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Design of a 2x2 Multivariable Control System with Monovariable Controllers and Standard Decoupler

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Abstract

In case of a 2X2 multivariable process having strong input-output (I-O) cross interactions, in order to design a control system using only two monovariable classical PID controllers, one for each I-O process channel, a 2X2 structure decoupler connected in front of the process has to be considered. The decoupler models on each I-O channel can have either a dedicated transfer function, which is based on the explicit model of the process or a standard transfer function (for instance, of first order on the cross I-O channels and of zero order on the direct channels). The purpose of this paper is to design and implement a control system for a heat transfer multivariable 2X2 process using two PID monovariable classical controllers and a 2X2 standard decoupler.

Key words: *multivariable control system, 2X2 standard decoupler design, PID controller design.*

Introduction

In most of the practical control applications the processes have typically more than two input variables and more than two output variables [1]. This is the case of the Multi-Input Multi-Output (MIMO) systems. The main feature of these systems are the process cross interactions between input and output variables. A particular and simple case of MIMO systems is the case of 2X2 systems, which have only two input variables and two output variables [8].

The design of a control system for any MIMO process has to consider a decoupler having the same number of input and output variables as the process, connected in front of the process, which goal is to cancel the process unwanted natural Input-Output (I-O) cross interactions. In this way, in the ideal case, the design of a multivariable control system is reduced to the design of several monovariable control systems [5, 9]. In reality, because the decoupler is designed using process models which are affected by modeling errors due to the simplifying assumptions, the decoupling and the entire control system performance will be affected.

Considering the serial connection between the decoupler and the process and taking account of the decoupling condition (the decoupled process has no I-O cross interactions [1]), the 2X2 decoupler can be designed in two different ways:

- using a process dedicated form, that has the explicit model of the process on each of the four I-O channels; this case leads to a complicated, difficult to design and to implement decoupler, as was shown in [3];
- using a standard form, that has on the direct I-O channels transfer functions equal to one and on the cross I-O channels first order transfer functions; the transfer functions gains and the time constants are found based on the process model parameter values, using some formulas; this case leads to a simpler, easy to design and implement decoupler, but with weaker decoupling performance, as will be showed in this paper.

The serial connection between the controller and the decoupler is named decoupling controller and the connection between the decoupler and the process is named decoupled process (pseudo-process). The decoupled process has only direct I-O interactions [7].

The objective of this paper is to present a practical control solution of a 2X2 multivariable temperature process using a simple decoupler having a standard form and two monovariable PID controllers.

Multivariable Temperature Process Identification

The multivariable process from figure 1 was presented and studied in detail in author's previous papers, see [2, 3]. The process consists of two chambers situated close one to each other, having one bulb each. The inputs are the two voltages that control the degree of the bulb light (U1 and U2) and the outputs are the two chamber's temperatures (Y1 and Y2) [2]. When we want to increase the temperature in one chamber, we increase the voltage (the degree of light) and the temperature increases but, unwanted, because of the small distance between the two chambers increases also the temperature in the other chamber [3].

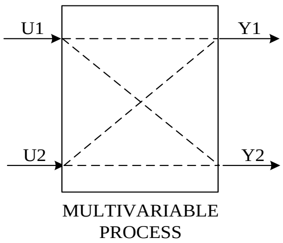

Fig. 1. Multivariable 2X2 process block diagram: U1 and U2 – process inputs (voltages), Y1 and Y2 – process outputs (temperatures).

Designing a control system for this MIMO process is difficult, as was discussed above. The solution is to consider a decoupler connected in front of the process which tends to cancel the natural and unwanted process cross interactions. In order to design the decoupler we need the process model, figure 2.

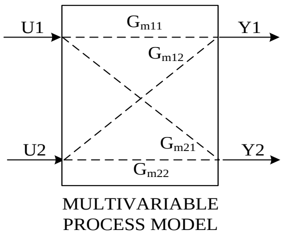

Fig. 2. Multivariable 2X2 process model block diagram: U1 and U2 – process inputs, Y1 and Y2 – process outputs, G_{m11} - the transfer function for U1-Y1 process channel, G_{m21} - the transfer function for U1-Y2 process channel, G_{m12} - the transfer function for U2-Y1 process channel and G_{m22} - the transfer function for U2-Y2 process channel.

