BULETINUL Universităţii Petrol – Gaze din Ploieşti	Vol. LXV No. 4/2013	121 - 124	Seria Tehnică

On Complex Trajectories Generation with a Robotic Arm with Spherical Structure

Dorin Bădoiu

Universitatea Petrol-Gaze din Ploieşti, Bd. Bucureşti 39, Ploieşti
e-mail: badoiu@mail.upg-ploiesti.ro

Abstract

The paper presents a method that allows the generation of complex trajectories using a robotic arm with spherical structure. The positional analysis of the robotic arm is realized using the rotation matrices method. The method is applied for generating a parabolic trajectory. A simulation program has been realized for this case. Finally, some simulation results are presented.

Key words: *robot, trajectory, spherical structure*

Introduction

Most of the industrial robots are sold having integrated programs that allow only linear trajectories generation. Very few industrial robotic systems contain programs that allow circular trajectories generation. In many cases, when the robots are used in different industrial operations (machining, welding, coating, painting) these ones must generate complex trajectories of different characteristic points of the objects manipulated by the robot. A solution in these cases is to use *teach and play* programming system [1], by storing the coordinates of a number of points on the trajectory. The number of the stored points is limited by the capacity of the robot memory. On the other hand, the trajectory between the stored points can not be controlled.

In this paper a method that allows the generation of complex trajectories using a robotic arm with spherical structure is presented. The positional analysis of the robotic arm is realized using the rotation matrices method [2]. The method is applied for generating a parabolic trajectory.

Theoretical Considerations and Simulation Results

In Figure 1, the cinematic scheme of the mechanism of a robotic arm with spherical structure is presented. The systems of coordinates $(O_i x_i y_i z_i), i=\overline{0,3}$, have been attached to each component module *i*, $i=\overline{0,3}$.

The rotation matrices ${}^{i}R_{i+1}, i=\overline{0,2}$, corresponding to the relative orientation of the component modules *i*+1 and *i* have the following expressions:

$$ {}^{0}R_{1} = R(z,q_{1}) = \begin{bmatrix} c1 & -s1 & 0 \\ s1 & c1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1) $$

$$ {}^{1}R_{2} = R(x,q_{2}) = \begin{bmatrix} c2 & -s2 & 0 \\ s2 & c2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2) $$

$$ {}^{2}R_{3} = I_{3} \quad (3) $$

where: I_3 is the unit matrix of rank three and:

$$ \begin{cases} si = \sin q_i \\ ci = \cos q_i \end{cases} \quad i = 1,2 \quad (4) $$

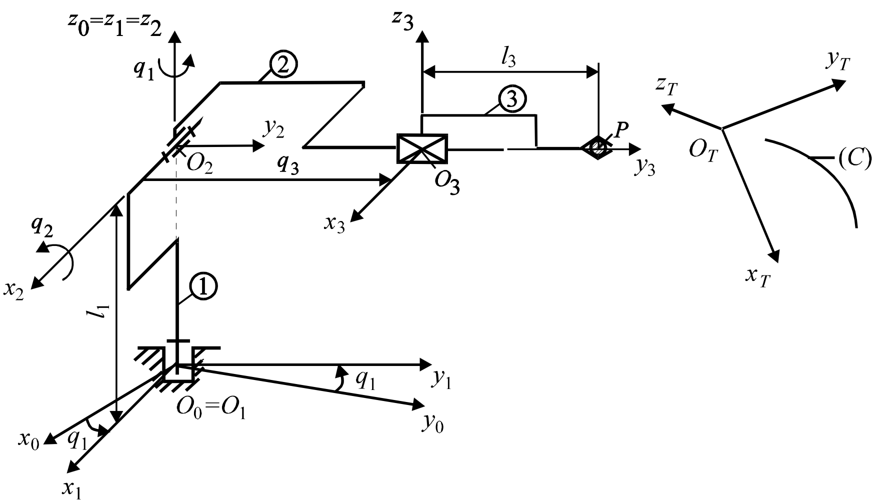

Fig. 1. Robotic arm with spherical structure

The position of the characteristic point P of the robot gripper (fig. 1) relative to the system of coordinates $(O_0x_0y_0z_0)$ can be determined with the following relation:

