ and trajectory ψ described by shear stress vector $\vec{C}$

Papadopoulos [29] considers that $\sqrt{J_{2a}}$ is equal to the radius C_a of the minimum circumscribed circle to loading path ψ (Fig. 4).

The mean shear stress is given by the magnitude of vector $\vec{w}$, which unites the origine with the center of the circle. One advantage in the calculation of $\sqrt{J_{2a}}$ is the transformation of the deviatoric stress tensor **S** into a five-dimension vector $\vec{s}$. This way the deviatoric stress tensor is defined by fewer components [35] (Eq. (5)).

During a load cycle, $\vec{s}(t)$ also describes a closed curve, representing the load contour or path.

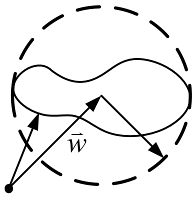

Fig. 4. Minimum circumscribed circle to lading path ψ [29]

Another method for determining $\sqrt{J_{2a}}$ has been proposed by Bin Li [32]. According to Bin Li, the circumscribed circle is replaced by an ellipse. $\sqrt{J_{2a}}$ can be found applying Eq. (6).

In Eq. (6) R_a and R_b represent the semiradii of the circumscribed ellipse to the loading path (Fig. 5). This approach takes into account the effect of nonproportionality opposite to the Dang Van and Papadopoulos criteria.

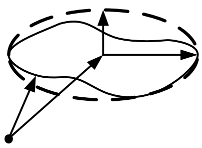

Fig. 5. Minimum circumscribed ellipse to lading path ψ [32]

Mamiya and Araujo [33] replace the circle or the ellipse with a prismatic hull. The hull circumscribes the loading path in the deviatoric plane (Fig. 6). $\sqrt{J_{2a}}$ can be found in this case applying Eq. (7).

$$s_1 = \sqrt{\frac{3}{2}}\left[\frac{2}{3}\sigma_x - \frac{1}{3}(\sigma_x + \sigma_z)\right];\; s_2 = \frac{\sqrt{2}}{6}(\sigma_y - \sigma_z);\; s_3 = \sqrt{2}\tau_{xy};\; s_4 = \sqrt{2}\tau_{yz};\; s_5 = \sqrt{2}\tau_{xz} \quad (5)$$

$$\sqrt{J_{2a}} = \sqrt{R_a^2 + R_b^2} \quad (6)$$

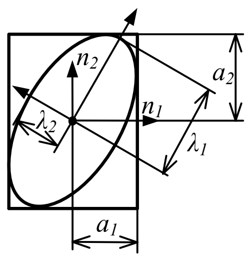

Fig. 6. Rectangular prism circumscribing an ellipsoid [33]

In Eq. (7) a_i is the amplitude of component $x_i(t)$ of the deviatoric stress tensor. In order to find a_i Eq. (8) can be applied.

The last part of this paper is dedicated to the comparison of these methods for a nonproportional loading case.

$$\sqrt{J_{2a}} = \left(\sum_{i=1}^{5} a_i^2 \right)^{1/2} \tag{7}$$

$$a_i = \max_i \left| x_i(t) \right| \tag{8}$$

3.6. Mesoscopic models

According to this concept, some material grains suffer plastic strains in time at mesoscopic scale (1 – 100) μm, even though the material has an elastic behavior at macroscopic scale (engineering scale ≈ (0,1 – 1) mm). This leads to the nucleation of the first microcracks. In HCF, crack nucleation usually takes place at the level of a single grain (or a few grains), this level being denoted “mesoscopic scale”.

At material grain level stresses and strains are difficult to quantify, hence the necessity for methods that allow the transition from macroscopic level to mesoscopic. The transition conditions have been detailed by Hill [36] and Mandel [37] by calculating the mesoscopic stresses as follows:

$$\underline{\underline{\sigma}} = \underline{\underline{\underline{A}}}(x) : \underline{\underline{\Sigma}} + \underline{\underline{\rho}} \tag{9}$$

where $\underline{\underline{\underline{A}}}(x)$ is the stress localization tensor, $\underline{\underline{\Sigma}}$ macroscopic stress tensor, $\underline{\underline{\rho}}$ local remanent stress tensor. In order to determine these values, simplifying hypotheses are needed, such as the Liu-Taylor hypothesis:

$$\underline{\underline{\sigma}} = \underline{\underline{\Sigma}} - 2G\underline{\underline{\varepsilon}}^p \tag{10}$$

where G is the elastic shear modulus and $\underline{\underline{\varepsilon}}^p$ is the mesoscopic plastic strain tensor.