There are two ways of designing the decoupler, using a process dedicated form or a standard form. The case of the dedicated decoupler was tested in [3]. The conclusion was that the decoupler has a complicated and difficult to implement structure. Much easier is to consider the second way, the standard decoupler.

In order to use this method we have the process models represented as first order transfer functions on each on the four I-O channels, as:

$$G_m(s) = \begin{bmatrix} G_{\mathrm{m}11}(s) & G_{12}(s) \\ G_{\mathrm{m}21}(s) & G_{\mathrm{m}22}(s) \end{bmatrix} = \begin{bmatrix} \dfrac{k_{11}}{T_{11}\cdot s+1} & \dfrac{k_{12}}{T_{12}\cdot s+1} \\ \dfrac{k_{21}}{T_{21}\cdot s+1} & \dfrac{k_{22}}{T_{22}\cdot s+1} \end{bmatrix}, \tag{1}$$

where G_{m11} is the transfer function for U1-Y1 process channel, G_{m21} is the transfer function for U1-Y2 process channel, G_{m12} is the transfer function for U2-Y1 process channel and G_{m22} is the transfer function for U2-Y2 process channel.

Using the data from the dynamic investigations presented in [2], the process model parameters (k_{11}, k_{21}, k_{11} and k_{21}) from (1) are computed as the process output percent variation divided by the process input percent variation [2]:

$$k_{11} = \frac{52\% - 20\%}{10\%} = 3.2, \quad k_{21} = \frac{25\% - 20\%}{10\%} = 0.5, \tag{2}$$

$$k_{12} = \frac{27\% - 20\%}{10\%} = 0.7, \quad k_{22} = \frac{47\% - 20\%}{10\%} = 2.7. \tag{3}$$

The transfer function time constants (T_{11}, T_{21}, T_{11} and T_{21}) are computed as the process channel settling time divided by 4. The process settling time is the time in which the process output reaches 98% from its steady-state value:

$$T_{11} = \frac{26.5\,\mathrm{min}}{4} = 6.7\,\mathrm{min}, \quad T_{21} = \frac{61\,\mathrm{min}}{4} = 15.3\,\mathrm{min}, \tag{4}$$

$$T_{12} = \frac{77.5\,\mathrm{min}}{4} = 19.4\,\mathrm{min}, \quad T_{22} = \frac{24.4\,\mathrm{min}}{4} = 6.1\,\mathrm{min}. \tag{5}$$

The process model (1) was validated by representing in the same figures, see figures 3 and 4, the identified process model and the real process responses to a step change in the two process inputs.

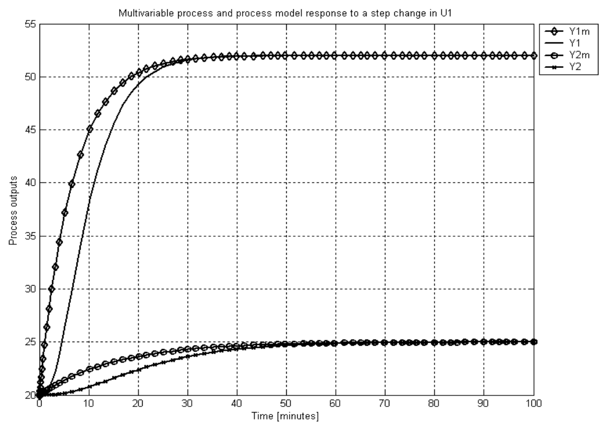

Fig. 3. Multivariable process response (Y1 and Y2 [°C]) and identified process model response (Y1m and Y2m [°C]) to a 10% step change in U1.