$$ {}^{(0)}O_0P = {}^{(0)}O_0O_1 + {}^{0}R_1 \cdot {}^{(1)}O_1O_2 + {}^{0}R_2 \cdot {}^{(2)}O_2O_3 + {}^{0}R_3 \cdot {}^{(3)}O_3P \quad (5) $$

where:

$$ \begin{cases} {}^{(0)}O_0O_1 = 0; \quad {}^{(1)}O_1O_2 = \begin{bmatrix} 0 & 0 & l_1 \end{bmatrix}^{\mathrm{T}} \\ {}^{(2)}O_2O_3 = \begin{bmatrix} 0 & q_3 & 0 \end{bmatrix}^{\mathrm{T}}; \quad {}^{(3)}O_3P = \begin{bmatrix} 0 & l_3 & 0 \end{bmatrix}^{\mathrm{T}} \end{cases} \quad (6) $$

$$ {}^{0}R_2 = {}^{0}R_1 \cdot {}^{1}R_2 = \begin{bmatrix} c1 & -s1 \cdot c2 & s1 \cdot s2 \\ s1 & c1 \cdot c2 & -c1 \cdot s2 \\ 0 & s2 & c2 \end{bmatrix} \quad (7) $$

$$ {}^{0}R_3 = {}^{0}R_2 \cdot {}^{2}R_3 = {}^{0}R_2 \cdot I_3 = {}^{0}R_2 \quad (8) $$

After performing the calculations, it results:

$$^{(0)}O_0P = \begin{bmatrix} ^{(0)}x_P \\ ^{(0)}y_P \\ ^{(0)}z_P \end{bmatrix} = \begin{bmatrix} -s1 \cdot c2 \cdot (q_3 + l_3) \\ c1 \cdot c2 \cdot (q_3 + l_3) \\ l_1 + s2 \cdot (q_3 + l_3) \end{bmatrix} \tag{9}$$

The characteristic point *P* is going to move along the curve (*C*) (fig. 1), in the plane $(O_T x_T y_T)$, by equation: $y_T = f(x_T)$. Are known the position of the origin O_T, given by the vector $^{(0)}O_0O_T$, and the orientation of the system of coordinates $(O_T x_T y_T z_T)$, given by the rotation matrix ^0R_T. So, when the point *P* is on a point *A* on the curve (*C*) the position vector $^{(0)}O_0P$ verifies the following relation:

$$^{(0)}O_0P = {}^{(0)}O_0A = {}^{(0)}O_0O_T + {}^0R_T \cdot {}^{(T)}O_TA \tag{10}$$

where: $^{(T)}O_TA = \begin{bmatrix} x_T^{(A)} & f(x_T^{(A)}) & 0 \end{bmatrix}^T$, $x_T^{(A)}$ is the *x* coordinate of the point *A* in the system of coordinates $(O_T x_T y_T z_T)$.

By introducing the values of the coordinates of the point *P,* given by relation (10), in the equations system (9), the following expressions for q_1, q_2 and q_3 are obtained:

$$\begin{cases} q_1 = \text{arctg}(-{}^{(0)}x_P / {}^{(0)}y_P) + k\pi;\ k \in Z \\ q_2 = ATAN2({}^{(0)}z_P - l_1,\ -{}^{(0)}x_P / \sin q_1) \\ q_3 = \sqrt{{}^{(0)}x_P^2 + {}^{(0)}y_P^2 + ({}^{(0)}z_P - l_1)^2} - l_3 \end{cases} \tag{11}$$

