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Design of a 2x2 Multivariable Control System with Two Monovariable Controllers and One Process Dedicated Decoupler

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Abstract

For a multivariable process with strong input-output cross-interactions, the control system design is difficult to do using only monovariable controllers, one for each input-output (I-O) process channel. The purpose of this paper is to present a practical solution for controlling a heat transfer multivariable 2X2 process using two PID monovariable classical controllers and a 2X2 decoupler connected in front of the process, in order to provide a decoupled process.

Key words: *multivariable process control, process identification, 2X2 decoupler design, PID controller design.*

Introduction

Design of a MIMO control system which can satisfy high control requirements or indices is often a difficult problem because of the process input-output cross-interactions. The solution is to consider a system having the same structure as the process, connected in series, in front of the process, which tends to cancel the unwanted I-O cross-interactions. This system is named decoupler. The decoupler is designed based on the process models of each input-output channel. If the decoupler is well designed, the process will be decoupled and will act as a reunion of monovariable systems. In this way, in the ideal case, controlling a multivariable system is reduced to controlling several monovariable systems [15]. In reality, because of the modeling errors, the decoupling performance will be affected and there will be a small and usually negligible cross interaction.

Dealing with multivariable systems, in terms of control, involves three steps:

- establishing the appropriate manipulated inputs-controlled outputs pairs; the interaction between these two is named direct interaction, and the process channel is the direct one; the other interactions are referred as cross interactions/cross process channels;
- modeling the process;
- designing a decoupler that aims to cancel the cross interactions, based on the process models;
- designing monovariable controllers (such PID type), one for each direct process channel.

The controller together with the decoupler is named decoupling controller [10].

The main advantage of this structure is that the tuning problem is reduced to tuning the independent monovariable controllers associated to each monovariable control loop [4, 5].

Obviously, the modeling errors will affect the decoupling performance and consequently the performance of the entire control system.

The objective of this paper is to present a practical control solution of a 2X2 multivariable temperature process using a decoupler and two monovariable PID controllers tuned based on process model parameter values found by an identification method.

Decoupling the Multivariable Temperature Process

The multivariable process considered for this study is a multivariable 2X2, having two input variables and two output variables as in fig.1. The process was presented and studied in detail in an author's previous paper, see [2]. The process consists of two chambers, with one bulb each, located close one to the other. The inputs are two voltages that control the degree of the light (U1 and U2) and the two process outputs are the chamber temperatures (Y1 and Y2). Because the chambers are close one to each other when we want to increase the temperature in one chamber, we increase the voltage, the degree of light increases and the temperature will increase but, unwanted will increase also the temperature in the other chamber [2].

In figure 1 is presented the multivariable process block diagram that has as inputs the two voltages (U1 and U2) and as outputs the two chamber temperatures (Y1 and Y2). Controlling this multivariable process is difficult to do because of the I-O cross interactions, as was described above.

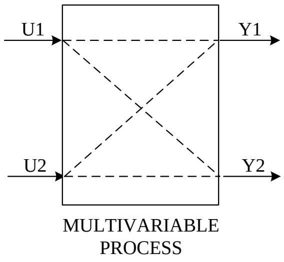

Fig. 1. Multivariable 2X2 process block diagram: U1 and U2 – process inputs (voltages), Y1 and Y2 – process outputs (temperatures).

The process dynamics was investigated considering step changes in the process inputs U1 and U2. Using the data results the process was identified. The process model block diagram is presented in figure 2 [2].

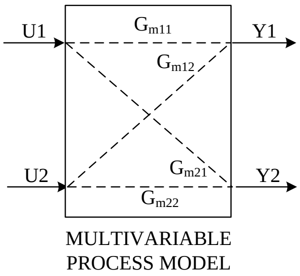

Fig. 2. Multivariable 2X2 process model block diagram: U1 and U2 – process inputs, Y1 and Y2 – process outputs, G_{m11} - the transfer function for U1-Y1 process channel, G_{m21} - the transfer function for U1-Y2 process channel, G_{m12} - the transfer function for U2-Y1 process channel and G_{m22} - the transfer function for U2-Y2 process channel.