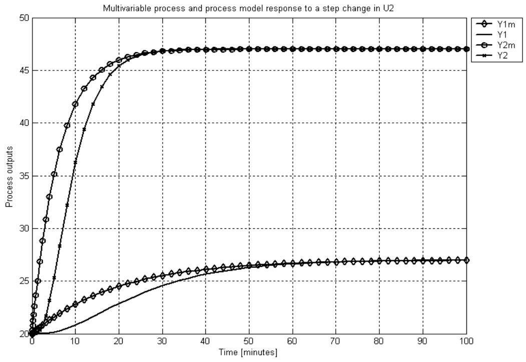

Fig. 4. Multivariable process response (Y1 and Y2 [°C]) and identified process model response (Y1m and Y2m [°C]) to a 10% step change in U2.

From figures 3 and 4 we can see that the process model behaves closer to the behaviour of the real process, in terms of gains and transient times, so it can be further used in a control stage.

Standard Decoupler and PID Controllers Design

The multivariable process with decoupler is presented in figure 5 [2].

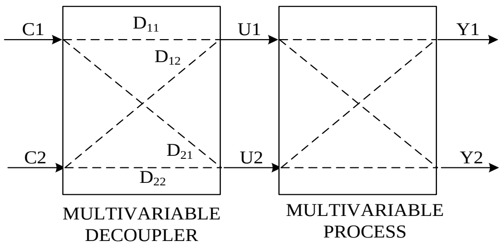

Fig. 5. Decoupled process block diagram: C1 and C2 – decoupler inputs, U1 and U2 – decoupler outputs/process inputs, Y1 and Y2 – process outputs.

The decoupler models on each of the four input output channels are found so that we obtain the following pseudo-process structure that in the ideal case has only direct I-O interactions [2].

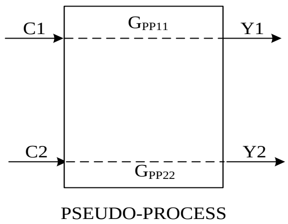

Fig. 6. Decoupled process/pseudo-process block diagram: C1 and C2 – decoupled process inputs, Y1 and Y2 – decoupled process outputs G_{pp11} - the transfer function for U1-Y1 pseudo-process channel, G_{pp22} - the transfer function for U2-Y2 pseudo-process channel.

The general decoupling condition is [1]:

$$G_{pp}(s) = G_m(s) \cdot D(s) = \begin{bmatrix} G_{\mathrm{m}11}(s) & G_{\mathrm{m}12}(s) \\ G_{\mathrm{m}21}(s) & G_{\mathrm{m}22}(s) \end{bmatrix} \cdot \begin{bmatrix} D_{11}(s) & D_{12}(s) \\ D_{21}(s) & D_{22}(s) \end{bmatrix} = \begin{bmatrix} G_{\mathrm{pp}11}(s) & 0 \\ 0 & G_{\mathrm{pp}22}(s) \end{bmatrix}. \quad (6)$$

Choosing the standard decoupler as [6]:

$$D(s)=\begin{bmatrix} D_{11}(s) & D_{12}(s) \\ D_{21}(s) & D_{22}(s) \end{bmatrix}=\begin{bmatrix} 1 & \dfrac{k_{D12}}{T_{D12}\cdot s+1} \\ \dfrac{k_{D21}}{T_{D21}\cdot s+1} & 1 \end{bmatrix}. \tag{7}$$

The steady-state decoupling condition becomes:

$$G_{pp}(0)=G_m(0)\cdot D(0)=\begin{bmatrix} G_{\mathrm{m11}}(0) & G_{\mathrm{m12}}(0) \\ G_{\mathrm{m21}}(0) & G_{\mathrm{m22}}(0) \end{bmatrix}\cdot\begin{bmatrix} D_{11}(0) & D_{12}(0) \\ D_{21}(0) & D_{22}(0) \end{bmatrix}=\begin{bmatrix} G_{\mathrm{pp11}}(0) & 0 \\ 0 & G_{\mathrm{pp22}}(0) \end{bmatrix}, \tag{8}$$

which implies

$$G_{pp}(0)=\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}\cdot\begin{bmatrix} 1 & k_{D12} \\ k_{D21} & 1 \end{bmatrix}=\begin{bmatrix} k_{\mathrm{pp11}} & 0 \\ 0 & k_{\mathrm{pp22}} \end{bmatrix}, \tag{9}$$

which leads to

$$k_{pp_{11}}=k_{11}\cdot k_{D12}+k_{12}=0\Rightarrow k_{D12}=-\frac{k_{12}}{k_{11}} \tag{10}$$

and

$$k_{pp_{22}}=k_{22}\cdot k_{D21}+k_{21}=0\Rightarrow k_{D21}=-\frac{k_{21}}{k_{22}} \tag{11}$$