where: ATAN*2(y,x)* calculates arctg*(y/x)* by taking into account the signs of the parameters *y* and *x*. When $\sin q_1 = 0$, q_2 can be calculated with the following relation:

$$q_2 = ATAN2({}^{(0)}z_P - l_1, {}^{(0)}y_P / \cos q_1) \tag{12}$$

The presented method has been applied for generating a parabolic trajectory: $y = 2x^2 + 2x + 1$. The position vector $^{(0)}O_0O_T$ and the rotation matrix ^0R_T have the following expressions: $^{(0)}O_0O_T = [1.5\ \ 2\ \ 1.2]^T$; $^0R_T = R(z,30^\circ) \cdot R(x,60^\circ) \cdot R(y,45^\circ)$. l_1 and l_3 have the following values: $l_1 = 1.8\ \text{m}$; $l_3 = 0.75\ \text{m}$. The hourly variation of the coordinate x_T is: $x_T(t) = 0.04 \cdot t$, when *t* varies from zero to ten seconds.

This application has been transposed into a computer program. In Figures 2, 3 and 4 the variation curves of q_1, q_2 and q_3, respectively, are presented for a solution of the analyzed case.

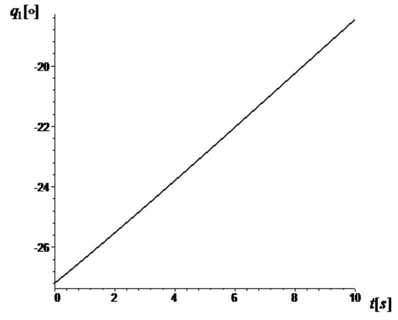

Fig. 2. The variation of q_1

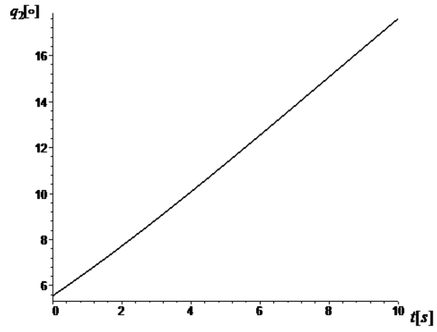

Fig. 3. The variation of q_2

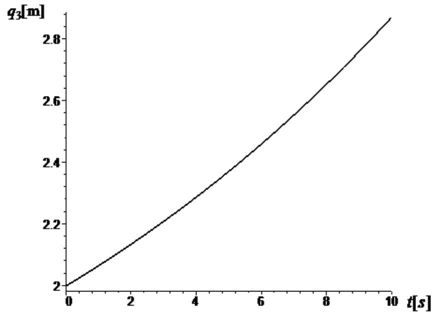

Fig. 4. The variation of q_3

Conclusions

In this paper a method that allows the generation of complex trajectories using a robotic arm with spherical structure has been presented. The positional analysis of the robotic arm was realized using the rotation matrices method. The method was applied for generating a parabolic trajectory. This application has been transposed into a computer program and the variation curves of the robot coordinates has been obtained.

References

1. B ă d o i u , D . – *Mecanica robotilor,* Ed. Universităţii Petrol-Gaze din Ploieşti, 2006.
2. B ă d o i u , D . – *Analiza structurală şi cinematică a mecanismelor,* Ed. Tehnică, Bucureşti, 2001.
3. C r a i g , J . J . – *Introduction to robotics: mechanics and control,* Addison-Wesley, 1986.
4. D o m b r e , E . , K h a l i l , W . - *Modélisation et commande des robots,* Ed. Hermès, Paris, 1988.

Asupra generării de traiectorii complexe cu un braţ robotic cu structură sferică

Rezumat

Articolul prezintă o metodă care permite generarea de traiectorii complexe folosind un braţ robotic cu structură sferică. Analiza poziţională a braţului robotic se realizează folosind metoda matricelor de rotaţie. Pentru acest caz s-a realizat un program de simulare. In final, o serie de rezultate ale simulărilor sunt prezentate.