The process is represented on each on the four channels using second order without dead time transfer functions; G_{m11} is the transfer function for U1-Y1 process channel, G_{m21} is the transfer function for U1-Y2 process channel, G_{m12} is the transfer function for U2-Y1 process channel and G_{m22} is the transfer function for U2-Y2 process channel, having the following expressions [2]:

$$G_{\mathrm{m}11}(s) = \frac{k_{11}}{(T_{11}\cdot s+1)^2} = \frac{3.2}{(4.4\cdot s+1)^2}, \tag{1}$$

$$G_{\mathrm{m}21}(s) = \frac{k_{21}}{(T_{21}\cdot s+1)^2} = \frac{0.5}{(10.9\cdot s+1)^2}, \tag{2}$$

$$G_{\mathrm{m}12}(s) = \frac{k_{12}}{(T_{12}\cdot s+1)^2} = \frac{0.7}{(12.9\cdot s+1)^2}, \tag{3}$$

$$G_{\mathrm{m}22}(s) = \frac{k_{22}}{(T_{22}\cdot s+1)^2} = \frac{2.7}{(4.1\cdot s+1)^2}, \tag{4}$$

In order to have a controllable process the process I-O cross interactions have to be reduced or even cancelled. The solution is to use a decoupler with the same structure as the process (two inputs and two outputs) connected in front of the process. The multivariable process with decouper is presented in figure 3 [2].

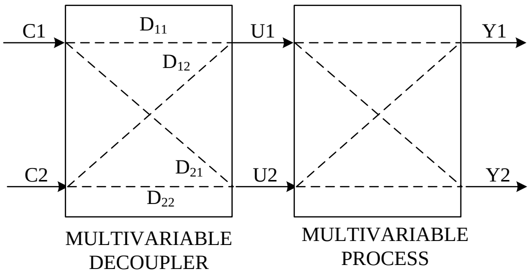

Fig. 3. Decoupled process block diagram: C1 and C2 – decoupler inputs, U1 and U2 – decoupler outputs/process inputs, Y1 and Y2 – process outputs.

The decoupler models on each of the four input output channels are found so that we obtain the following pseudo-process structure that in the ideal case, has only direct I-O interactions.

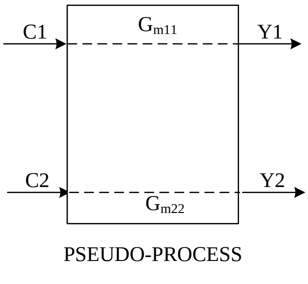

Fig. 4. Decoupled process/pseudo-process block diagram: C1 and C2 – decoupled process inputs, Y1 and Y2 – decoupled process outputs.

Considering the decoupling condition:

$$G_m(s)\cdot D(s) = \begin{bmatrix} G_{\mathrm{m}11}(s) & G_{\mathrm{m}12}(s) \\ G_{\mathrm{m}21}(s) & G_{\mathrm{m}22}(s) \end{bmatrix} \cdot \begin{bmatrix} D_{11}(s) & D_{12}(s) \\ D_{21}(s) & D_{22}(s) \end{bmatrix} = \begin{bmatrix} G_{\mathrm{m}11}(s) & 0 \\ 0 & G_{\mathrm{m}22}(s) \end{bmatrix}, \tag{5}$$

we obtain the dedicated decoupler models [2]:

$$D_{11}(s)=\frac{G_{m11}(s)\cdot G_{m22}(s)}{G_{m11}(s)\cdot G_{m22}(s)-G_{m12}(s)\cdot G_{m21}(s)}, \tag{6}$$

$$D_{21}(s)=-\frac{G_{m11}(s)\cdot G_{m21}(s)}{G_{m11}(s)\cdot G_{m22}(s)-G_{m12}(s)\cdot G_{m21}(s)}, \tag{7}$$

$$D_{12}(s)=-\frac{G_{m22}(s)\cdot G_{m12}(s)}{G_{m11}(s)\cdot G_{m22}(s)-G_{m12}(s)\cdot G_{m21}(s)}, \tag{8}$$

$$D_{22}(s)=\frac{G_{m11}(s)\cdot G_{m22}(s)}{G_{m11}(s)\cdot G_{m22}(s)-G_{m12}(s)\cdot G_{m21}(s)}, \tag{9}$$

Because the dedicated decoupler model depends on the process model, which has unavoidable modeling errors, the decoupling operation is not perfect, but the process interaction is negligible; so, we can consider, with sufficient precision, that the process is decoupled, and we can use two monovariable PID controllers, one for each direct I-O process channel.