In transient time, (6) leads to

$$G_{pp_{11}}(s)=G_{m11}(s)\cdot D_{12}(s)+G_{m12}(s)=0, \tag{12}$$

and

$$G_{pp_{22}}(s)=G_{m22}(s)\cdot D_{21}(s)+G_{m21}(s)=0. \tag{13}$$

Considering (1) and (7):

$$G_{pp_{11}}(s)=\frac{k_{11}}{T_{11}\cdot s+1}\cdot\frac{k_{D12}}{T_{D12}\cdot s+1}+\frac{k_{12}}{T_{12}\cdot s+1}=0, \tag{14}$$

$$G_{pp_{22}}(s)=\frac{k_{22}}{T_{22}\cdot s+1}\cdot\frac{k_{D21}}{T_{D21}\cdot s+1}+\frac{k_{21}}{T_{21}\cdot s+1}=0, \tag{15}$$

and taking account of (10) and (11) we have the decoupler time constants

$$T_{D21}=T_{21}-T_{22}, \tag{16}$$

$$T_{D12}=T_{12}-T_{11}. \tag{17}$$

From (10), (11), (16) and (17) and considering the numerical values from (2), (3), (4) and (5) we obtain the standard decoupler as:

$$D(s)=\begin{bmatrix} 1 & D_{12}(s) \\ D_{21}(s) & 1 \end{bmatrix}=\begin{bmatrix} 1 & \dfrac{-0.19}{9.2\cdot s+1} \\ \dfrac{-0.22}{12.7\cdot s+1} & 1 \end{bmatrix}. \tag{18}$$

The behavior of the multivariable process with decoupling (fig. 5) was investigated further in figures 7-10, to step changes in the two inputs C1 and C2.

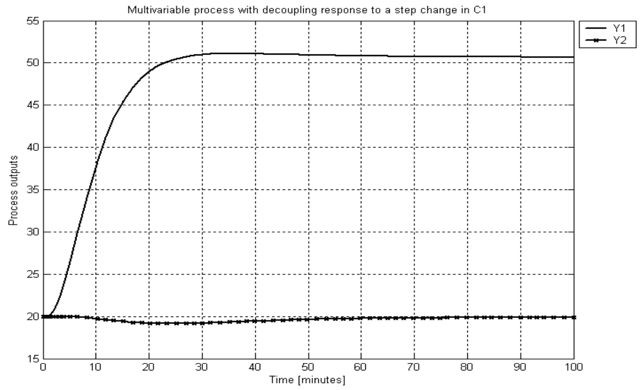

Fig. 7. Decoupled process response (Y1 and Y2 [°C]) to a 10% step change in C1.

As we can see from figure 7 the decoupling is not perfect (see fig.8), which magnifies the trend of the output Y2 to the change of input C1, from figure 7.

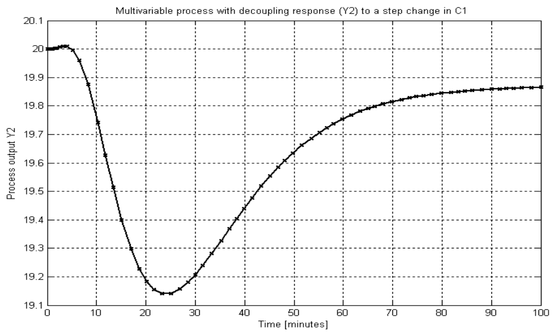

Fig. 8. Decoupled process response (Y2 [°C]) to a 10% step change in C1.