The most complicated calculations arise when determining $\underline{\underline{\varepsilon}}^p$. The formulation of the first mesoscopic criterion belongs to Dang Van [38] and it is ulilized today in the French automotive industry. The criterion was developed for polycrystalline materials in the infinite life domain. According to the latest Dang Van criterion, at the level of a single grain only one active slip system exists. Dang Van postulates on this basis that crack nucleation will not occur while the following condition is respected:

$$\left\| \vec{\tau}_n \right\| - \tau_y \leq 0 \tag{11}$$

where τ_y is the critical shear stress and $\vec{\tau}_n$ the shear vector on a side with normal $\vec{n}$.

Besides mesoscopic shearing, Dang Van also proposes that crack nucleation at stress levels close to the fatigue limit is strongly influenced by the mesoscopic hydrostatic stress, which in many cases is considered to be equal to its macroscopic value p_H. The (*(D.P)*) obtained by double maximization is shown in Table 1.

Another widely utilized mesoscopic criterion was proposed by Morel [39][40][41]. Morel considers that fatigue crack nucleation occurs when the accumulated mesoscopic plastic strain Γ reaches a material specific critical value Γ_{cr}. Furthermore, Morel accepts that only macroscopic slip mechanisms are active during the cracking process and studies different stages of material behavior on this basis. Finally a Wöhler curve equation is deduced for crack nucleation. If *(D.P)* ≤ 1, the number of cycles necessary for crack nucleation can also be determined.

Proposition of a new *(D.P)* and comparison with existing criteria

The authors propose a correction to Crossland's criterion as follows:

$$\sqrt{J_2} + p_H^{\alpha} = \beta \tag{12}$$

where α and β can be obtained from simple fatigue tests, symmetrical torsion and tension respectively. When torsion is considered, the following can be written:

$$\sigma(t) = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tau_a \sin \omega t \tag{13}$$

yielding $p_H(t) = \frac{1}{3} tr(\sigma(t)) = 0 \Rightarrow \max\limits_t \{p_H(t)\} = 0$.

By transforming the deviatoric stress tensor, the equivalent shear stress amplitude will be:

$$\tau_a = \sqrt{\frac{1}{2} s(t) : s(t)} = \sqrt{\frac{1}{2} 2(\tau_a \sin \omega t)^2} = \tau_a \sin \omega t \tag{14}$$

Maximal value of the amplitude:

$$\tau_{a\max} = \max_t (\tau_a \sin \omega t) = \tau_a \tag{15}$$

Introducing in Eq. (12) and considering $\sqrt{J_{2a}} = \tau_a$, it yields $\tau_{-1}^2 = \beta^2 \Rightarrow \beta = \tau_{-1}$.

Analogically, when tension is considered the following can be written:

$$\sigma(t) = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \sigma_a \sin \omega t \tag{16}$$

yielding $p_H(t) = \frac{1}{3} tr(\sigma(t)) = \frac{1}{3} \sigma_a \sin \omega t \Rightarrow \max\limits_t \{p_H(t)\} = \max\limits_t \left\{\frac{1}{3} \sigma_a \sin \omega t\right\} = \frac{1}{3} \sigma_a$.

If the expression of $s(t)$ is considered:

$$s(t) = \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix} \tag{17}$$

It results in:

$$\tau_{a\max} = \max_t \sqrt{\frac{1}{2} s(t) : s(t)} = \max_t \sqrt{\frac{1}{2}\left\{\left(\frac{2}{3}\sigma_a \sin \omega t\right)^2 + 2\left(-\frac{1}{3}\sigma_a \sin \omega t\right)^2\right\}} = \frac{1}{\sqrt{3}} \sigma_a \tag{18}$$

Introducing in Eq. (12) the following is obtained:

$$\frac{1}{\sqrt{3}} \sigma_a + \left(\frac{1}{3}\sigma_a\right)^{\alpha} = \beta \Rightarrow \alpha = \frac{\log\left(\tau_{-1} - \sigma_{-1}/\sqrt{3}\right)}{\log(\sigma_{-1}/3)} \tag{19}$$

Considering the above, the proposed criterion has the final form:

$$\sqrt{J_2} + p_H^{\frac{\log\left(\tau_{-1} - \sigma_{-1}/\sqrt{3}\right)}{\log(\sigma_{-1}/3)}} = \beta \tag{20}$$

In Table 2 the relative errors obtained by applying the Crossland, Papadopoulos, Mamiya-Araujo and the proposed model are compared, in case of a steel with the characteristics σ_{-1} = 313.9 MPa and τ_{-1} = 196.2 MPa [42][43].