Monovariable PID Controllers Design

The Proportional-Integral-Derivative (PID) algorithm is the most widely used control algorithm in the practical industrial applications.

The PID control algorithm has the expression [3]:

$$c(t)=c_0+k_R\cdot(e(t)+\frac{1}{T_i}\int_0^t e dt+T_d\frac{de}{dt}), \tag{10}$$

where c_0 is the initial control value, e(t) - error value, k_R - controller gain, T_i - integral time constant and T_d is the derivative time constant.

The transfer function of the PID controller is:

$$G_{\mathrm{PID}}(s)=k_R\cdot(1+\frac{1}{T_i\cdot s}+T_d\cdot s). \tag{11}$$

The parameters k_R, T_i and T_d from the control law (10) are named controller tuning parameters, and the process of finding their values is referred as tuning action/method.

There are several tuning methods, which can be classified into two main categories:

- classical methods, that use formulas based on the process model parameters (gain, time constants, dead time) eg. Ziegler-Nichols, Oppelt, Cohen-Coon, Cirtoaje [1, 3, 8, 9, 16] or that use formulas based on the tuning parameter values which bring the process to its limit of stability, eg. Ziegler-Nichols, Seborg, Offereins [13].
- computational or optimization methods, which are based on the optimization of a cost function; such methods are the ones based on neural networks [6], on genetic algorithms [12] or on differential evolution method [7].

For this study was used a simple tuning method based on the identified process model parameter values.

Generally, if a process step response is

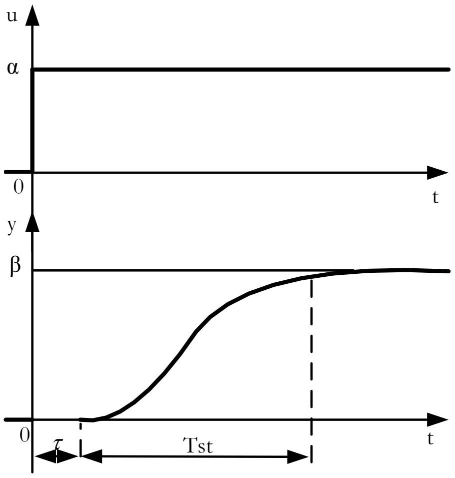

Fig. 5. Process step response: u – the system input variable with the steady-state value α, y – the system output variable with the steady-state value β, T_{st} – the system settling time and τ – the dead time.

According to this method the tuning parameters are found using the following formulas [2]:

$$k_R = \frac{0.9}{k_P \cdot (1 + \frac{6 \cdot \tau}{T_{st}})}, \quad T_i = \frac{T_{st}}{4}. \tag{12}$$

In case of using also the derivative term, the derivative time constant value is chosen experimentally to have the best dynamic response.

The proposed control system is represented in figure 6.

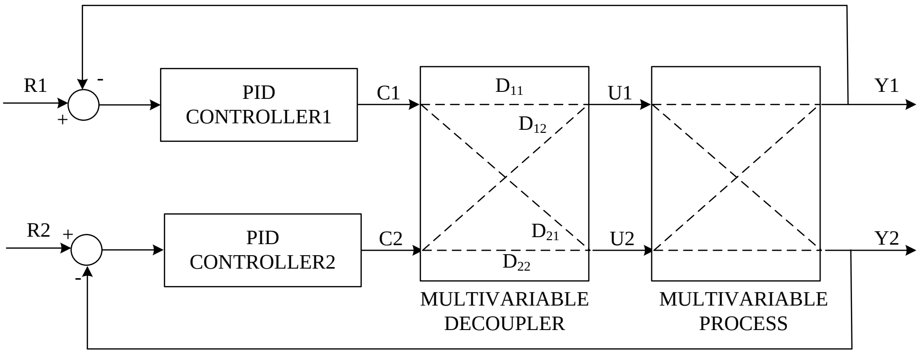

Fig. 6. Multivariable control system with two monovariable PID controllers and process decoupler: R1 and R2 – controller setpoints [°C], C1 and C2 – controller outputs/decoupler inputs [%], U1 and U2 – decoupler outputs/process inputs [%], Y1 and Y2 – process outputs [°C].