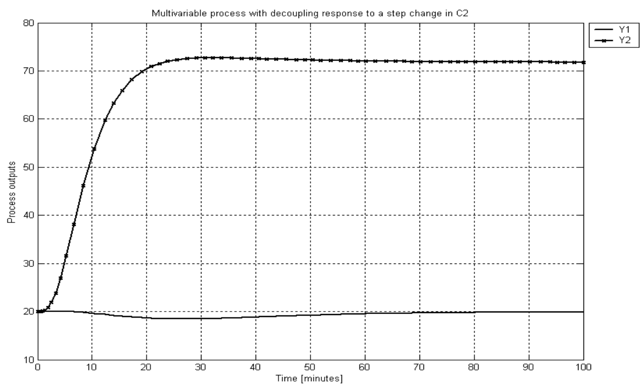

Fig. 9. Multivariable process with decoupling response (Y1 and Y2 [°C]) to a 10% step change in C2.

Also in fig.10 we have the magnified trend of the output Y1 to the change of input C2, from fig. 9 which shows us that the decoupling is not perfect because of the modeling errors.

As we see from figures 7-10, we can consider with sufficient precision that the obtained decoupled process behaves approximately as two monovariable processes; in order to control it we can use two monovariable PID controllers, one for each direct I-O process channel.

In comparison with the results from [3] we can see that the decoupling performance is weaker in case of using the standard form, against the process dedicated form, but the decoupler has the big advantage of being extremely simple, easy to design and implement.

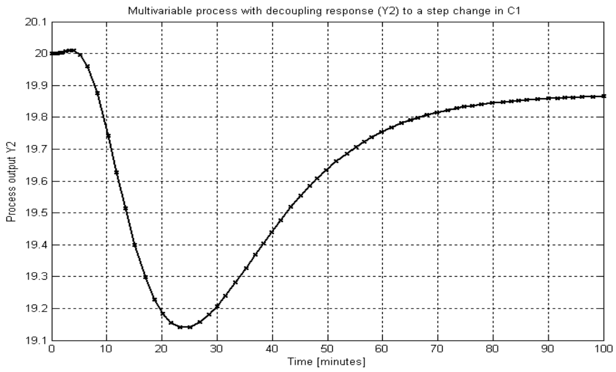

Fig. 10. Decoupled process response (Y1 [°C]) to a 10% step change in C2.

The proposed control system is represented in fig. 11 [3].

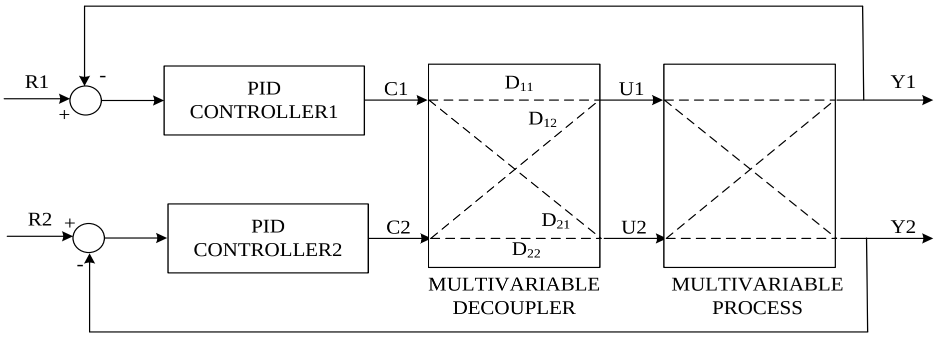

Fig. 11. Decoupled process response and two monovariable PID controllers block diagram: R1 and R2 – controller setpoints [°C], C1 and C2 – controller outputs/decoupler inputs [%], U1 and U2 – decoupler outputs/process inputs [%], Y1 and Y2 – process outputs [°C].

The PID control algorithm has the expression [4]:

$$c(t) = c_0(t) + k_R \cdot (e(t) + \frac{1}{T_i} \int_0^t e dt + T_d \frac{de}{dt}), \tag{19}$$

where $c_0(t)$ is the initial control value, e(t) - error value, k_R - controller gain; T_i - integral time constant and T_d is the derivative time constant.