Table 2. Relative errors for different models

No.	σ_{xa} [MPa]	σ_{xm} [MPa]	τ_{xya} [MPa]	τ_{xym} [MPa]	σ_{VMa} [MPa]	σ_{VMm} [MPa]	Φ [°]	e^C [%]	e^P [%]	$e^{M\text{-}A}$ [%]	e^a [%]
1	138.1	0	167.1	0	320.6	0	0	-2.27	-2.3	-2.28	-2.5
2	140.4	0	169.9	0	326.0	0	30	-2.6	-0.6	-0.64	0.91
3	145.7	0	176.3	0	338.3	0	60	-3.61	3.1	3.1	-3.16
4	150.2	0	181.7	0	349.0	0	90	-3.74	6.3	6.27	-2.19
5	245.3	0	122.6	0	324.4	0	0	1.44	1.5	1.44	1.92
6	249.7	0	124.8	0	330.2	0	30	0.01	3.3	3.26	0.92
7	252.4	0	126.2	0	333.9	0	60	-8.35	4.4	4.39	-7.75
8	258.0	0	129.0	0	341.3	0	90	-17.81	6.5	6.70	-16.34
9	299.1	0	62.8	0	318.3	0	0	0.92	0.9	0.92	0.45
10	304.5	0	63.9	0	324.0	0	90	-2.99	2.7	2.74	-2.80

C – Crossland; *P* – Papadopoulos; *M-A* – Mamiya-Araujo; *a* – Authors

Conclusions

This paper presents the multiaxial HCF damage parameters in the finite and infinite life domain. Besides the damage parameters, the corresponding fatigue calculation methods have been also presented, i.e. equivalent stress criteria, critical plane criteria, the integral approach, stress invariant models, strain energy criteria and mesoscopic models.

The last part of the paper presents a new model proposed by the authors based on a correction to the Crossland criterion. The model has been compared with three other models using experimental data obtained by Nishihara and Kawamoto. The new model is in better correlation with experimental data than the Crossland criterion and it is easier to apply than other existing models.

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Nomenclature

σ_a – amplitude of a tension-compression cycle
σ_m – mean stress in a tension-compression cycle
τ_a – amplitude of a torsion cycle
τ_m – mean stress in a torsion cycle
f_a – amplitude of a bending cycle
f_m – mean stress in a bending cycle
σ_{-1} – symmetrical tension fatigue limit
τ_{-1} – symmetrical torsion fatigue limit
f_{-1} – symmetrical bending fatigue limit
R_m – ultimate tensile strength
R_p – tensile yield limit
τ_r – ultimate shear strength
τ_c – torsion yield limit
σ_0 – pulsating tension fatigue limit
τ_0 – pulsating torsion fatigue limit
f_0 – pulsating bending fatigue limit
σ_1, σ_2, σ_3 – normal principal stresses
$\boldsymbol{\sigma}_{ij}$ – stress tensor
$\boldsymbol{\varepsilon}_{ij}^{e}$ – elastic strain tensor
U_f – distorsion component of specific strain energy
p_H – hydrostatic pressure
G – elastic shear modulus
I_1 – first stress invariant
J_2 – second deviatoric stress invariant
$\boldsymbol{S}$ – deviatoric stress tensor
$\vec{s}$ – five-dimension vector in the Euclidian space E5

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Asupra parametrilor de degradare la oboseală multiaxială cu numere mari de cicluri

Rezumat

La ora actuală este binecunoscut faptul că majoritatea componentelor mecanice sunt supuse unor solicitări multiaxiale ciclice. Pe această bază în ultimii ani au fost efectuate numeroase studii teoretice şi experimentale pentru elucidarea mecanismelor de degradare la solicitări variabile în condiţii de multiaxialitate. În acest context în cadrul lucrării se face o analiză critică a principalilor parametri de degradare folosiţi pentru studiul durabilităţii la oboseală multiaxială cu numere mari de cicluri (domeniile durabilităţii limitate şi nelimitate). Lucrarea prezintă şi un parametru nou de degradare propus de autori, care a fost verificat dovedind o bună concordanţă cu unele rezultate experimentale din literatura de specialitate.