In order to obtain the PID controllers tuning parameter values using (12), the pseudo-process (decoupled process/ the process together with the dedicated decoupler) step responses have to be used [11], figures 7 and 8.

Using the decoupled process response to the input step changes from fig. 7 and 8, the tuning parameter for the two PI controllers have the following values:

$$k_{R1} = \frac{0.9}{3.2} = 0.28, \quad T_{i1} = \frac{28.4\,\text{min}}{4} = 7.1\,\text{min}, \tag{13}$$

$$k_{R2} = \frac{0.9}{2.7} = 0.33, \quad T_{i2} = \frac{25.6\,\text{min}}{4} = 6.4\,\text{min}, \tag{14}$$

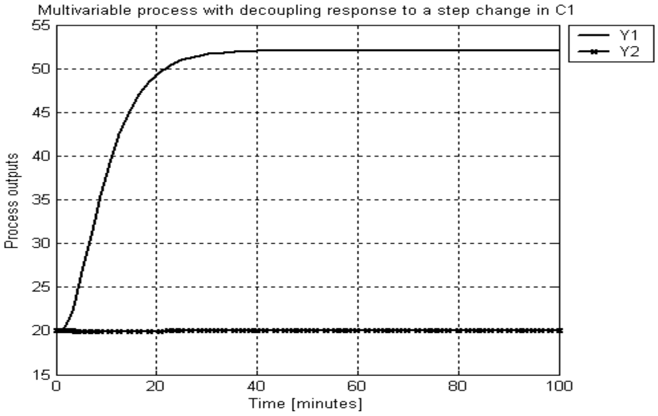

Fig. 7. Decoupled process response (Y1 and Y2 [°C]) to a 10% step change in C1 [2].

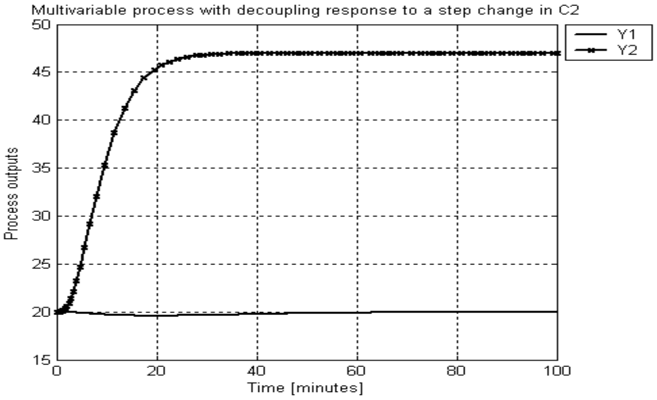

Fig. 8. Decoupled process response (Y1 and Y2 [°C]) to a 10% step change in C2 [2].

Results

Further, the control system was investigated for step changes in the two controller's setpoints for different values of the PID tuning parameters, see figures 9-14.

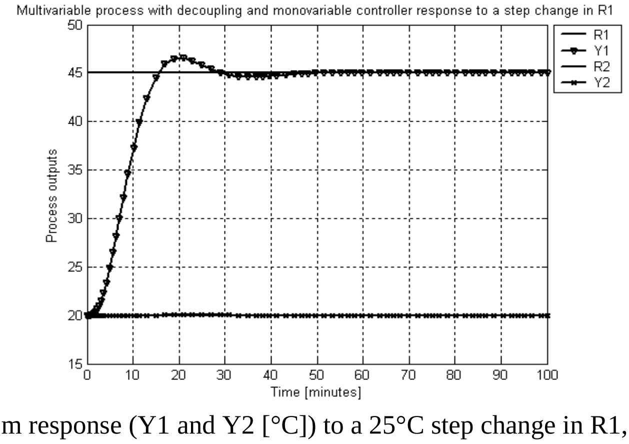

Fig. 9. Control system response (Y1 and Y2 [°C]) to a 25°C step change in R1, from 20 to 45°C, having k_{R1}=0.28, T_{i1}=7.1 min and T_{d1}=0 min.