Using the pseudo-process time response to the input variables step changes, from fig. 7 and 9, and the PID tuning formulas from [3]:

$$k_R = \frac{0.9}{k_P \cdot (1 + \frac{6 \cdot \tau}{T_{st}})}, \; T_i = \frac{T_{st}}{4}, \tag{20}$$

the PID tuning parameter values for the two controllers (fig. 11) are:

$$k_{R1} = \frac{0.9}{3.08} = 0.29, \;\; T_{i1} = \frac{23.6\,\text{min}}{4} = 5.9\,\text{min}, \tag{21}$$

$$k_{R2} = \frac{0.9}{2.59} = 0.35, \;\; T_{i2} = \frac{20.7\,\text{min}}{4} = 5.2\,\text{min}. \tag{22}$$

Results

Further, the control system was investigated for step changes in the two controller's setpoints for different values of the PID tuning parameters (see figs. 12-16).

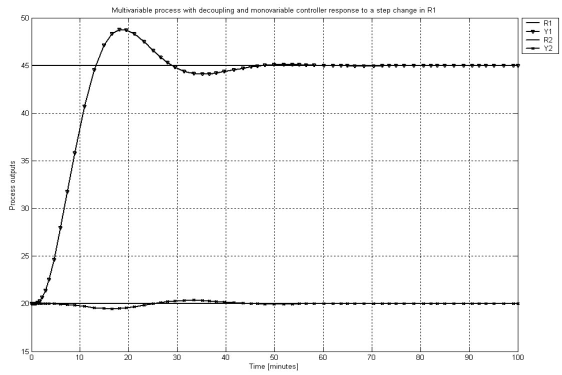

Fig. 12. Control system response (Y1 and Y2 [°C]) to a 25°C step change in R1, from 20 to 45°C, having k_{R1}=0.29, T_{i1}=5.9 min and T_{d1}=0 min.

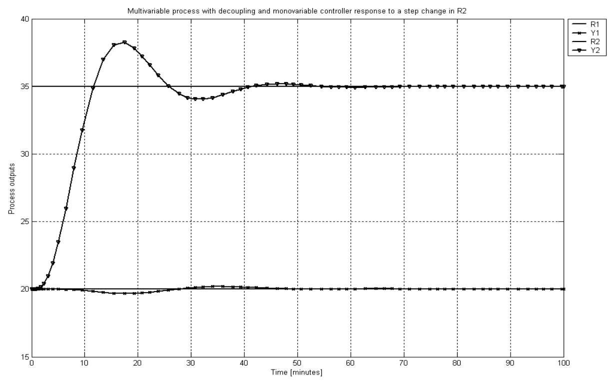

Fig. 13. Control system response (Y1 and Y2 [°C]) to a 15°C step change in R2, from 20 to 35°C, having k_{R2}=0.35, T_{i2}=5.2 min and T_{d2}=0 min.

Considering also the derivative term from PID and more refined values for the integral term, we have the following dynamic responses, see fig. 14 and 15.

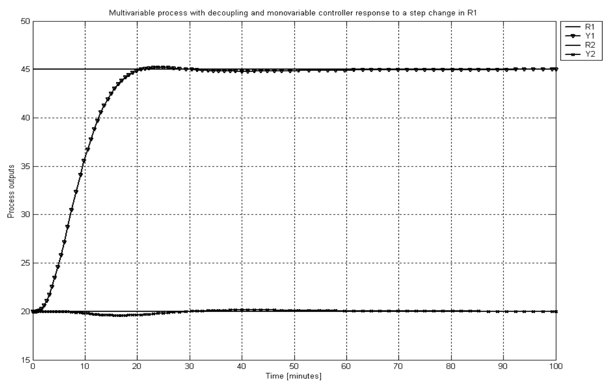

Fig. 14. Control system response (Y1 and Y2 [°C]) to a 25°C step change in R1, from 20 to 45°C, having k_{R1}=0.29, T_{i1}=8.1 min and T_{d1}=1.1 min.