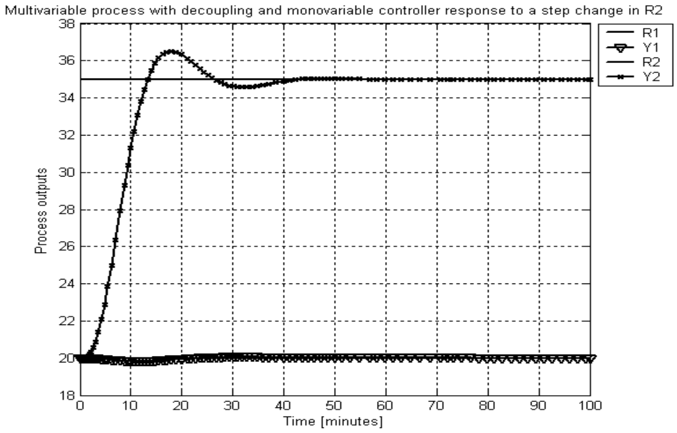

Fig. 10. Control system response (Y1 and Y2 [°C]) to a 15°C step change in R2, from 20 to 35°C, having k_{R2}=0.33, T_{i2}=6.4 min and T_{d2}=0 min.

Considering also the derivative term from PID, we have the following dynamic responses, see figures 11 and 12.

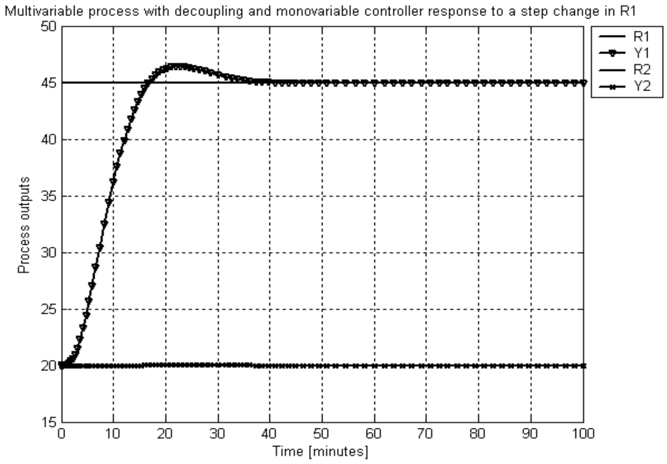

Fig.11. Control system response (Y1 and Y2 [°C]) to a 25°C step change in R1, from 20 to 45°C, having k_{R1}=0.28, T_{i1}=7.1 min and T_{d1}=1.1 min.

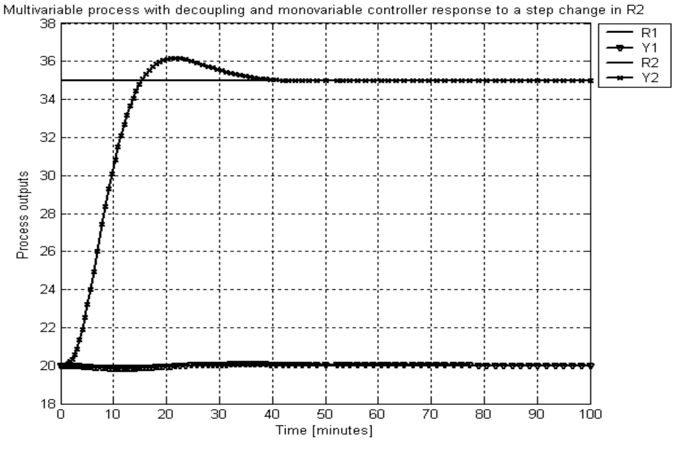

Fig.12. Control system response (Y1 and Y2 [°C]) to a 15°C step change in R2, from 20 to 35°C, having k_{R2}=0.33, T_{i2}=6.4 min and T_{d2}=1.8 min.

As we can see from the above figures 11 and 12, if we consider the derivative term we obtain better control systems dynamic responses, in comparison with figures 8 and 9.

If we modify the integral time constant in order to have the best static and dynamic control systems performance (no static error, small transient time and no output overshoot), we have the following dynamic responses, see figures 13 and 14.

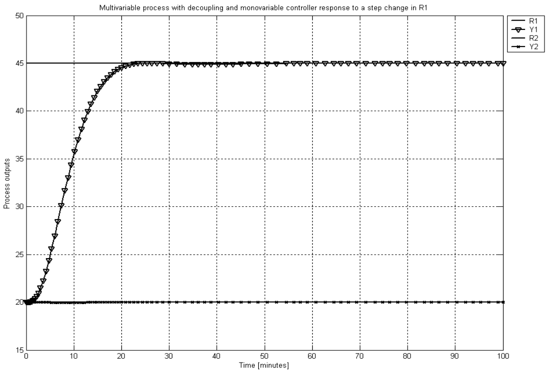

Fig.13. Control system response (Y1 and Y2 [°C]) to a 25°C step change in R1, from 20 to 45°C, having k_{R1}=0.28, T_{i1}=8.3 min and T_{d1}=1.1 min.

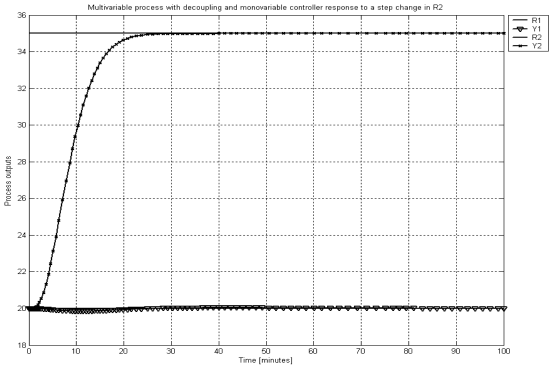

Fig.14. Control system response (Y1 and Y2 [°C]) to a 15°C step change in R2, from 20 to 35°C, having k_{R2}=0.33, T_{i2}=8.1 min and T_{d2}=1.8 min.

As we can observe from the above figures (figs. 9 - 14) when the set point for one chamber temperature is changed, the temperature for that chamber changes and the temperature from the other chamber remains approximately unchanged.

In fig. 15 was considered the case when the set points, for the two chamber temperatures, are changed at once. As we can see the system works with good results.

Conclusions

The main paper objective is the design of a simple control structure using only two PID monovariable controllers in order to control a heat transfer multivariable process. Because of the unwanted I-O cross interactions that characterize any MIMO system, the control action is difficult to do using only monovariable controllers. The solution was to consider a dedicated decoupler which was designed using the studied MIMO process model, in order to reduce or even cancel the process I-O cross interactions. The process was identified using the process output trends to the input variables step changes. Also the process was decoupled, using a dedicated decoupler, in a previous work.

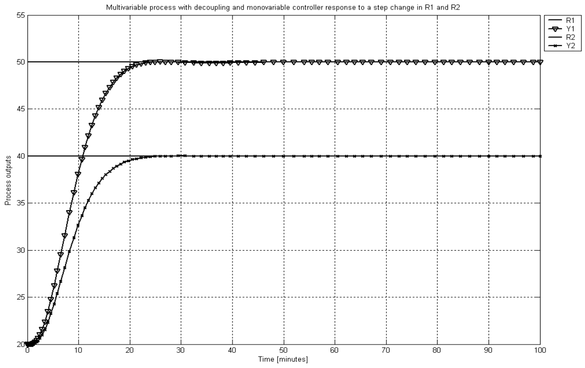

Fig.15. Control system response (Y1 and Y2 [°C]) to a 30°C step change in R1, from 20 to 50°C and to a 20°C step change in R2,from 20 to 40°C, having k_{R1}=0.28, T_{i1}=8.3 min, T_{d1}=1.1 min and k_{R2}=0.33 and T_{i2}=8.1 min, T_{d1}=1.8 min.

The proposed temperature control structure uses two monovariable PID controllers, one for each direct I-O channel; the PID controllers were tuned using the model parameter values of the decoupled process.

The proposed control structure was tested for validation considering the two controllers set point step changes, observing that the control system has good steady-state and dynamic performance.

Because the proposed dedicated process decoupler has a complicated model, based on the process model (1)…(4) and relations (6)…(9), a more practical and effective solution is to use a decoupler with a simple standard model (of first or second order).