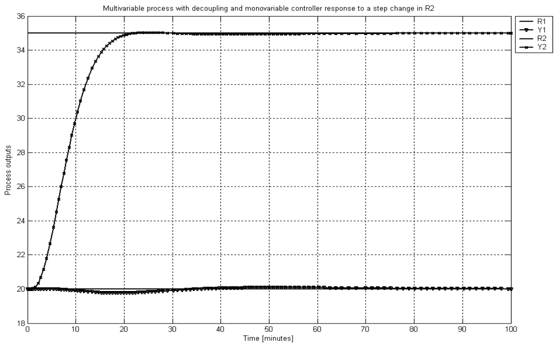

Fig. 15. Control system response (Y1 and Y2 [°C]) to a 15°C step change in R2, from 20 to 35°C, having k_{R2}=0.35, T_{i2}=7.9 min and T_{d2}=1.8 min.

As we can observe from the above figures (figs. 12 - 15) when the setpoint for one chamber temperature is changed, the temperature for that chamber changes and the temperature from the other chamber remains approximately unchanged.

In fig. 16 was considered the case when the setpoits, for the two chamber temperatures, are changed at once. As we can see the system works with good results.

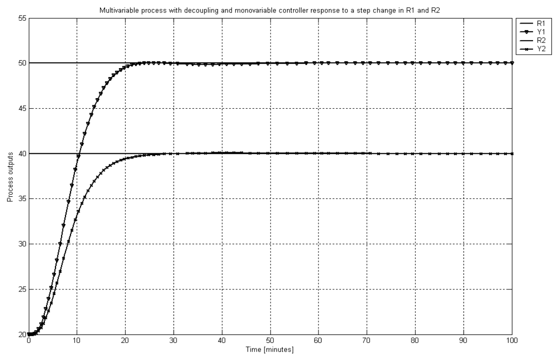

Fig.16. Control system response (Y1 and Y2 [°C]) to a 30°C step change in R1, from 20 to 50°C and to a 20°C step change in R2,from 20 to 40°C, having k_{R1}=0.29, T_{i1}=8.1 min, T_{d1}=1.1 min and k_{R2}=0.35 and T_{i2}=7.9 min, T_{d2}=1.8 min.

Conclusions

This paper objective was to present the results that can be obtained when a 2X2 multivariable control system is designed and implemented for a multivariable 2X2 temperature process. The multivariable system structure consists of two classical PID monovariable controllers and one 2X2 standard form decoupler.

The decoupler can have a process dedicated form, which contains on each of the four I-O channels the explicit model of the process, or a standard form, which contains on each of the four I-O channels standard transfer functions, equal to one on the direct decoupler's channels and equal to a first order transfer function, on the cross decoupler's channels.

The first case was tested in other author's research [3]. The conclusion was that the proposed dedicated process decoupler has a complicated form, difficult to design and implement; a more practical and effective solution is to use a decoupler with a simple standard model (of first order) easy to be designed and implemented.

The results obtained using this type of decoupler are weaker than the ones obtained with the dedicated decoupler form, but having in mind the degree of simplicity this method is recommend to be used for the particular process that was tested.

The proposed control structure was tested for validation considering the two controllers set point step changes, observing that the control system has good steady-state and dynamic performance.

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Proiectarea unui sistem de reglare multivariabil 2x2 utilizând două regulatoare monovariabile şi un decuplor standard

Rezumat

În cazul proceselor multivariabile 2X2, din cauza interacţiunilor încrucişate intrare-ieşire (I-E), proiectarea unui sistem de reglare multivariabil utilizând numai două regulatoare PID monovariabile, câte unul pentru fiecare canal I-E al procesului, trebuie considerat un decuplor 2X2 conectat în faţa procesului. Modelul decuplorului pentru fiecare dintre cele patru canale I-E poate avea o forma dedicată procesului, ce conţine modelul explicit al procesului sau o formă standard, caracterizată prin funcţii de transfer de ordinul întâi pe canalele încrucişate ale decuplorului şi egale cu unu pe canalele directe. Scopul acestei lucrări este să prezinte rezultatele proiectării şi implementării unui sistem de reglare a unui proces de transfer termic multivariabil 2X2 utilizând două regulatoare monovariabile PID şi un decuplor standard 2X2.