References

1. Băieşu, A. – *Tehnica Reglarii Automate*, Editura Matrix Rom, Bucuresti, 2012, p. 47, 121.
2. Băieşu, A.C., Băieşu, A.S. – Dynamic Investigation, Identification and Decoupling of a Multivariable 2X2 Temperature Process, *Buletinul Universităţii Petrol-Gaze din Ploieşti, Seria Tehnică*, Vol. LXVI, No. 2, 2014, pp. 91-99.
3. Cîrtoaje, V. – *Teoria Sistemelor Automate - Analiza in domeniul complex*, Editura Universitatii Petrol-Gaze din Ploiesti, 2013.
4. Cîrtoaje, V., Frâncu, S., Băieşu, A. – A practical control method for multivariable systems, *SINTES 11 Proceedings: The International Symposium on System Theory, Automation, Mechatronic Systems*, XIth, Vol.I. Craiova, 2003a.
5. Cîrtoaje, V., Guţu, A., Frâncu, S. – A practical decoupling method for linear multivariable systems, *Proceedings of International Symposium on Process Control, SPC 2003, Buletinul UPG Ploieşti, Seria Tehnică*, Vol.LV, 2003b.
6. Doležel, P., Mareš, J. – Self-tuning PID Control using Genetic Algorithm and Artificial Neural Networks, *ASR 2009 Instruments and Control*, 2009, pp. 33- 39.
7. Gao, F., Tong, H. – Differential Evolution, An Efficient Method in Optimal PID Tuning and Online Tuning, *Proc. of the International Conference on Complex Systems and Applications*, 2006, pp. 785-789.
8. Luyben, L.M., Luyben, L.W. – *Essentials of Process Control*, McGraw-Hill, 1997.
9. Marlin, E.Th. – *Process Control. Designing Processes and Control Systems*, New York, Mc Graw-Hill, 1995.
10. Mihalache, S., Cîrtoaje, V., Popescu, M., Băieşu, A. – Distillation Decoupling: A New Procedure For Two Point Composition Control, *Proceedings of the 6th WSEAS Int. Conf. on Systems Theory & Scientific Computation*, Elounda, Greece, August 21-23, 2006, pp. 174-179.
11. Popescu, M. – Multivariable feedback control of a binary fractionation column, *Buletinul Universităţii Petrol-Gaze din Ploieşti*, Vol. LXII, *Seria Tehnică*, Nr. 4B/2010, pp. 75-82.

12. Salami, M., Cain, G. – An Adaptive PID Controller Based on Genetic Algorithm Processor, *IEE Genetic Algorithms in Engineering Systems, Innovations and Applications Conference*, Publication No. 414, 1995.
13. Seborg, E.D., Edgar F.TH., Mellichamp, A.D. – *Process Dynamics and Control*, New York, John Wiley&Sons, 1989.
14. Tham, M. T. – *Multi-variable Control: An Introduction to Decoupling Control.*, Department of Chemical and Process Engineering, University of Newcastle upon Tyne, 1999.
15. Waller, M. Waller, J.B., Waller, K.V. – Decoupling Revisited, *Ind. Eng. Chem. Res.*, Vol. 42, 2003, pp. 4575-4577.
16. Willems, J.C. – Models for Dynamics, *Dynamics Reported*, **2**, 1989.

Proiectarea unui sistem multivariabil de reglare 2x2 utilizând două regulatoare monovariabile şi un decuplor dedicat procesului

Rezumat

În cazul proceselor multivariabile cu interacţiuni pe canalele încrucişate intrare-ieşire de intensitate comparabilă cu cea a canalelor directe, reglarea este dificil de realizat utilizând numai regulatoare monovariabile, câte unul pentru fiecare canal intrare-ieşire al procesului. Scopul acestei lucrări este să prezinte o soluţie practică de reglare a unui proces de transfer termic multivariabil 2X2 utilizând două regulatoare monovariabile clasice, PID şi un decuplor cu structură 2X2, înseriat în faţa procesului, proiectat în funcţie de modelele pe cele patru canale I-O ale procesului, având ca scop obţinerea unui proces cvasidecuplat.